AMSI AUSTRALIAN MATHEMATICAL SCIENCES INSTITUTE

# ICE-EM MATHEMATICS 7

## THIRD EDITION

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INTERNATIONAL CENTRE OF EXCELLENCE FOR EDUCATION IN MATHEMATICS

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# Contents


| | | | |
|---|---|---|---|
| | Preface | | ix |
| | Author biographies | | x |
| | How to use this resource | | xii |
| | The Interactive Textbook and the Online Teaching Suite | | xiii |
| | Acknowledgements | | xiv |
| **Chapter 1** | **Whole numbers** | | **1** |
| | 1A | The number line | 2 |
| | 1B | Addition | 4 |
| | 1C | The standard addition algorithm | 6 |
| | 1D | Subtraction | 8 |
| | 1E | Multiplication | 12 |
| | 1F | Combinations of operations and the distributive law | 15 |
| | 1G | Place value | 18 |
| | 1H | The standard multiplication algorithms | 21 |
| | 1I | Division | 23 |
| | 1J | The short division algorithm | 28 |
| | 1K | The long division algorithm | 30 |
| | 1L | Order of operations | 32 |
| | | Review exercise | 34 |
| | | Challenge exercise | 35 |
| **Chapter 2** | **Factors, multiples, primes and divisibility** | | **38** |
| | 2A | Factors and multiples | 39 |
| | 2B | Odd and even numbers | 43 |
| | 2C | Prime and composite numbers | 45 |
| | 2D | Powers of numbers | 47 |
| | 2E | Using mental strategies to multiply and divide | 51 |
| | 2F | Using powers in factorisation | 54 |
| | 2G | Squares and square roots | 57 |
| | 2H | Lowest common multiple and highest common factor | 59 |
| | 2I | Divisibility tests | 61 |
| | | Review exercise | 64 |
| | | Strengthening multiplication tables skills | 66 |
| | | Challenge exercise | 67 |



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**Chapter 3 An introduction to algebra 69**

3A Using algebra 70
3B Algebraic notation 74
3C Substitution 77
3D Addition and subtraction of like terms 79
3E Brackets 82
3F Multiplying terms 86
3G Describing arrays, areas and number patterns 87
Review exercise 92
Challenge exercise 94

**Chapter 4 Fractions – part 1 96**

4A What is a fraction? 97
4B Equivalent fractions and simplest form 103
4C Mixed numerals and division by whole numbers 108
4D Comparison of fractions 112
4E Addition and subtraction of fractions 114
4F Word problems involving addition and subtraction of fractions 118
4G Subtraction of mixed numerals 120
Review exercise 122
Challenge exercise 124

**Chapter 5 Fractions – part 2 125**

5A Multiplication of fractions 126
5B Division of fractions 129
5C Multiplication and division of mixed numerals 134
5D Word problems involving fractions 136
5E Order of operations with fractions 138
Review exercise 140
Challenge exercise 141

**Chapter 6 An introduction to geometry 143**

6A Points, lines and planes 144
6B Intervals, rays and angles 148
6C Measuring angles 151
6D Angles at a point – geometric arguments 157
6E Angles associated with transversals 163



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6F Further problems involving parallel lines 172
6G Proving that two lines are parallel 178
Review exercise 183
Challenge exercise 185

**Chapter 7 Algebra with fractions 187**

7A Division in algebra 188
7B Multiplication and division in algebra 190
7C Dividing and cancelling 194
Review exercise 197
Challenge exercise 199

**Chapter 8 Decimals 201**

8A Place value and comparison of decimals 202
8B Converting decimals to fractions and fractions to decimals 207
8C Addition and subtraction of decimals 211
8D Multiplication and division by powers of 10 213
8E Multiplication of one decimal by another 217
8F Division of decimals 219
8G Recurring decimals 223
8H Rounding of decimals 225
Review exercise 227
Challenge exercise 230

**Chapter 9 Measurement 231**

9A Units of measurement 232
9B Other units 237
9C The unitary method 238
9D Perimeter 240
9E Area 243
9F Areas by addition and subtraction 247
9G Areas of triangles and parallelograms 251
9H Volume of rectangular prisms 259
9I Time 262
9J Speed 266
Review exercise 269
Challenge exercise 272


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**Chapter 10 Review and problem-solving 275**

10A Review 276
10B Problem-solving 288
10C Number bases 293
10D Binary numbers 296

**Chapter 11 Integers 299**

11A Negative integers 300
11B Addition and subtraction of a positive integer 304
11C Addition and subtraction of a negative integer 306
11D Multiplication involving negative integers 310
11E Division involving negative integers 312
11F Indices and order of operations 315
Review exercise 318
Challenge exercise 320

**Chapter 12 Algebra and the Cartesian plane 322**

12A Substitution with integers 323
12B The Cartesian plane 326
12C Completing tables and plotting points 330
12D Finding rules 333
Review exercise 339
Challenge exercise 340

**Chapter 13 Triangles and constructions 343**

13A Angles in triangles 344
13B Circles and compasses 351
13C Isosceles and equilateral triangles 355
13D Constructions with compasses and a straight edge 360
13E Quadrilaterals 363
Review exercise 367
Challenge exercise 370

**Chapter 14 Negative fractions and decimals 372**

14A Addition and subtraction of negative fractions 373
14B Multiplication and division of negative fractions 376
14C Negative decimals 380
14D Substitution involving negative fractions and decimals 382



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Review exercise 384
Challenge exercise 385

**Chapter 15 Percentages and ratios 386**

15A Percentages, fractions and decimals 387
15B One quantity as a percentage of another 392
15C Percentage of a quantity 394
15D Ratios 395
15E Solving problems with ratios 399
15F Best buys 400
Review exercise 402
Challenge exercise 404

**Chapter 16 Solving equations 405**

16A An introduction to equations 406
16B Equivalent equations 408
16C Solving equations involving more than one step 412
16D Equations with negative solutions 414
16E Expanding brackets and solving equations 415
16F Collecting like terms and solving equations 418
16G Equations with pronumerals on both sides 421
16H Solving problems using equations 422
Review exercise 425
Challenge exercise 427

**Chapter 17 Probability 428**

17A An introduction to probability 429
17B Experiments and counting 431
Review exercise 439
Challenge exercise 440

**Chapter 18 Transformations and symmetry 441**

18A Translations 442
18B Rotations 446
18C Reflections 451
18D Combinations of transformations 454
18E Transformations in the Cartesian plane 459
18F Symmetry 462
18G Regular polygons 465


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Review exercise 468
Challenge exercise 471

**Chapter 19 Graphs and tables 473**

19A Reading tables 474
19B The pictogram 477
19C Column graphs 481
19D Divided bar charts and pie charts 485
19E Line graphs 492
19F Applications of the line graph 495
Review exercise 501

**Chapter 20 Statistics 505**

20A Data and dot plots 506
20B Mode 509
20C Stem-and-leaf plots 512
20D Median, mean and range 514
Review exercise 520

**Chapter 21 Polyhedra and three-dimensional drawing 522**

21A Polyhedra 523
21B Drawing a solid 527
Review exercise 531
Challenge exercise 532

**Chapter 22 Review and problem-solving 533**

22A Review 534
22B Tessellations 544

**Chapter 23 Sets and Venn diagrams** Interactive Textbook only

23A Describing and naming sets
23B Finite and infinite sets
23C Subsets of a set
23D Sets and the number line
23E Union and intersection
23F Set complements and Venn diagrams
23G Keeping count with Venn diagrams
23H Problem-solving using Venn diagrams

**Answers 548**


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# Preface

*ICE-EM Mathematics Third Edition* is a series of textbooks for students in years 5 to 10 throughout Australia who study the Australian Curriculum and its state variations.

The program and textbooks were developed in recognition of the importance of mathematics in modern society and the need to enhance the mathematical capabilities of Australian students. Students who use the series will have a strong foundation for work or further study.

## Background

The International Centre of Excellence for Education in Mathematics (ICE-EM) was established in 2004 with the assistance of the Australian Government and is managed by the Australian Mathematical Sciences Institute (AMSI). The Centre originally published the series as part of a program to improve mathematics teaching and learning in Australia. In 2012, AMSI and Cambridge University Press collaborated to publish the Second Edition of the series to coincide with the introduction of the Australian Curriculum, and we now bring you the Third Edition.

## The series

*ICE-EM Mathematics Third Edition* provides a progressive development from upper primary to middle secondary school. The writers of the series are some of Australia's most outstanding mathematics teachers and subject experts. The textbooks are clearly and carefully written, and contain background information, examples and worked problems.

For the Third Edition, the series has been carefully edited to present the content in a more streamlined way without compromising quality. There is now one book per year level and the flow of topics from chapter to chapter and from one year level to the next has been improved.

The year 10 textbook incorporates all material for the 10A course, and selected topics in earlier books carefully prepare students for this. *ICE-EM Mathematics Third Edition* provides excellent preparation for all of the Australian Curriculum's year 11 and 12 mathematics courses.

For the Third Edition, *ICE-EM Mathematics* now comes with an Interactive Textbook: a cutting-edge digital resource where all textbook material can be answered online (with students' working-out), additional quizzes and features are included at no extra cost. See "The Interactive Textbook and Online Teaching Suite" on page xiii for more information.


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# Author biographies

## Peter Brown

Peter Brown studied Pure Mathematics and Ancient Greek at Newcastle University, and completed postgraduate degrees in each subject at the University of Sydney. He worked for nine years as a mathematics teacher in NSW State schools. Since 1990, he has taught Pure Mathematics at the School of Mathematics and Statistics at the University of New South Wales (UNSW). He was appointed Director of First Year Studies at UNSW from 2011 to 2015. He specialises in Number Theory and History of Mathematics and has published in both areas. Peter regularly speaks at teacher inservices, Talented Student days and Mathematics Olympiad Camps. In 2008 he received a UNSW Vice Chancellor's Teaching Award for educational leadership.

## Michael Evans

Michael Evans has a PhD in Mathematics from Monash University and a Diploma of Education from La Trobe University. He currently holds the honorary position of Senior Consultant at the Australian Mathematical Sciences Institute at the University of Melbourne. He was Head of Mathematics at Scotch College, Melbourne and has also taught in public schools, returning to classroom teaching in recent years. He has been very involved with curriculum development at both state and national levels. In 1999, Michael was awarded an honorary Doctor of Laws by Monash University for his contribution to mathematics education, and in 2001 he received the Bernhard Neumann Award for contributions to mathematics enrichment in Australia.

## Garth Gaudry

Garth Gaudry was Head of Mathematics at Flinders University before moving to UNSW, where he became Head of School. He was the inaugural Director of AMSI before he became the Director of AMSI's International Centre of Excellence for Education in Mathematics. Previous positions include membership of the South Australian Mathematics Subject Committee and the Eltis Committee appointed by the NSW Government to enquire into Outcomes and Profiles. He was a life member of the Australian Mathematical Society and Emeritus Professor of Mathematics, UNSW.

## David Hunt

David Hunt graduated from the University of Sydney in 1967 with an Honours degree in Mathematics and Physics. He then obtained a master's degree and a doctorate from the University of Warwick. He was appointed to a lectureship in Pure Mathematics at UNSW in early 1971, where he is currently an honorary Associate Professor. David has taught courses in Pure Mathematics from first year to Master's level and was Director of First Year Studies in Mathematics for five years. Many of David's activities outside UNSW have centred on the Australian Mathematics Trust. In 2016 David was awarded the Paul Erdos medal, in recognition of his contributions to education, as well as his work with the International Mathematical Olympiad movement.


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## Janine McIntosh

Janine McIntosh works at The Australian Mathematical Sciences Institute, where she manages AMSI Schools. Janine leads a professional development and schools visit program for teachers across the country. Through clusters of schools supported by industry and government partners, Janine's aim is to encourage more Australians to enjoy and study mathematics. Janine has developed a suite of online and careers materials in her time at AMSI and was one of the writers for the Australian Curriculum: Mathematics F – 10. She is an experienced primary teacher, who has worked as a lecturer in mathematics education at the University of Melbourne and serves on the Maths Challenge and AMOC Committees of the Australian Mathematics Trust.

## Bill Pender

Bill Pender has a PhD in Pure Mathematics from Sydney University and a BA (Hons) in Early English from Macquarie University. After a year at Bonn University, he taught at Sydney Grammar School from 1975 to 2008, where he was Subject Master for many years. He has been involved in the development of NSW Mathematics syllabuses since the early 1990s, and was a foundation member of the Education Advisory Committee of AMSI. He has also lectured and tutored at Sydney University and at UNSW, and given various inservice courses. Bill is the lead author of the NSW calculus series *Cambridge Mathematics*.

## Jacqui Ramagge

Jacqui Ramagge is currently Head of the School of Mathematics and Statistics at the University of Sydney. After graduating in 1993 with a PhD in Mathematics from the University of Warwick (UK) she worked at the University of Newcastle (Australia) until 2007 and then at the University of Wollongong until 2015, when she moved to the University of Sydney. She has served on the Australian Research Council College of Experts, including as Chair of Australian Laureate Fellowships Selection Advisory Committee. She teaches mathematics at all levels from primary school to PhD courses and has won a teaching award. She contributed to the Vermont Mathematics Initiative (USA) and is a founding member of the Australian Mathematics Trust Primary Problems Committee. In 2013 she received a BH Neumann Award from the Australian Mathematics Trust for her significant contribution to the enrichment of mathematics learning in Australia.


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# How to use this resource

## The textbook

Each chapter in the textbook addresses a specific Australian Curriculum content strand and set of sub-strands. The exercises within chapters take an integrated approach to the concept of proficiency strands, rather than separating them out. Students are encouraged to develop and apply Understanding, Fluency, Problem-solving and Reasoning skills in every exercise.

The series places a strong emphasis on understanding basic ideas, along with mastering essential technical skills. Mental arithmetic and other mental processes are major focuses, as is the development of spatial intuition, logical reasoning and understanding of the concepts.

Problem-solving lies at the heart of mathematics, so *ICE-EM Mathematics* gives students a variety of different types of problems to work on, which help them develop their reasoning skills. Challenge exercises at the end of each chapter contain problems and investigations of varying difficulty that should catch the imagination and interest of students. Further, two 'Review and Problem-solving' chapters in each 7–10 textbook contain additional problems that cover new concepts for students who wish to explore the subject even further.

## The Interactive Textbook and Online Teaching Suite

Included with the purchase of the textbook is the Interactive Textbook. This is the online version of the textbook and is accessed using the 16-character code on the inside cover of this book.

The Online Teaching Suite is the teacher version of the Interactive Textbook and contains all the support material for the series, including tests, worksheets, skillsheets, curriculum documentation and more.

For more information on the Interactive Textbook and Online Teaching Suite, see page xiii.

The Interactive Textbook and Online Teaching Suite are delivered on the *Cambridge HOTmaths* platform, providing access to a world-class Learning Management System for testing, task management and reporting. They do not provide access to the *Cambridge HOTmaths* stand-alone resource that you or your school may have used previously. For more information on this resource, contact Cambridge University Press.

## AMSI's TIMES and SAM modules

The TIMES and SAM web resources were developed by the *ICE-EM Mathematics* author team at AMSI and are written around the structure of the Australian Curriculum. These resources have been mapped against your *ICE-EM Mathematics* book and are available to teachers and students via the AMSI icon on the dashboard of the Interactive Textbook and Online Teaching Suite.


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# The Interactive Textbook and the Online Teaching Suite

## Interactive Textbook

The Interactive Textbook is the online version of the print textbook and comes included with purchase of the print textbook. It is accessed by first activating the code on the inside cover. It is easy to navigate and is a valuable accompaniment to the print textbook.

### Students can show their working

All textbook questions can be answered online within the Interactive Textbook. Students can show their working for each question using either the Draw tool for handwriting (if they are using a device with a touch-screen), the Type tool for using their keyboard in conjunction with the pop-up symbol palette, or by importing a file using the Import tool.

Once a student has completed an exercise they can save their work and submit it to the teacher, who can then view the student's working and give feedback to the student, as they see appropriate.

### Auto-marked quizzes

The Interactive Textbook also contains material not included in the textbook, such as a short auto-marked quiz for each section. The quiz contains 10 questions which increase in difficulty from question 1 to 10 and cover all proficiency strands. There is also space for the student to do their working underneath each quiz question. The auto-marked quizzes are a great way for students to track their progress through the course.

### Additional material for Years 5 and 6

For Years 5 and 6, the end-of-chapter Challenge activities as well as a set of Blackline Masters are now located in the Interactive Textbook. These can be found in the 'More resources' section, accessed via the Dashboard, and can then easily be downloaded and printed.

## Online Teaching Suite

The Online Teaching Suite is the teacher's version of the Interactive Textbook. Much more than a 'Teacher Edition', the Online Teaching Suite features the following:

- The ability to view students' working and give feedback. When a student has submitted their work online for an exercise, the teacher can view the student's work and can give feedback on each question.
- For Years 5 and 6, access to Chapter tests, Blackline Masters, Challenge exercises, curriculum support material, and more.
- For Years 7 to 10, access to Pre-tests, Chapter tests, Skillsheets, Homework sheets, curriculum support material, and more.
- A Learning Management System that combines task-management tools, a powerful test generator, and comprehensive student and whole-class reporting tools.


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CHAPTER

# 1

Number and Algebra

# Whole numbers

The numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, ... are called the **whole numbers**. Whole numbers are used for counting objects, such as the number of the people in a room.

Zero (0) is the first of these numbers. Zero is important because it is used to describe some common situations:

- The room is empty. (There are 0 people in the room.)
- There are no frogs in my bathroom. (There are 0 frogs in my bathroom.)

There is no last whole number, as every whole number is followed by another whole number. The next whole number is obtained by adding 1 to the previous whole number. The list of whole numbers is infinite – it never ends.

The whole numbers are also known as the **counting numbers**. We use them every day to talk about ideas and describe events and achievements.

The following are some world records from *Guinness World Records*. Each one is expressed in terms of a whole number.

- The greatest number of step-ups completed in 1 hour is 4135.
- The greatest number of dominoes stacked end-to-end vertically is 726.
- The greatest number of drum beats in 1 minute is 1080.
- The greatest number of pancakes tossed in 2 minutes is 416.
- The highest first-class cricket score ever is 501, scored by Brian Lara.

*continued over page*



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The operations of addition, subtraction, multiplication and division help to answer further questions about such numbers. For example:

- Estimate the record for the greatest number of drum beats in a second. This is found by dividing the number of drum beats in a minute by 60.
- How many pancakes could be tossed in 6 minutes if the record-holder could keep going at the same rate? This is found by multiplying 416 by 3.

Many of the calculations in this chapter should be carried out mentally. Mental calculations will help you build your mathematical skills.

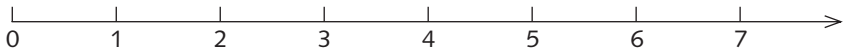
### Whole numbers

- The whole numbers (or counting numbers) are the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, ...
- Zero is the first whole number.
- There is no last whole number – the list of whole numbers is infinite.
- Counting any collection of objects gives the same answer, whatever order they are counted in.

# 1A The number line

The whole numbers can be represented by points on a line.

Label a point 0 and then mark off equal intervals of any chosen length, always moving to the right. Label the points 0, 1, 2, 3, 4, ... as shown. The arrow shows that the line continues in the same direction forever. This line is called the **number line**.




## Less than and greater than

Any two numbers can be compared with each other. For example, if I have \$2 and you have \$6, then I have less than you and you have more than me.

On the number line, 2 is to the left of 6. This is written as $2 < 6$. It is read as '2 is **less than** 6'.

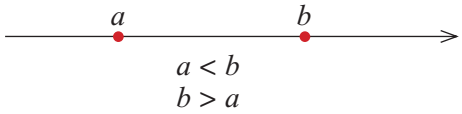
The sharp end of the new symbol $<$ points to the smaller number, 2, and the open end faces the larger number, 6.

We can also say that 6 is greater than 2. This means that 6 is to the right of 2 on the number line. This is written as $6 > 2$ and is read as '6 is **greater than** 2'.


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## Less than and greater than

- A whole number, $a$, is **less than** another whole number, $b$, if $a$ lies to the left of $b$ on the number line. The symbol $<$ is used for **less than**. For example, $2 < 6$.
- We can also say that a whole number, $b$, is **greater than** another whole number, $a$, if $b$ lies to the right of $a$ on the number line. The symbol $>$ is used for **greater than**. For example, $11 > 4$.




$a < b$
$b > a$

- Zero is less than every other whole number.

### Example 1

**a** List all the whole numbers less than 5.

**b** List all the whole numbers less than 10 and greater than 1.

#### Solution

**a** 0, 1, 2, 3, 4

**b** 2, 3, 4, 5, 6, 7, 8, 9

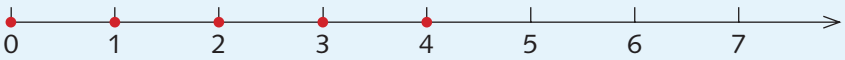
### Example 2

**a** Draw a number line and on it mark with dots all the whole numbers less than 5.

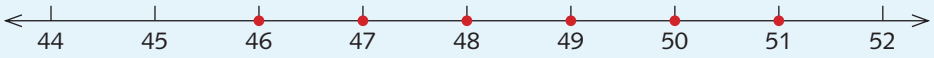
**b** Draw a number line and on it mark all the whole numbers greater than 45 and less than 52.

#### Solution

**a**




**b**




## Exercise 1A

Example 1

**1** **a** List the whole numbers less than 11.

**b** List the whole numbers greater than 52 and less than 61.


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Example 2

**2** For each of the following, draw a number line from 0 to 10.

**a** Mark the numbers 2, 4, 6 and 8 on it.

**b** Mark the numbers 1, 3, 5 and 7 on it.

**c** Mark the whole numbers less than 5 on it.

**d** Mark the whole numbers less than 8 and greater than 2 on it.

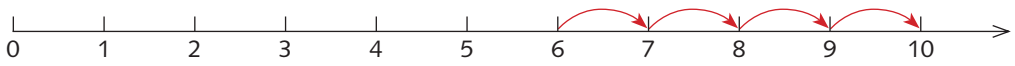
**3** The manager of an underground railway system decides to save time in the mornings by having one particular train only stop at every third station between stations 1 and 19. The stations are all 1 km apart. Show the stations on a number line and mark with a dot each station where the train stops.

# 1B Addition

Addition is an operation that is carried out on **two numbers**. You have learned about addition in earlier years, but we will talk about it here to be complete.

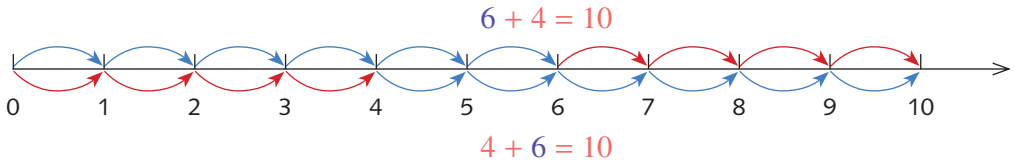
The **sum** is the result of the addition of two numbers.

The sum of two whole numbers, for example, 6 + 4 can be obtained by starting at the number 6 and counting 4 more numbers to the right, as shown on the number line below.




## Making mental addition simpler

The order in which we perform addition does not matter. For example, 6 + 4 = 4 + 6. This can be shown on the number line.




This property is called the **commutative law for addition**.

The word *commutative* is related to the word *commute*. Both words come from the Latin word *commutare*, which means 'to interchange'.

In mathematics, it is a rule that operations contained in brackets are performed first. When three or more numbers are added together two at a time, it does not matter which two are added together first.

This property is called the **associative law for addition**. For example:

$$(2 + 4) + 5 = 2 + (4 + 5)$$

When the commutative law and the associative law are used together for addition, the result can be described as the 'any-order property for addition'.


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### Any-order property for addition

A list of whole numbers can be added two at a time in any order to give the same result.

## Using mental addition

The any-order property for addition is a great help in simplifying arithmetic calculations. Many of them can be done in your head, or mentally.

### Example 3

Calculate each of the following using the any-order property for addition. (They are set out so that you can see the strategy.)

**a** $23 + 41 + 7 + 9$ **b** $27 + 55 + 445 + 23 + 7$

### Solution

**a** $23 + 41 + 7 + 9 = (23 + 7) + (41 + 9)$
$= 30 + 50$
$= 80$

**b** $27 + 55 + 445 + 23 + 7 = (27 + 23) + (55 + 445) + 7$
$= 50 + 500 + 7$
$= 557$

## Exercise 1B

**1** Carry out these additions mentally.

| | | | |
|---|---|---|---|
| **a** $15 + 5$ | **b** $8 + 22$ | **c** $13 + 7$ | **d** $74 + 6$ |
| **e** $7 + 58$ | **f** $6 + 38$ | **g** $8 + 89$ | **h** $32 + 9$ |
| **i** $35 + 27$ | **j** $42 + 19$ | **k** $29 + 36$ | **l** $57 + 86$ |

Example 3a

**2** Carry out these additions.

| | | | |
|---|---|---|---|
| **a** $1 + 9 + 33$ | **b** $2 + 38 + 5$ | **c** $27 + 6 + 3$ | **d** $16 + 24 + 5$ |
| **e** $61 + 9 + 24$ | **f** $4 + 42 + 38$ | **g** $16 + 55 + 27$ | **h** $72 + 19 + 26$ |

Example 3b

**3** Do these computations using the techniques introduced so far.

| | | |
|---|---|---|
| **a** $22 + 17 + 18 + 23$ | **b** $14 + 18 + 76 + 92$ | **c** $13 + 27 + 64 + 6$ |
| **d** $25 + 32 + 15 + 18$ | **e** $15 + 34 + 26 + 35$ | **f** $12 + 19 + 18 + 1$ |

**4** Carry out these additions.

| | | | |
|---|---|---|---|
| **a** $243 + 57$ | **b** $567 + 43$ | **c** $328 + 22$ | **d** $786 + 24$ |
| **e** $435 + 25$ | **f** $963 + 57$ | **g** $486 + 524$ | **h** $364 + 251$ |


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**5** Three cows produced 29 litres, 47 litres and 23 litres of milk in one day.
How much milk did they produce in total?

**6** A tiler laid 267 tiles in the kitchen, 20 tiles in the laundry and 113 tiles in the bathroom.
How many tiles did he lay in total?

**7** On the first day of my holidays, I travelled 85 kilometres from my home in Victor Harbor to Adelaide, then 516 kilometres from Adelaide to Broken Hill. The next day I travelled 298 kilometres from Broken Hill to Mildura.
How many kilometres did I travel in the first two days of my trip?

**8** A busker collected \$8.00, \$13.00, \$4.00 and \$12.00 over four days. How much did she earn?

**9** In three Year 7 classes, 27 students, 31 students and 26 students attended roll call one morning. How many Year 7 students were present?

**10** I picked 18 daffodils from my garden on Monday, 3 on Tuesday, 27 on Wednesday, 6 on Thursday and 12 on Friday. How many daffodils did I pick over the five days?

**11** In one week, Sam read four books. The first book had 312 pages, the second 175, the third 48 and the fourth 98. How many pages did Sam read in the week?

**12** **a** By appropriately pairing numbers, carry out the addition $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9$.

**b** Use the same idea to find the sum of numbers from 1 to 99 inclusive.

# 1C The standard addition algorithm

An **algorithm** is a set of procedures or steps for performing a task. In this section we look at an algorithm for addition.

We will use the two-digit numbers 15 and 27 to demonstrate the addition algorithm.

$$\begin{array}{rl} \phantom{+}\ 1\ \ 5 & \text{(1 ten and 5 ones)} \\ +\ 2_{1}\ 7 & \text{(2 tens and 7 ones)} \\ \hline 4\ \ 2 & \text{(4 tens and 2 ones)} \end{array}$$

This is explained by saying, as we add the ones:

5 ones + 7 ones = 12 ones
and 12 ones is 1 ten and 2 ones

Write 2 in the ones column and carry 1 ten to the tens column.

As we add the tens, we say:

1 ten + 2 tens + 1 ten = 4 tens

We then write 4 in the tens column.

The final answer is 42.

This technique can be extended to three or more numbers with different numbers of digits.


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### Example 4

Complete the following additions.

**a**
$$\begin{array}{r} 935 \\ 2382 \\ +\quad 97 \\ \hline \end{array}$$

**b**
$$\begin{array}{r} 7136 \\ +8095 \\ \hline \end{array}$$

### Solution

**a**
$$\begin{array}{r} 935 \\ 2382 \\ +\quad 97 \\ \hline 3414 \end{array}$$

**b**
$$\begin{array}{r} 7136 \\ +8095 \\ \hline 15231 \end{array}$$

## Exercise 1C

Example 4

**1** Work out the answers to these additions by using the standard algorithm.

**a** 721 + 630 **b** 235 + 549 **c** 109 + 872 **d** 468 + 951

**e** 973 + 296 **f** 2107 + 989 **g** 1432 + 3791 **h** 793 + 274 + 473

**i** 8693 + 7392 **j** 9999 + 32

**2** Work out the answers to these additions.

**a** 37 + 129 + 1647 **b** 9230 + 839 + 61 **c** 829 + 1083 + 437

**d** 72 + 274 + 391 + 28 **e** 254 + 194 + 482 + 53 **f** 632 + 106 + 7 + 270

**3** For each of the following, find the missing digits (★) to make the addition correct.

**a**
$$\begin{array}{r} 6\bigstar8 \\ +\bigstar39 \\ \hline 997 \end{array}$$

**b**
$$\begin{array}{r} 8\bigstar4 \\ +\bigstar9\bigstar \\ \hline \bigstar03 \end{array}$$

**c**
$$\begin{array}{r} 919 \\ +89\bigstar \\ \hline 1\bigstar18 \end{array}$$

**d**
$$\begin{array}{r} 6\bigstar3 \\ +\bigstar89 \\ \hline 1262 \end{array}$$

**e**
$$\begin{array}{r} 7\bigstar1 \\ +\bigstar97 \\ \hline 109\bigstar \end{array}$$

**f**
$$\begin{array}{r} 3\bigstar3 \\ +77\bigstar \\ \hline \bigstar110 \end{array}$$

**4** Make six different three-digit numbers from the digits 4, 7 and 8. What is their sum?

**5** Find the sum of:

**a** three hundred and sixty-seven and six hundred and twenty-seven

**b** four hundred and twenty-four and five thousand, three hundred and twenty-six

**6** The odometer of a car read 54 987 kilometres at the beginning of a journey of 765 kilometres. What did it read at the end of the journey?

**7** A shop had three employees. The manager received $976 per week and the senior assistant received $654 per week. The junior assistant earned $443 per week. What was the total weekly wages bill for the shop?


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**8** The Amazon River is 6436 kilometres in length and the Nile River is 234 kilometres longer. What is the length of the Nile?

**9** The populations of five ant farms were determined at the end of each of three months.

| | Ant farm A | Ant farm B | Ant farm C | Ant farm D | Ant farm E |
|---|---|---|---|---|---|
| March | 97 | 597 | 143 | 89 | 13 |
| April | 113 | 603 | 89 | 432 | 28 |
| May | 214 | 411 | 17 | 729 | 164 |

**a** List the ant farms in order of increasing size of their populations at the end of March.

**b** What was the population of Ant farm A at the end of May?

**c** Which of the ant farms had a population greater than 100 at the end of April?

**d** What was the total population of all five ant farms at the end of May?

**10** Complete the following to make the addition correct.

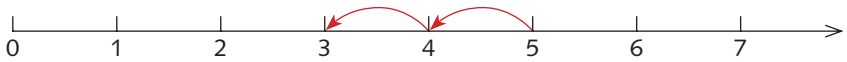
$$\begin{array}{r} \square\ 6\ 3 \\ +\ 7\ \square\ 2 \\ 5\ 8\ \square \\ \hline \square\ 0\ 4\ 2 \end{array}$$

# 1D Subtraction

A whole number can be subtracted from a larger whole number. The result is called the **difference** of the two numbers. For example, $8 - 5 = 3$. The difference of 8 and 5 is 3.

## Subtracting as 'taking away'

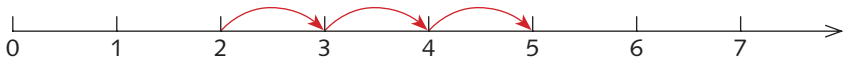
If you have 5 items and you take 2 away, the number remaining is $5 - 2 = 3$.
For example, I had 5 pencils but my sister took 2 away, so now I have 3.
On a number line, this is illustrated by moving two numbers to the left.




$5 - 2 = 3$

## Subtracting as 'adding on'

Another way of thinking about the subtraction $5 - 2$ is to ask 'What is the difference?' or 'What do you add on to 2 to get to 5?' Subtraction can also be thought of as an alternative way of expressing the addition $2 + 3 = 5$.




$2 + 3 = 5$ or $5 - 2 = 3$

One example of this is 'I have 5 pencils and my brother has 2, so I have 3 more pencils than my brother'.


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### Subtraction of whole numbers

- A whole number can always be subtracted from a larger whole number to give the **difference** of the two numbers.
- 'I had 8 but someone took away 5' can be expressed as $8 - 5$.
- 'I have 8 and she has 5, so I have 3 more' can also be expressed as $8 - 5$.
- Subtraction is the reverse process of addition. For example, $8 - 5 = 3$ is equivalent to saying $3 + 5 = 8$.

## Mental subtractions

Here are three ways to perform the subtraction $100 - 67$.

**1** Count on from the smaller number to the larger number.

Going from 67 to 70 requires counting on 3.

Going from 70 to 100 requires counting on 30.

Therefore $100 - 67 = 33$.

**2** Subtract the 60 and then the 7.

$$\begin{aligned} 100 - 67 &= 100 - 60 - 7 && \text{(First subtract the 60.)} \\ &= 40 - 7 && \text{(Then subtract the 7.)} \\ &= 33 \end{aligned}$$

**3** Take away 70 and then add 3.

$$\begin{aligned} 100 - 67 &= 100 - 70 + 3 \\ &= 30 + 3 && \text{(First subtract 70, which is 3 too many,} \\ &= 33 && \text{so then we add the 3.)} \end{aligned}$$

### Example 5

Carry out these subtractions mentally.

**a** $39 - 31$ **b** $27 - 8$ **c** $183 - 96$

### Solution

**a** One way you might choose to do this is to subtract the 30, then the 1.

$$\begin{aligned} 39 - 31 &= 39 - 30 - 1 \\ &= 9 - 1 \\ &= 8 \end{aligned}$$

**b** One way you might choose to do this is to first subtract 7 from 27, then subtract 1.

$$\begin{aligned} 27 - 8 &= 27 - 7 - 1 \\ &= 20 - 1 \\ &= 19 \end{aligned}$$

**c** One way you might choose to do this is to take away 100, then add 4.

$$\begin{aligned} 183 - 96 &= 183 - 100 + 4 \\ &= 83 + 4 \\ &= 87 \end{aligned}$$


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## Zero

The number zero is very important in arithmetic. When you add 0 to a number or subtract 0 from a number, the result is the original number.

For example, $2 + 0 = 2$ and $2 - 0 = 2$. When a whole number is subtracted from itself, the answer is zero. For example, $5 - 5 = 0$.

## The standard subtraction algorithms

We can set out subtraction in the familiar column form.

$$\begin{array}{r} 5\ 6 \\ -\ 2\ 4 \\ \hline 3\ 2 \end{array}$$

In the previous example, $6 > 4$ and $5 > 2$ so the subtraction is very easy. What can we do if this is not the case?

Consider $54 - 26$. We set this out in column form and we work on the units column before the tens column. (In general, we work from right to left across the columns.)

### Method 1: Equal addition

$$\begin{array}{r} 5\ {}^{1}4 \\ -\ 2_{1}\ 6 \\ \hline 2\ 8 \end{array}$$

Since $6 > 4$, we add 10 to the 54 by changing the 4 to 14, which we write as ${}^{1}4$. To restore the correct answer, we add 10 to the 26, making it 36; we do this by writing 3 as $2_{1}$. We can now subtract in each column to get 28.

### Method 2: Trading or decomposition

$$\begin{array}{r} {}^{4}\not{5}\ {}^{1}4 \\ -\ 2\ 6 \\ \hline 2\ 8 \end{array}$$

Think of 54 as 5 tens and 4 ones, and 26 as 2 tens and 6 ones.

Now write 54 as 4 tens and 14 ones, and subtract 2 tens and 6 ones to get 2 tens and 8 ones.

It does not matter which of the two methods you use – it is your choice.

**Example 6**

Carry out the following subtractions.

**a** $456 - 278$ **b** $20\,007 - 7986$

**Solution**

**a Method 1**

$$\begin{array}{r} 4\ {}^{1}5\ {}^{1}6 \\ -\ 2_{1}\ 7_{1}\ 8 \\ \hline 1\ 7\ 8 \end{array}$$

**a Method 2**

$$\begin{array}{r} {}^{3}\not{4}\ {}^{14}\not{5}\ {}^{1}6 \\ -\ 2\ 7\ 8 \\ \hline 1\ 7\ 8 \end{array}$$

**b Method 1**

$$\begin{array}{r} 2\ {}^{1}0\ {}^{1}0\ {}^{1}0\ 7 \\ -\ {}_{1}\ 7_{1}\ 9_{1}\ 8\ 6 \\ \hline 1\ 2\ 0\ 2\ 1 \end{array}$$

**b Method 2**

$$\begin{array}{r} {}^{1}\not{2}\ {}^{1}\not{0}\ {}^{9}\not{0}\ {}^{1}\not{0}\ {}^{9}\not{0}\ {}^{1}0\ 7 \\ -\ \ 7\ 9\ 8\ 6 \\ \hline 1\ 2\ 0\ 2\ 1 \end{array}$$


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## Exercise 1D

Example 5

**1** Carry out these subtractions mentally.

**a** 28 – 22 **b** 273 – 68 **c** 350 – 47 **d** 94 – 66

**e** 137 – 53 **f** 167 – 59 **g** 475 – 95 **h** 1008 – 17

**2** Do each of these computations by working from left to right.

**a** 8 + 7 – 7 **b** 8 + 12 – 12 – 8 **c** 19 – 7 + 8 – 19

**d** 56 – 11 + 11 **e** 38 – 18 – 1 + 19 **f** 101 – 11 + 11

Example 6a

**3** Carry out these subtractions.

**a** 562 – 387 **b** 921 – 428 **c** 405 – 286 **d** 813 – 619

Example 6b

**4** Carry out these subtractions.

**a** 3456 – 234 **b** 18 502 – 7862 **c** 22 788 – 19 999

**5** Copy and then complete each subtraction by finding a digit for each ★.

**a**
$$\begin{array}{r} 7\ 6 \\ -\ 2\ \bigstar \\ \hline \bigstar\ 7 \end{array}$$

**b**
$$\begin{array}{r} 9\ \bigstar \\ -\ \bigstar\ 7 \\ \hline 4\ 6 \end{array}$$

**c**
$$\begin{array}{r} 2\ 3\ \bigstar\ \bigstar \\ -\ \bigstar\ \bigstar\ 9\ 7 \\ \hline 3\ 9 \end{array}$$

**d**
$$\begin{array}{r} 1\ \bigstar\ 5\ \bigstar \\ -\ \ 4\ \bigstar\ 9 \\ \hline \ \ 7\ 3\ 4 \end{array}$$

**e**
$$\begin{array}{r} \bigstar\ 7\ 6\ \bigstar \\ -\ \ \bigstar\ 4\ 3 \\ \hline 1\ 9\ 1\ 8 \end{array}$$

**f**
$$\begin{array}{r} 5\ 3\ 7\ \bigstar \\ -\ 2\ 1\ 4\ 8 \\ \hline 3\ 2\ \bigstar\ 8 \end{array}$$

**6** Complete each statement by filling in the boxes with numbers that make the statement true.

**a** 3 + 7 = 10 is equivalent to $\square$ – 3 = 7

**b** 18 + $\square$ = 27 is equivalent to $\square$ – 18 = 9

**c** 42 + 26 = $\square$ is equivalent to $\square$ – 26 = 42

**d** $\square$ + $\square$ = 144 is equivalent to 144 – $\square$ = 81

**7** Bill has $456 more than Anna. Bill has $3789. How much does Anna have?

**8** During the last school holidays, Stephen drove from Brisbane to Cairns along the Bruce Highway. He set his trip meter to zero when he left home. It showed 1236 km at Townsville and 1586 km on his arrival in Cairns. How far did he drive between Townsville and Cairns?

**9** Melissa had invited 1534 people to attend a fundraising event, and 204 people indicated they would not be able to attend. How many people did Melissa expect to come to the event?

**10** Australia scored 246 and England scored 196 in a one-day cricket match. What was the winning margin? That is, how many more runs did Australia score than England?

**11** A town with a population of 34 827 has 18 439 adults. How many children are there?

**12** 65 376 tickets were sold for a concert. The venue had seating for 75 000 people. How many more tickets could be sold to fill every seat?


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# 1E Multiplication

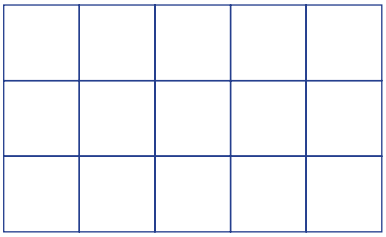
Any two numbers can be multiplied together. The result is called the **product** of the numbers.

The product of 5 and 3 is $5 \times 3 = 15$.

This product can be represented by a rectangular array of counters with 3 rows and 5 columns. (An array of 5 rows of 3 columns will do just as well.)



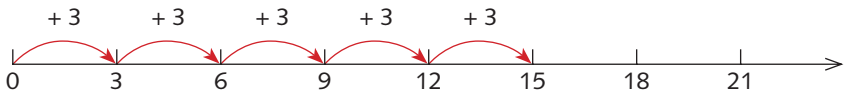
Multiplication can also be represented using areas. For example, the rectangle shown has side lengths of 5 cm and 3 cm. Each of the smaller squares shown has side length 1 cm. The area of the rectangle is 15 $cm^2$.



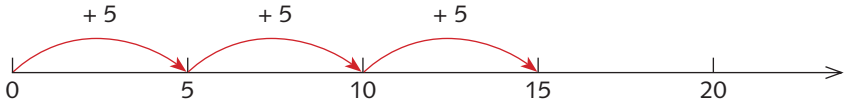
We have used centimetres here, but any unit of length could be used.

Multiplication by whole numbers can also be illustrated on a number line by using repeated addition. For example:

$5 \times 3 = 3 + 3 + 3 + 3 + 3$




or $3 \times 5 = 5 + 5 + 5$




## Zero and one

When you multiply any number by 0, the result is 0. When you multiply any number by 1, the result is the number you started with. For example:

$7 \times 0 = 0$ and $2 \times 1 = 2$

## Multiplying by 10, 100, 1000, ...

Multiplying by powers of 10 – that is, 10, $10 \times 10 = 100$, $10 \times 10 \times 10 = 1000$ and so on – is straightforward.

$2 \times 10 = 20$ $\quad$ $32 \times 10 = 320$
$2 \times 100 = 200$ $\quad$ $32 \times 100 = 3200$
$2 \times 1000 = 2000$ $\quad$ $32 \times 1000 = 32\,000$
$2 \times 10\,000 = 20\,000$ $\quad$ $32 \times 10\,000 = 320\,000$


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### Example 7

Perform each multiplication.

**a** $23 \times 0$ **b** $33 \times 1$ **c** $43 \times 10$

**d** $73 \times 100$ **e** $93 \times 1000$

### Solution

**a** $23 \times 0 = 0$ **b** $33 \times 1 = 33$ **c** $43 \times 10 = 430$

**d** $73 \times 100 = 7300$ **e** $93 \times 1000 = 93\,000$

## Laws for multiplication

From the diagrams on the previous page, you can see that $5 \times 3 = 3 \times 5$. This is an example of the **commutative law for multiplication**.

Remember that the convention in mathematics is that operations inside brackets are performed first. For three or more numbers multiplied together, two at a time, it does not matter which multiplication you do first. For example:

$$(5 \times 3) \times 4 = 15 \times 4 \qquad 5 \times (3 \times 4) = 5 \times 12$$
$$= 60 \qquad = 60$$

That is:

$$(5 \times 3) \times 4 = 5 \times (3 \times 4)$$

This illustrates that the **associative law** also holds for multiplication. The two rules together show that for strings of multiplications, order does not matter. This result can be described as the 'any-order property for multiplication'.

### Any-order property for multiplcation

Whole numbers can be multiplied two at a time in any order to give the same result.

For example:

$$4 \times 7 \times 5 = 4 \times 5 \times 7$$
$$= 20 \times 7$$
$$= 140$$

### Example 8

Use the any-order property for multiplication to work out the following calculations.

**a** $5 \times 7 \times 3 \times 2$ **b** $20 \times 12 \times 5$

### Solution

**a** $5 \times 7 \times 3 \times 2 = 5 \times 2 \times 7 \times 3$
$= 10 \times 21$
$= 210$

**b** $20 \times 12 \times 5 = 20 \times 5 \times 12$
$= 100 \times 12$
$= 1200$


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## Properties of multiplication

- Multiplication of whole numbers can be thought of as repeated addition:

  $5 \times 3 = 3 + 3 + 3 + 3 + 3$ or $5 \times 3 = 5 + 5 + 5$

- The **commutative law** and the **associative law** hold for multiplication:

  $3 \times 4 = 4 \times 3$ (commutative law)

  $(5 \times 2) \times 3 = 5 \times (2 \times 3)$ (associative law)

- The **any-order property for multiplication** says that a list of whole numbers can be multiplied two at a time in any order to give the same result:

$$2 \times 6 \times 100 \times 4 \times 7 \times 25 = (7 \times 6 \times 2) \times (4 \times 25) \times 100$$
$$= 840\,000$$

# Exercise 1E

Example 7

**1** Perform each multiplication.

**a** $17 \times 10$ **b** $89 \times 0$ **c** $100 \times 1$

**d** $18 \times 1000$ **e** $120 \times 100$ **f** $100 \times 100$

**g** $1000 \times 73$ **h** $10\,000 \times 100$ **i** $67\,430 \times 1000$

Example 8

**2** Carry out each calculation, using the any-order property for multiplication.

**a** $25 \times 4 \times 6$ **b** $5 \times 26 \times 2$ **c** $5 \times 43 \times 20$

**d** $50 \times 49 \times 2$ **e** $16 \times 5 \times 40$ **f** $13 \times 6 \times 0$

**g** $1 \times 34 \times 20$ **h** $6 \times 10 \times 10 \times 2$ **i** $3 \times 7 \times 4 \times 5$

**3** Fill in each box with a number to make the statements true.

**a** $(3 \times 2) \times 7 = 3 \times (\square \times 7)$

**b** $(5 \times 9) \times (2 \times 8) = 9 \times 8 \times (\square \times 2)$

**4** Ian has 13 jars, each containing 20 olives. If he decides to redistribute the olives equally among 20 jars, how many olives will there be in each jar?

**5** Six friends buy a large box of jelly snakes. The snakes come in 5 different colours. How many snakes are needed so that every person has two of each colour?

**6** Bricks are arranged on a concrete floor in 12 rows of 25, and stacked 4 bricks high. How many bricks are there in total?

**7** Holly has prepared 28 bags of lollies for her birthday party. Each bag has 9 lollies in it. She makes these into 14 new bags when some of her friends do not turn up. How many lollies will each person now receive?


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# 1F Combinations of operations and the distributive law

As soon as there is more than one type of operation in a calculation (for example, both addition *and* multiplication), we need a set of rules about the order in which the operations are to be performed. These are some of the rules.

### Order of operations

- Work out the calculations inside brackets first.
- In the absence of brackets, carry out operations in the following order:
  - multiplication from left to right, then
  - addition and subtraction from left to right.

This means that:

$$\begin{aligned} 5 \times 3 + 2 &= 15 + 2 \\ &= 17 \end{aligned}$$

Brackets can be used to force the addition to be done first. For example:

$$\begin{aligned} 5 \times (3 + 2) &= 5 \times 5 \\ &= 25 \end{aligned}$$

We will revisit and add to these rules in later sections of this chapter.

### Example 9

Carry out each of the following calculations.

**a** $3 \times 7 - 4$ **b** $3 \times (7 - 4)$ **c** $5 \times 6 + 8$

**d** $7 \times (11 + 4)$ **e** $3 + 4 \times 2$ **f** $25 - 6 \times 3$

### Solution

**a** $\begin{aligned} 3 \times 7 - 4 &= 21 - 4 \\ &= 17 \end{aligned}$

**b** $\begin{aligned} 3 \times (7 - 4) &= 3 \times 3 \\ &= 9 \end{aligned}$

**c** $\begin{aligned} 5 \times 6 + 8 &= 30 + 8 \\ &= 38 \end{aligned}$

**d** $\begin{aligned} 7 \times (11 + 4) &= 7 \times 15 \\ &= 105 \end{aligned}$

**e** $\begin{aligned} 3 + 4 \times 2 &= 3 + 8 \\ &= 11 \end{aligned}$

**f** $\begin{aligned} 25 - 6 \times 3 &= 25 - 18 \\ &= 7 \end{aligned}$

## The distributive law

When multiplying, it is sometimes useful to express one of the numbers you are multiplying as a sum of two other numbers. For example:

$$\begin{aligned} 6 \times 105 &= 6 \times (100 + 5) \\ &= 6 \times 100 + 6 \times 5 \\ &= 600 + 30 \\ &= 630 \end{aligned}$$


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This is an example of the **distributive law for multiplication over addition**. Using the distributive law can often help you to do multiplications more easily.

Using the **distributive law for multiplication over subtraction** can also help to make a multiplication easier. For example:

$$\begin{aligned} 85 \times 98 &= 85 \times (100 - 2) \\ &= (85 \times 100) - (85 \times 2) \\ &= 8500 - 170 \\ &= 8330 \end{aligned}$$

## Example 10

Carry out each of the following computations, using the distributive law.

**a** $106 \times 8$

**b** $43 \times 7 + 43 \times 3$

**c** $97 \times 88$

### Solution

The following are possible methods.

**a** $\begin{aligned} 106 \times 8 &= (100 + 6) \times 8 \\ &= 100 \times 8 + 6 \times 8 \\ &= 800 + 48 \\ &= 848 \end{aligned}$ (distributive law)

**b** $\begin{aligned} 7 \times 43 + 3 \times 43 &= (7 + 3) \times 43 \\ &= 430 \end{aligned}$ (distributive law)

**c** $\begin{aligned} 97 \times 88 &= 88 \times (100 - 3) \\ &= 88 \times 100 - 88 \times 3 \\ &= 8800 - 264 \\ &= 8536 \end{aligned}$ (distributive law)

## Example 11

For each of the following, put a whole number in the box to make the statement true.

**a** $6 \times (7 + \square) = 6 \times 7 + 6 \times 5$

**b** $13 \times 7 + 13 \times 8 = \square \times (7 + 8)$

**c** $10 \times (4 + 7) = 10 \times 4 + 10 \times \square$

**d** $8 \times \square = 8 \times 10 + 8 \times 7$

### Solution

**a** $6 \times (7 + \boxed{5}) = 6 \times 7 + 6 \times 5$

**b** $13 \times 7 + 13 \times 8 = \boxed{13} \times (7 + 8)$

**c** $10 \times (4 + 7) = 10 \times 4 + 10 \times \boxed{7}$

**d** $8 \times \boxed{17} = 8 \times 10 + 8 \times 7$


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### The distributive law

- The distributive law holds for multiplication over addition:
  $2 \times (3 + 4) = 2 \times 3 + 2 \times 4$
- The distributive law holds for multiplication over subtraction:
  $2 \times (8 - 4) = 2 \times 8 - 2 \times 4$

## Exercise 1F

Example 9

**1** Carry out these calculations mentally.

**a** $5 \times 6 - 3$ **b** $5 \times (6 - 3)$ **c** $6 \times 5 + 7$

**d** $6 \times (5 + 7)$ **e** $11 \times (2 + 3)$ **f** $10 + 2 \times 7$

**g** $(10 + 2) \times 7$ **h** $19 - 9 \times 2$ **i** $(19 - 9) \times 2$

Example 10

**2** Carry out these calculations mentally, using the distributive laws.

**a** $6 \times 87 + 4 \times 87$ **b** $64 \times 77 + 36 \times 77$ **c** $23 \times 78 + 77 \times 78$

**d** $27 \times 4$ **e** $9 \times 102$ **f** $87 \times 101$

Example 11

**3** Put a whole number in the box to make each statement true.

**a** $11 \times (7 + \square) = 11 \times 7 + 11 \times 5$ **b** $15 \times 7 + 15 \times 8 = \square \times (7 + 8)$

**c** $11 \times \square = 11 \times 20 + 11 \times 3$ **d** $21 \times 6 + 21 \times 8 = \square \times (6 + 8)$

**4** Use the distributive law to carry out these calculations.

**a** $6 \times 87 - 4 \times 87$ **b** $123 \times 77 - 23 \times 77$

**c** $23 \times 78 - 13 \times 78$ **d** $8 \times 120 - 13 \times 120$

**5** Put a whole number in each box to make the statements true.

**a** $11 \times (7 - \square) = 11 \times 7 - 11 \times 5$ **b** $15 \times 8 - 15 \times 7 = \square \times (8 - 7)$

**c** $9 \times \square = 9 \times 20 - 9 \times 1$ **d** $8 \times \square = 8 \times 100 - 8 \times 1$

**6** There are 40 passengers on a bus when the bus stops. Eleven passengers leave the bus and 7 passengers get on the bus. How many passengers are there on the bus?

**7** Daniel has 6 boxes of chocolates, each containing 20 chocolates, and he also has 54 loose chocolates. How many chocolates does he have altogether?

**8** Deborah has 5 hair clips while Christine has 11.

**a** What is the total number of hair clips?

**b** Jane has three times as many hair clips as Deborah, and Leanne has three times as many hair clips as Christine. What is the total number of hair clips that Jane and Leanne have?


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**9** John and Minh walk each day for 12 days to keep fit. John walks 9 km a day, and Minh walks 5 km a day.

**a** What is the total distance walked by John and Minh in the 12 days?

**b** How many more kilometres than Minh has John walked in the 12 days?

**10** James has 10 fewer novels than Janine, and Ainesh has 5 times the number of novels that James has. Janine has 13 novels. How many novels does Ainesh have?

**11** Becky earns $5 less than Ben each week, and Jake earns 4 times the amount that Becky earns each week. Ben earns $17 each week. How much does Jake earn each week?

**12** **a** Seven is multiplied by 5, and 6 is added. What is the result?

**b** Six is added to 7 and the result is multiplied by 5. What is the final result?

**c** Eleven is subtracted from 20 and the result is multiplied by 2. What is the final result?

**d** The result of multiplying 7 by 8 is added to the result of multiplying 7 by 2. What is the final result?

**e** The result of multiplying 9 by 3 is subtracted from the result of multiplying 9 by 11. What is the final result?

# 1G Place value

The symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 are called **digits**. For example, 421 is a three-digit number and 40 000 is a five-digit number.

The **place value** of a digit in a number means its value according to its place in that number.

We can break apart any number and write it showing its place-value parts. For example:

$$\begin{aligned} 3721 &= 3 \times 1000 + 7 \times 100 + 2 \times 10 + 1 \\ &= 3000 + 700 + 20 + 1 \end{aligned}$$

For the number 3721, we say that:

- the place value of 3 is 3000
- the place value of 7 is 700
- the place value of 2 is 20
- the place value of 1 is 1.

A number can be represented in a place-value table. In this table, the **places** are thousands, hundreds, tens and ones. The ones place is sometimes called the units place.

| Thousands | Hundreds | Tens | Ones |
|---|---|---|---|
| 3 | 7 | 2 | 1 |


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## Powers of 10

Recall that:

$$10 \times 10 = 100 \text{ and } 10 \times 10 \times 10 = 1000 \text{ and } 10 \times 10 \times 10 \times 10 = 10\,000$$

We can record that there is 1 factor of 10 in 10, 2 factors of 10 in 100, 3 factors of 10 in 1000 and so on, by writing:

$$10 = 10^1$$
$$100 = 10^2$$
$$1000 = 10^3$$
$$10\,000 = 10^4$$

### Example 12

Find the correct number for each box.

**a** $120 = 12 \times 10^{\square}$ **b** $700 = 7 \times 10^{\square}$ **c** $340\,000 = 34 \times 10^{\square}$

#### Solution

**a** $120 = 12 \times 10 = 12 \times 10^1$

**b** $700 = 7 \times 100 = 7 \times 10^2$

**c** $340\,000 = 34 \times 10\,000 = 34 \times 10^4$

Notation using powers of 10 is very useful in describing the place values of digits. It is particularly useful for large numbers. When we use powers of 10 to show the place value of the digits in a number, we say that the number is written in **expanded form**.

For example, 30 721 is written in expanded form as:

$$3 \times 10^4 + 7 \times 10^2 + 2 \times 10^1 + 1$$

### Example 13

Write each of the following numbers in expanded form and give the place value of the digit 5.

**a** 523 **b** 750 987

#### Solution

**a** $523 = 500 + 20 + 3 = 5 \times 10^2 + 2 \times 10 + 3$
The place value of the digit 5 is 500.

**b** $750\,987 = 7 \times 10^5 + 5 \times 10^4 + 9 \times 10^2 + 8 \times 10^1 + 7$
The place value of the digit 5 is 50 000.

### Example 14

Write down all the three-digit numbers that can be formed from the digits 3, 7 and 9 (use each digit only once in each number formed), and list the numbers from largest to smallest.

#### Solution

There are 6 such numbers. They are 973, 937, 793, 739, 397, 379.


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## Large numbers

There are common names for some large numbers:

$1\,000\,000 = 10^6$ is 1 million
$1\,000\,000\,000 = 10^9$ is 1 billion
$1\,000\,000\,000\,000 = 10^{12}$ is 1 trillion

Large numbers are often used in astronomy. Here are some examples:

- The average distance from the Earth to the Sun is approximately 150 million kilometres.
- There are between 100 billion and 2000 billion stars in the Milky Way.
- The star Sirius is approximately 75 684 billion kilometres from Earth.

### Place value

- Each digit in a number has a place value.

  For example, in the number 567, the place value of 5 is 500, the place value of 6 is 60 and the place value of 7 is 7.
- A number can be written in expanded form to show all the place values. For example: $567 = 5 \times 10^2 + 6 \times 10^1 + 7$.

## Exercise 1G

Example 12

**1** Find the correct number for each box.

**a** $90 = 9 \times 10^{\square}$
**b** $500 = 5 \times 10^{\square}$
**c** $1800 = 18 \times 10^{\square}$
**d** $20\,000 = 2 \times 10^{\square}$
**e** $45\,000 = 45 \times 10^{\square}$
**f** $234\,000 = 234 \times 10^{\square}$
**g** $7\,000\,000 = 7 \times 10^{\square}$
**h** $25\,000\,000 = 25 \times 10^{\square}$

Example 13

**2** Write each of the following numbers in expanded form and give the place value of the digit 6.

**a** 46 **b** 623 **c** 569 **d** 63 **e** 286 **f** 760

**3** Write each of the following numbers in expanded form and give the place value of the digit 3.

**a** 2083 **b** 3758 **c** 5036 **d** 43 170
**e** 50 732 **f** 235 678 **g** 23 678 978

Example 14

**4** Write down all of the three-digit numbers that can be formed from the digits 2, 5 and 9 (use each digit only once in each number formed), and list them from largest to smallest.

**5** Write down all of the three-digit numbers that can be formed from the digits 2, 5 and 9 (each digit can be used more than once in each number formed), and list them from largest to smallest.


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# 1H The standard multiplication algorithms

In Section 1F we saw that the distributive law can be used to help perform multiplications. The distributive law can also be used to explain multiplication algorithms.

## Multiplication by a single digit (the short multiplication algorithm)

The multiplication $43 \times 5$ can be done mentally. Here is the explanation of how to do this:

$$\begin{aligned} 43 \times 5 &= (40 + 3) \times 5 \\ &= 40 \times 5 + 3 \times 5 \\ &= 200 + 15 \\ &= 215 \end{aligned}$$

This can also be set out using the **short multiplication algorithm**:

$$\begin{array}{r} 4\ 3 \\ \times\ \ 5 \\ \hline 2\ 1\ 5 \end{array}$$

### Example 15

Multiply 27 by 8, using the short multiplication algorithm.

**Solution**

$$\begin{array}{r} 2\ 7 \\ \times\ \ 8 \\ \hline 2\ 1\ 6 \end{array}$$

## Multiplication by more than one digit (the long multiplication algorithm)

Consider $378 \times 37$. The distributive law gives:

$$\begin{aligned} 378 \times (30 + 7) &= 378 \times 30 + 378 \times 7 \\ &= 11\,340 + 2646 \\ &= 13\,986 \end{aligned}$$

An efficient setting out for this is as follows.

$$\begin{array}{rr} & 3\ 7\ 8 \\ \times & \ \ 3\ 7 \\ \hline & 2\ 6\ 4\ 6 \\ 1\ & 1\ 3\ 4\ 0 \\ \hline 1\ & 3\ 9\ 8\ 6 \end{array}$$

(Multiply by 7.)
(Multiply by 30. That is why the 0 is here.)


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Multiplication by three digits is carried out in a similar way.

$$\begin{array}{rr} & 378 \\ \times & 237 \\ \hline & 2646 \\ 1134 & 0 \\ 756 & 00 \\ \hline 89586 & \end{array}$$

(Multiply by 7.)
(Multiply by 30. That is why the 0 is here.)
(Multiply by 200. That is why the 00 is here.)

To make the examples easy to read, we have left out all of the carry digits.

**Example 16**

**a** Multiply 389 by 46.

**b** Multiply 667 by 667.

**Solution**

**a**
$$\begin{array}{rr} & 389 \\ \times & 46 \\ \hline & 2334 \\ 1556 & 0 \\ \hline 17894 & \end{array}$$

**b**
$$\begin{array}{rr} & 667 \\ \times & 667 \\ \hline & 4669 \\ 4002 & 0 \\ 400200 & \\ \hline 444889 & \end{array}$$

## Exercise 1H

Example 15

**1** Carry out each calculation, using the short multiplication method.

**a** $53 \times 4$ **b** $19 \times 8$ **c** $64 \times 7$ **d** $85 \times 4$

**e** $513 \times 4$ **f** $819 \times 8$ **g** $235 \times 7$ **h** $2006 \times 7$

**i** $6543 \times 7$ **j** $8159 \times 4$ **k** $91\,370 \times 9$ **l** $43\,987 \times 6$

Example 16

**2** Carry out each calculation, using the long multiplication method.

**a** $453 \times 24$ **b** $179 \times 86$ **c** $614 \times 47$ **d** $895 \times 45$

**e** $135 \times 27$ **f** $506 \times 68$ **g** $235 \times 34$ **h** $5646 \times 73$

**i** $91\,270 \times 39$ **j** $762 \times 549$ **k** $936 \times 564$ **l** $91\,370 \times 109$

**3** Calculate each of the following using either short or long multiplication.

**a** Each student in a class is given 9 coloured pencils by the teacher. How many pencils does the teacher need to supply 26 students?

**b** A packaging machine in a factory packs 893 boxes per hour. How many boxes are packed in a 12-hour day?

**c** A brick wall has 43 rows of 723 bricks. How many bricks are in the wall?

**d** A publishing company packages books in boxes of 125. How many books are there in 298 boxes?


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**4** A hall has 86 rows of 34 seats. How many seats are there in the hall?

**5** A machine makes 257 doughnuts in an hour. How many doughnuts can it make in 13 hours?

**6** Copy and complete the following by finding a digit for each ★.

**a**
$$\begin{array}{r} \bigstar\ 6 \\ \times\quad 7 \\ \hline 6\ 0\ \bigstar \end{array}$$

**b**
$$\begin{array}{r} \bigstar\bigstar 9 \\ \times\quad 3 \\ \hline 5\ 0\ \bigstar \end{array}$$

**c**
$$\begin{array}{r} \bigstar\ \bigstar\ \bigstar \\ \times\quad \bigstar\ 3 \\ \hline 6\ 4\ 8 \\ \bigstar\ \bigstar\ \bigstar\ 0 \\ \hline 4\ 9\ 6\ 8 \end{array}$$

**7** A particular brand of lollies comes in packets of 26. A carton contains 34 packets.

**a** How many lollies are there in one carton?

**b** How many lollies are there in 30 cartons?

**8** A trolley at an airport is loaded with 15 cases, each with the maximum allowable weight of 20 kilograms. The trolley weighs 115 kilograms. What is the maximum possible weight of the trolley and the cases?

**9** If 25 people each own 7 pairs of shoes, and 32 people each own 8 pairs of shoes, then how many *shoes* do the 57 people own in total?

**10** Calculate your age in:

**a** months **b** weeks **c** days **d** hours **e** seconds

# 1I Division

## Division without remainder

Division is an operation on two numbers that tells **how many equal groups** a number can be divided into. It can also tell **how many are in each equal group**.

Division without remainder is the reverse of multiplication. This is shown in the following example.

**Example 17**

Fill in each box to give the equivalent multiplication or division statement.

**a** $60 \div 5 = 12$ is equivalent to $60 = 12 \times \square$. **b** $24 \div \square = 4$ is equivalent to $24 = 6 \times 4$.

**Solution**

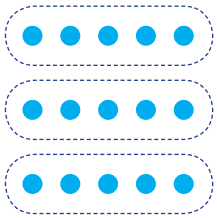
**a** $60 \div 5 = 12$ is equivalent to $60 = 12 \times \boxed{5}$. **b** $24 \div \boxed{6} = 4$ is equivalent to $24 = 6 \times 4$.


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Division without remainder can answer questions such as:

> 'How many equal groups of 5 objects can 15 objects be divided into?'

This is shown in the diagram below.



There are 3 such groups. The 15 objects are divided into 3 equal groups of 5, so $15 \div 5 = 3$.

The diagram also shows that $15 = 5 \times 3$.

In addition, the diagram shows the answer to this question:

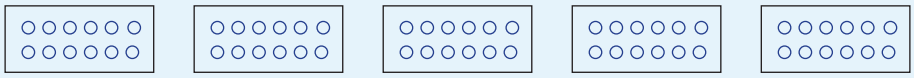
> 'If 15 objects are divided into three equal groups, how many objects are in each group?'
>
> There are 5 objects in each group. The 15 objects are divided into three groups, each containing 5 objects.

**Example 18**

There are 60 chocolates to be packed into boxes so that each box has 12 chocolates in it. How many boxes are needed?

**Solution**

As $60 = 12 \times 5$, 5 boxes are needed. That is, $60 \div 12 = 5$.



**Example 19**

A box of 72 chocolates is to be divided equally between 9 people. How many chocolates does each person get?

**Solution**

Each person gets $72 \div 9 = 8$ chocolates.

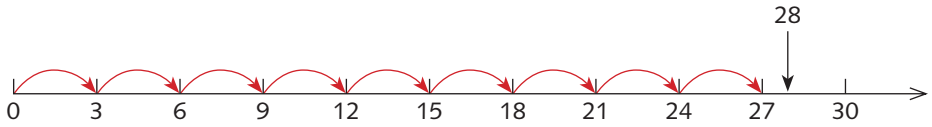
## Division with remainder

If there are 28 marbles and you wish to form them into 3 equal groups, then the 28 marbles can be broken up into 3 groups of 9 marbles with 1 left over.




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It is not possible to divide 28 into 3 equal groups, because 28 lies between $3 \times 9 = 27$ and $3 \times 10 = 30$. The best we can do is take 9 groups of 3, with 1 left over. We can see this on a number line.




This shows that $28 = 3 \times 9 + 1$. We say '28 ÷ 3 equals 9 with remainder 1'.

In this process, 28 is called the **dividend**, 3 is the **divisor**, 9 is the **quotient** and 1 is the **remainder**. The remainder must be less than the divisor.

**Example 20**

Put the quotient in the first box and the remainder in the second box to make each statement true.

**a** $26 = \square \times 4 + \square$ **b** $34 = \square \times 3 + \square$

**Solution**

**a** $26 = \boxed{6} \times 4 + \boxed{2}$ **b** $34 = \boxed{11} \times 3 + \boxed{1}$

## Another notation for division

Up to this point we have only used the sign ÷ for division. There is another way of writing division. For example:

$24 \div 6$ can also be written as $\frac{24}{6}$

Other examples using this notation are:

$\frac{0}{3} = 0$, $\frac{72}{9} = 8$ and $\frac{108}{12} = 9$

For the time being, we will only use this notation for division without remainder. We will have more to say about this in Chapter 4 when we study fractions.

## One and zero

Any number divided by 1 gives the original number. For example:

$3 \div 1 = 3$, and the equivalent multiplication statement is $1 \times 3 = 3$

Any number divided by itself gives 1. For example:

$7 \div 7 = 1$

Dividing by 0 does not make sense. For example, if $4 \div 0$ is a number, then that number multiplied by 0 is 4. But multiplying any number by 0 gives 0, so no such number exists.

However, 0 divided by any non-zero number is 0. For example:

$0 \div 13 = 0$, since $13 \times 0 = 0$


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**Example 21**

Evaluate:

**a** $\frac{25}{5}$ **b** $\frac{240}{3}$

**Solution**

**a** $\frac{25}{5} = 25 \div 5$
$= 5$

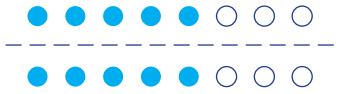
**b** $\frac{240}{3} = 240 \div 3$
$= 80$

## The distributive law with division

When dividing, it is sometimes useful to express the number you are dividing as a sum of two other numbers.

Here is a simple example, with a dot diagram to illustrate it. It uses the distributive law of division over addition.

$16 \div 2 = (10 + 6) \div 2$
$= 10 \div 2 + 6 \div 2$
$= 5 + 3$
$= 8$



Here is another example, this time involving subtraction. It uses the distributive law of division over subtraction.

$196 \div 4 = (200 - 4) \div 4$
$= 200 \div 4 - 4 \div 4$
$= 50 - 1$
$= 49$

The **distributive law for division over addition and subtraction** makes it easier to carry out some divisions.

**Example 22**

Use the distributive law to evaluate each of the following.

**a** $(100 + 55) \div 5$ **b** $(200 - 15) \div 5$ **c** $540 \div 5$

**Solution**

**a** $(100 + 55) \div 5 = 100 \div 5 + 55 \div 5$
$= 20 + 11$
$= 31$

**b** $(200 - 15) \div 5 = 200 \div 5 - 15 \div 5$
$= 40 - 3$
$= 37$

**c** $540 \div 5 = (500 + 40) \div 5$
$= 500 \div 5 + 40 \div 5$
$= 100 + 8$
$= 108$


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## Properties of division

- The expression $96 \div 8$ can mean: 'How many equal groups of 8 objects can be made from 96 objects?'
- The expression $96 \div 8$ can also mean: 'If 96 objects are divided into 8 equal groups, how many objects are there in each group?'
- Division without remainder is the reverse operation of multiplication. For example, $8 \times 12 = 96$ is equivalent to both $96 \div 8 = 12$ and $96 \div 12 = 8$.
- In $43 = 6 \times 7 + 1$ and $43 \div 6 = 7$ remainder 1, the number 43 is the **dividend**, 6 is the **divisor**, 7 is the **quotient** and 1 is the **remainder**.
- The distributive law works for division over addition. For example:

$$\begin{aligned}(700 + 25) \div 5 &= 700 \div 5 + 25 \div 5\\ &= 140 + 5\\ &= 145\end{aligned}$$

- The distributive law works for division over subtraction. For example:

$$\begin{aligned}(700 - 25) \div 5 &= 700 \div 5 - 25 \div 5\\ &= 140 - 5\\ &= 135\end{aligned}$$

## Exercise 1I

Example 17

**1** Fill in each box to give the equivalent multiplication or division statement.

**a** $108 \div 9 = 12$ is equivalent to $108 = 12 \times \square$.

**b** $200 \div 10 = 20$ is equivalent to $\square = 10 \times 20$.

**c** $72 \div \square = 12$ is equivalent to $72 = 12 \times \square$.

**2** Work from left to right to calculate the following.

**a** $24 \times 3 \div 3$ **b** $10 \times 2 \div 2$ **c** $36 \div 4 \times 4$

**d** $56 \div 8 \times 8$ **e** $18 \div 3 \times 3$ **f** $24 \div 12 \times 12$

Example 18, 19

**3** There are 28 chocolates to be divided equally among 4 people. How many chocolates does each person get?

**4** There are 84 people at a club meeting. The organiser wishes to form 7 equal groups. How many people will there be in each group?

Example 20

**5** Fill in the boxes to make each statement true, with the smallest possible remainder.

**a** $17 = \square \times 3 + \square$ **b** $37 = \square \times 5 + \square$

**c** $13 = \square \times 2 + \square$ **d** $87 = \square \times 8 + \square$

**e** $41 = \square \times 5 + \square$ **f** $148 = \square \times 12 + \square$


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**6** Draw a dot diagram to show $30 \div 8 = 3$ with remainder 6 or, equivalently, $30 = 8 \times 3 + 6$.

**7** Draw a dot diagram to show $20 \div 6 = 3$ with remainder 2 or, equivalently, $20 = 6 \times 3 + 2$.

**8** Illustrate each expression on a number line.

**a** $7 \div 2$ **b** $13 \div 3$

Example 21

**9** Evaluate:

**a** $\frac{20}{10}$ **b** $\frac{30}{6}$ **c** $\frac{42}{7}$

**d** $\frac{144}{12}$ **e** $\frac{36}{4}$ **f** $\frac{120}{3}$

**10** Perform each calculation by using the method indicated.

**a** $448 \div 32$ (divide by 2 five times)

**b** $640 \div 80$ (divide by 10 and then by 8)

**c** $805 \div 35$ (divide by 7 and then by 5)

Example 22

**11** Evaluate each expression by using the distributive law.

**a** $(600 + 35) \div 5$ **b** $(300 - 25) \div 5$ **c** $390 \div 5$

**d** $(600 + 27) \div 3$ **e** $(300 - 24) \div 3$ **f** $390 \div 3$

# 1J The short division algorithm

Consider 64 divided by 4 and 91 divided by 7:

$$\begin{aligned} 64 \div 4 &= (40 \div 4) + (24 \div 4) \\ &= 10 + 6 \\ &= 16 \end{aligned}$$

$$\begin{aligned} 91 \div 7 &= (70 \div 7) + (21 \div 7) \\ &= 10 + 3 \\ &= 13 \end{aligned}$$

We can set this out as follows:

$$\begin{array}{r} 1\ 6 \\ 4\overline{)6\ {}^{2}4} \end{array} \qquad \begin{array}{r} 1\ 3 \\ 7\overline{)9\ {}^{2}1} \end{array}$$

**Example 23**

Find $763 \div 4$.

**Solution**

$$\begin{array}{r} 1\ 9\ 0 \\ 4\overline{)7\ {}^{3}6\ 3} \end{array} \text{ remainder } 3$$

$763 \div 4 = 190$ remainder 3


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In this example, 4 is first divided into 700 to give 100 with 300 remainder. We write 1 in the hundreds column to show this.

The remaining 300 is added to the 60, then 4 is divided into 360 to give 90 exactly. We write 9 in the tens column.

Finally, 4 is divided into 3 to give 0 and 3 remainder. We write 0 in the ones column and a remainder of 3 at the end.

### Example 24

Find $473 \div 4$.

#### Solution

$$4\overline{)4\ 7^{3}3}\quad \begin{matrix}1\ 1\ 8\\ \text{remainder } 1\end{matrix}$$

$473 \div 4 = 118$ remainder 1

### Example 25

If \$6755 is to be divided equally among 5 people, how much will each person receive?

#### Solution

$$5\overline{)6^{1}7^{2}5\ 5}\quad 1\ 3\ 5\ 1$$

Each person will receive \$1351.

## Exercise 1J

Example 23

**1** Use short division to calculate:

**a** $556 \div 2$ **b** $869 \div 7$ **c** $4536 \div 8$

**d** $8624 \div 8$ **e** $1089 \div 9$ **f** $5472 \div 6$

**g** $1496 \div 11$ **h** $33\,552 \div 12$ **i** $39\,240 \div 9$

Example 24

**2** Work out each of the following, using short division.

**a** $524 \div 4$ **b** $1095 \div 3$ **c** $498 \div 6$

**d** $431 \div 8$ **e** $740 \div 11$ **f** $9756 \div 12$

**g** $67\,543 \div 6$ **h** $19\,005 \div 7$

Example 25

**3** If \$4250 is to be divided equally among 5 people, how much will each person receive?


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**4** There are 542 tennis balls to be packed into boxes of 12. How many boxes will be filled and how many tennis balls will be left over?

**5** A biscuit company packages its biscuits into tins of 96. The biscuits are arranged in rectangular arrays. How many rows with how many biscuits in each could there be? (Give four different answers.)

**6** If 231 children from a school are to be transported on 7 buses, how many children will there be on each bus, if the number of children on each bus is the same?

**7** There are 11 buses available to transport 407 people on an outing. How many people will there be on each bus, if the passengers are to be distributed equally?

**8** A library of 3458 books is to be divided equally among 7 organisations. How many books will each organisation receive?

**9** Eggs are sold in cartons of 12. If there are 345 eggs to be packaged, how many full cartons will there be and how many eggs will be left over?

**10** Cans of lemonade are to be packaged together in groups of 6. The factory has 4567 cans to be packaged. How many packages of 6 cans are there and how many are left over?

**11** 2552 people arrive at a film studio for a tour. The film studio decides that there should be exactly 8 people in a tour group.

**a** How many tour groups are there?

**b** How many people are left waiting to form the next group of 8?

# 1K The long division algorithm

The long division algorithm is the short division algorithm with the subtractions set out. The long division algorithm is an efficient and clear way to set out division, particularly with larger divisions.

$$
\begin{array}{r}
1\ 2\ 3 \\
7\overline{)8\ 6\ 1} \\
-\ 7\phantom{\ 6\ 1} \\
\hline
1\ 6\phantom{\ 1} \\
-\ 1\ 4\phantom{\ 1} \\
\hline
2\ 1 \\
-\ 2\ 1 \\
\hline
0
\end{array}
$$

(Bring down 6.)

(Bring down 1.)

The steps correspond to $861 - 700 = 161$,

then $161 - 140 = 21$,

and finally $21 - 21 = 0$.

The answer, 123, is recorded on the top line.


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Long division by larger numbers is more difficult because we do not know the multiples of large numbers in our heads. One of many possible methods is illustrated below. A table of the 9 multiples of the divisor is written on the right and then the appropriate multiple can easily be found.

### Example 26

Find 8618 ÷ 27, using the long division algorithm.

### Solution

$$
\begin{array}{rl}
 & 3\ 1\ 9 \\
27\overline{)} & 8\ 6\ 1\ 8 \\
-\ & 8\ 1 \\
\hline
 & 5\ 1 \quad \text{(Bring down 1.)} \\
-\ & 2\ 7 \\
\hline
 & 2\ 4\ 8 \quad \text{(Bring down 8.)} \\
-\ & 2\ 4\ 3 \\
\hline
 & \quad\ 5
\end{array}
$$

$27 \times 1 = 27$
$27 \times 2 = 54$
$27 \times 3 = 81$
$27 \times 4 = 108$
$27 \times 5 = 135$
$27 \times 6 = 162$
$27 \times 7 = 189$
$27 \times 8 = 216$
$27 \times 9 = 243$

8618 ÷ 27 = 319 remainder 5

*Note*: Once the table is written down, the only arithmetic to be done is subtraction.

## Exercise 1K

Example 26

**1** Use the long division algorithm to calculate:

**a** 728 ÷ 13 **b** 1050 ÷ 14 **c** 1344 ÷ 16 **d** 4047 ÷ 19

**2** Use the long division algorithm to calculate:

**a** 2982 ÷ 71 **b** 5244 ÷ 57 **c** 3268 ÷ 43 **d** 1743 ÷ 102

**3** If $4260 is divided equally among 15 people, how much will each person receive?

**4** If $11 572 is divided equally among 22 people, how much will each person receive?

**5** A piece of string that is 1170 cm long is to be cut into 26 equal lengths. How long is each piece?

**6** There are 5420 golf balls to be packed into boxes of 25. How many boxes will be filled and how many golf balls will be left over?

**7** If 1598 school children are to be transported on 34 buses, how many children will there be on each bus, if each bus contains the same number of children?

**8** There are 27 buses available to transport 1107 fans to a football match. How many people will there be on each bus, if each bus contains the same number of people?

**9** A computer program runs for 7568 seconds. Convert this to hours, minutes and seconds.


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# 1L Order of operations

There are important conventions for the order in which we carry out the arithmetic operations.

## Order of operations

- Evaluate expressions inside brackets first.
- In the absence of brackets, carry out the operations in the following order:
  - powers
  - multiplication and division from left to right
  - addition and subtraction from left to right.

### Example 27

**a** $3 \times (5 - 1)$ **b** $7 \times 10^2$ **c** $4 \times 10 \div 2$

**d** $6 + 3 \times 4$ **e** $4 + 6 - 5 + 11$ **f** $7 + 84 \div 7$

### Solution

**a** $3 \times (5 - 1) = 3 \times 4$ (brackets first)
$= 12$

**b** $7 \times 10^2 = 7 \times 100$ (powers first)
$= 700$

**c** $4 \times 10 \div 2 = 40 \div 2$ (multiplications and divisions from left to right)
$= 20$

**d** $6 + 3 \times 4 = 6 + 12$ (multiplication before addition)
$= 18$

**e** $4 + 6 - 5 + 11 = 10 - 5 + 11$ (addition and subtraction from left to right)
$= 5 + 11$
$= 16$

**f** $7 + 84 \div 7 = 7 + 12$ (division before addition)
$= 19$

## Exercise 1L

Example 27

**1** Evaluate:

**a** $6 + 7 + 11 + 8$ **b** $6 + 7 + 8 - 9$ **c** $4 - 3 + 6 - 2$

**d** $12 - 4 - 3 + 2$ **e** $7 - 1 - 3 + 6$ **f** $26 - 14 - 4 + 12$

**g** $56 - 28 - 20 + 2$ **h** $30 + 50 - 20 - 60$ **i** $32 + 8 - 40$


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Example 27

**2** Evaluate:

**a** $34 + 5 \times 3$ **b** $60 - 4 \times 10$ **c** $52 + 45 \div 9$

**d** $45 - 45 \div 9$ **e** $66 + 23 \times 2$ **f** $24 - 144 \div 12$

Example 27

**3** Evaluate:

**a** $4 \div 10^3$ **b** $5 + 7 - 4 + 13$ **c** $5 \times 6 \div 3 + 7$

**d** $(11 - 7) \times (12 - 5)$ **e** $4 + 28 \div 4$ **f** $64 \div 8 + 42 \div 7$

**g** $7 + 11 \times (5 + 7)$ **h** $(14 + 11) \div 5$ **i** $10^4 \times (2 + 11)$

**j** $(24 + 56) \div (7 + 3)$

Example 27

**4** Evaluate:

**a** $(4 - 3) \times 5$ **b** $24 - 15 \div 3$

**c** $3 \times (5 - 3) - 6$ **d** $(32 - 16) + (54 - 12) \div 6$

**e** $25 \div 5 \times 5 \div 25$ **f** $4 \times 11 \div 2 \times (12 + 8)$

**g** $(75 - 45) \times 3 + (11 + 9) \times 5$ **h** $(7 - 4) + 9 \div 3$

**i** $(11 + 7) \div 3 + 8 \times (11 + 19)$ **j** $(11 + 7) \div 3 + 8 \times 11 + 19$

**5** Insert brackets in each expression to make the resulting statement true.

**a** $3 \times 6 + 4 = 30$ **b** $3 \times 7 - 6 \div 3 = 1$ **c** $8 \times 7 + 30 \div 5 = 104$

**d** $7 \times 3 \times 2 + 8 = 210$ **e** $5 - 2 \times 1 + 23 \div 6 = 12$ **f** $6 + 7 \times 11 + 1 = 156$

**6** Evaluate:

**a** $(4 - 3) \times 10^2$ **b** $(340 - 140) - 10^2$ **c** $3 \times 5 - (13 - 6)$

**d** $10^3 \div 5 \times 5 \div 25$ **e** $4 \times 10^2 \div 2 \times (13 + 7)$

**7** Perform these calculations.

**a** Divide 36 by 3 and then add 6. **b** Add 6 to 36 and then divide by 3.

**c** Subtract 12 from 64 and then divide by 4. **d** Add 15 to 210 and then divide by 5.

**8** Crates of bananas have 60 bananas in each. A market store owner buys 12 crates and 23 loose bananas. How many bananas does he buy?

**9** Taj has 568 chocolates to give out at a party. He first divides the chocolates into 8 equal parcels. He then takes 3 of these parcels of chocolates and gives them to his friend Jane. How many chocolates does Jane receive?

**10** Large crates of soft drinks each contain 56 bottles. It is decided that these are too heavy, so 8 bottles are removed from each crate. How many bottles are there in 15 of the lighter crates?

**11** David divides $4250 equally between 5 bank accounts. He then adds another $32 to each of these accounts. How much money has he put into each account?


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# Review exercise

**1** Calculate:

**a** $226 + 601 + 478$

**b** $72 \div 3$

**c** $163 - 136$

**d** $20 \div (9 - 4) + 6$

**e** $8 \times 321 + 6$

**f** $68 - 42 + 12 \times 2$

**g** $382 - 792 \div 3$

**h** $268 \times (3 + 7)$

**i** $(96 \div 3) + (258 \div 3)$

**2** The contents of a tin of chocolates weigh 6500 grams. The chocolates are divided into packets of 250 grams. How many packets are there?

**3** There are 4000 apples to be divided into boxes so that each box holds 75 apples. How many boxes are required?

**4** A club started the year with 125 members. During the year, 23 people left and 68 people joined. How many people belonged to the club at the end of the year?

**5** If a bus can carry 45 passengers, how many buses are needed to transport 670 school students to a hockey game?

**6** A supermarket takes delivery of 54 cartons of soft drink cans. Each carton contains 48 cans. How many cans are delivered?

**7** On a school excursion, 17 buses each carry 42 students. How many students are transported?

**8** A school day is 6 hours long. How many minutes are there in a school day?

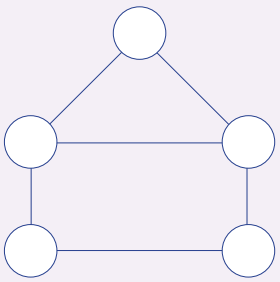
**9** Find the sum of eighty-six and fifty-four and then subtract sixty-eight.

**10** The manager of the school canteen orders 1000 hot dogs for the week. On Monday 384 are sold and on Tuesday 239 are sold. How many hot dogs does the school have left for the rest of the week?


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# Challenge exercise

**1** Place the numbers 1, 2, 3, 4 and 5 in the circles of the following figure so that no two adjacent numbers (that is, numbers with a difference that is 1) are connected by a line.

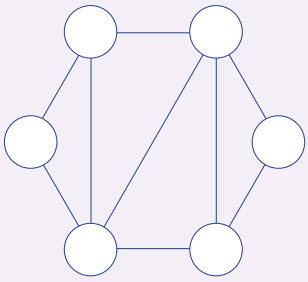


**2** A palindromic number is a whole number that is unchanged when the order of the digits is reversed, for example, 131 and 34 543. The number 39 793 is palindromic. Find the next 5 palindromic numbers.

**3** Complete the following magic square, in which each row, column and diagonal must add up to the same sum.

| | | |
|---|---|---|
| 19 | | |
| | 15 | |
| 22 | | 11 |

**4** Place the numbers 1, 2, 3, 4, 5 and 6 in the circles of the following figure so that no two adjacent numbers are connected by a line.

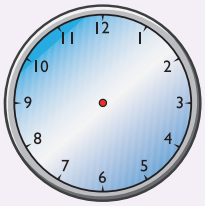


**5** Use five 6s and a selection of the symbols

(, ), + , − , × and ÷

to write a statement with 66 as the result.

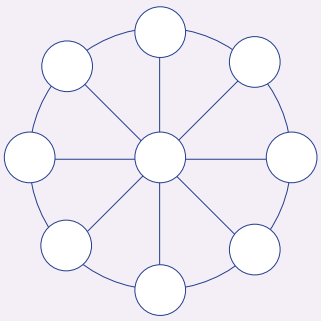
**6** Using two straight lines, divide the clock face into three parts so that the sums of the numbers in each part are equal.





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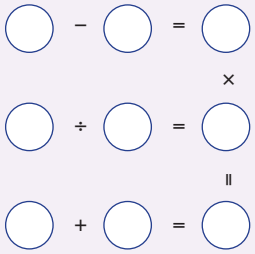
**7** Place the numbers 1 to 9 in each of the circles in the following figure so that the sums of the numbers on each straight line are equal.



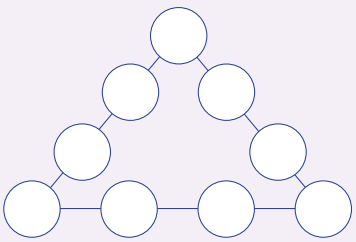
**8** Find a two-digit number that is twice the product of its digits.

**9** Use all of the numbers 1, 2, 3, 4, 5, 6 and 7 once and the symbols $+, -, \times$ and $\div$ to make a number sentence that results in 100.

**10** Place the numbers 1 to 9 in the circles to make each of the equations true.




**11** Place the numbers 1 to 9 in each of the circles in the following figure so that the sums of the numbers on each straight line are equal.



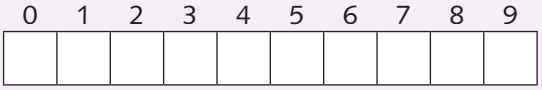
**12** To protect his money from pickpockets, a merchant keeps his coins in several pouches so that he can pay any amount without revealing how much money he has, just by handing over the correct pouches. On the first day of trading he has six coins, so he places one coin in the first pouch, two coins in the second, and three coins in the third. This allows him to pay any value from one to six coins without having to open a pouch. The next day of trading he has 23 coins and five pouches. How should he distribute the coins to ensure that he can pay any amount from 1 to 23 coins?

**13** Find the missing digits in the following multiplication.

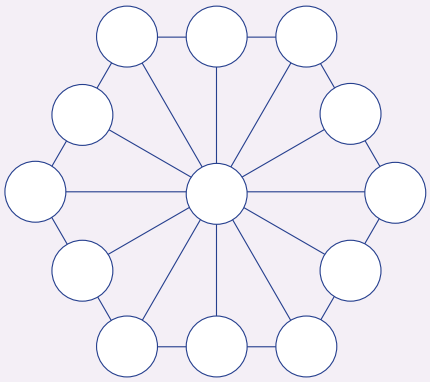
$$\begin{array}{r} \bigstar\bigstar\bigstar \\ \times\ \bigstar 2\bigstar \\ \hline \bigstar\bigstar\bigstar \\ \bigstar\bigstar\bigstar\bigstar 0 \\ \bigstar 8\bigstar 0\,0 \\ \hline \bigstar\bigstar 9\bigstar 2\bigstar \end{array}$$


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**14** In each box below, place any number between 0 and 9 so that the number in the first box is the number of 0s in all the boxes, the number in the second box is the number of 1s in all the boxes, and so on.




**15** Place the numbers 1 to 13 in the circles in the following figure so that the sums of the numbers on each straight line are equal.



**16** It is possible to choose four numbers such that any value between 1 and 40 can be made by taking one or more of these numbers and adding or subtracting them from each other. Find the four numbers, and show how the values from 1 to 40 can be made.

**17** Using all the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9, form two 5-digit numbers so that their sum is:

**a** the greatest possible

**b** the smallest possible

**18** A fast food store sells nuggets in boxes of 5 and 8. You can buy 31 nuggets at a time since $3 \times 5 + 2 \times 8 = 31$. What is the largest whole number of nuggets that **cannot** be purchased?

**19** Find the sum of whole numbers from 1 to 100.


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CHAPTER

# 2

Number and Algebra

# Factors, multiples, primes and divisibility

There are many interesting connections and patterns to be found among numbers. This chapter looks at some of them, especially those that involve multiplication and division. These ideas are useful for arithmetic and calculating mentally, and can be applied in many areas of mathematics.

We all know that $6 = 3 \times 2$. Here, 3 and 2 are primes and 6 is a composite.

**Class discussion**

Six stars can be arranged in rectangular arrays in two different ways.



3 rows 2 columns



1 row 6 columns

For this topic, 3 rows of 2 stars will be considered to be the same as 2 rows of 3 stars.

Draw rectangular arrays for each of the numbers from 1 to 20. Some numbers will have more than one possible arrangement. Use coins or counters to help you find the possibilities. What conclusions can you make about your arrangements?



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# 2A Factors and multiples

As the example in the class discussion shows, it is possible to arrange 6 items into 2 rows of 3 (or 3 rows of 2), but they can also be arranged into 6 rows of 1 (or 1 row of 6).

We can illustrate this by writing:

$6 = 2 \times 3$ or $6 = 3 \times 2$
$6 = 6 \times 1$ or $6 = 1 \times 6$

We call 1, 2, 3 and 6 the **factors** of 6 because each divides into 6 exactly. There is no remainder.

Look at your arrays for the number 12. In how many ways can you arrange 12 items in rectangular arrays? What are the factors of 12?

## Arrays and factors

Find the factors of 18 by looking at the possible rectangular arrays that can be formed:



$18 = 6 \times 3$
$= 3 \times 6$



$18 = 9 \times 2$
$= 2 \times 9$



$18 = 18 \times 1$
$= 1 \times 18$

From these rectangular arrays, we can make the following statements about 18.

- The factors of 18 are 1, 2, 3, 6, 9 and 18.
- The numbers 1, 2, 3, 6, 9 and 18 divide into 18 exactly, with no remainder.

## Multiples

A **multiple** of a whole number is a product of that whole number with any other whole number.

The multiples of 4 can be found by counting in fours.

0, 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, ...

The multiples of 7 can be found by counting in sevens.

0, 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, ...

## Common multiples

If we want to know which numbers are multiples of 4 *and* 7, we need to look at both counting patterns. We see that 0, 28 and 56 are the first three numbers that are multiples of both 4 and 7. These are called **common multiples** of 4 and 7.


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## Factors and multiples

- The **factors** of a number are the numbers that divide into it exactly.
- Every whole number is a factor of itself, and 1 is a factor of every whole number.
- A **multiple** of a number is a product of that whole number with any other whole number.
- A **common multiple** of two or more numbers is a number that is a multiple of those numbers.

### Example 1

Represent 16 using as many different rectangular arrays as possible.

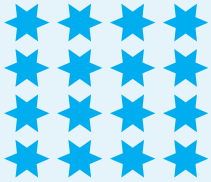
#### Solution



$16 \times 1$



$8 \times 2$



$4 \times 4$

### Example 2

Find the factors of 128.

#### Solution

We can pair factors to make sure we have them all.

Start with $128 = 1 \times 128$
$= 2 \times 64$
$= 4 \times 32$
$= 8 \times 16$

The factors of 128 are 1, 2, 4, 8, 16, 32, 64 and 128.


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**Example 3**

List the first 11 multiples of:

**a** 3 **b** 17

**Solution**

**a** 0, 3, 6, 9, 12, 15, 18, 21, 24, 27, 30

**b** 0, 17, 34, 51, 68, 85, 102, 119, 136, 153, 170

## Exercise 2A

Example 1 **1** Represent these numbers using as many different rectangular arrays as possible for each. What are the factors of these numbers?

**a** 6 **b** 12 **c** 14 **d** 15 **e** 11

Example 2 **2** Use pairing of factors to find all the factors of:

**a** 8 **b** 14 **c** 11 **d** 32 **e** 25 **f** 12

**g** 26 **h** 13 **i** 33 **j** 81 **k** 30 **l** 42

**3** Which of the following numbers have 3 as a factor?

6, 9, 10, 22, 27, 37, 43, 51, 52

**4** Which of the following numbers have 12 as a factor?

32, 144, 158, 192, 210, 222, 228

Example 3 **5** List the first 11 multiples of:

**a** 6 **b** 8 **c** 11 **d** 13 **e** 15 **f** 19

**6** Which of the following numbers are multiples of 17?

57, 68, 85, 135, 152, 170

**7** Which of these numbers have 72 as a multiple?

3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 21, 24, 28, 32, 36

**8** Express each of the following numbers as the product of two numbers, both of which are greater than 10.

**a** 312 **b** 392 **c** 143 **d** 540 **e** 221 **f** 408

**9** List the factors of 12 and 18. Which factors are common to both numbers?

**10** List the first 11 multiples of 6 and 8. Which numbers are common to both lists?


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**11** Find the multiple of 7 that is closest to:

**a** 50 **b** 100 **c** 200 **d** 300

**12** Find the number that is divisible by 11 and closest to:

**a** 100 **b** 200 **c** 125 **d** 50 **e** 500 **f** 1000

**13** **a** Find the quotient and remainder when each of the following numbers is divided into 30.
1, 2, 3, 4, 5

**b** Write down all the factors of 30.

**14** **a** Find the quotient and remainder when each of the following numbers is divided into 35.
1, 2, 3, 4, 5, 6

**b** Write down all the factors of 35.

**15** **a** 'The number 1 is a factor of every number.' Explain why this statement is true.

**b** 'Every number is a factor of zero.' Explain why this statement is true.

**c** List all the factors of 1.

**d** 'Zero is a multiple of every number.' Explain why this statement is true.

**16** Write down the factors of 6, excluding 6 itself, and then add them up. Do the same for 28. A number that is the sum of its own factors (excluding the number itself) is called a **perfect number**.

The next perfect number is 496. Find and add the factors of 496 to show that it is a perfect number.

**17** **a** Explain why 25 has an odd number of factors.

**b** What is the next whole number to have an odd number of factors?

**c** What is the largest two-digit number with an odd number of factors?

### Did you know?

Pairs of numbers are called **amicable** if each is equal to the sum of the factors of the other, excluding the number itself. The numbers 220 and 284 are the first amicable pair.

The sum of the factors of 220 is:

$1 + 2 + 4 + 71 + 142 = 220$

The sum of the factors of 284 is:

$1 + 2 + 4 + 5 + 10 + 11 + 20 + 22 + 44 + 55 + 110 = 284$


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# 2B Odd and even numbers

You will have met odd and even numbers previously. A whole number is **even** if it is a multiple of 2. The even numbers are 0, 2, 4, 6, 8, …

The **odd** numbers are the whole numbers that are not even. An odd number is one greater than or one less than an even number. The odd numbers are 1, 3, 5, 7, …

Every whole number is either odd or even.

From your rectangular arrays for the numbers 1 to 20, you can see that even numbers can always be represented by arrays with two rows or two columns.

For example:



$12 = 2 \times 6$



$20 = 2 \times 10$

Odd numbers cannot be represented by rectangular arrays with 2 rows or columns.

For example:



$13 = 2 \times 6 + 1$



$21 = 2 \times 10 + 1$

There is always one lone dot.

### Odd and even numbers

- Every whole number is either odd or even.
- **Odd** numbers end in the digits 1, 3, 5, 7 or 9.
- **Even** numbers are multiples of 2 and end in the digits 0, 2, 4, 6 or 8.

### Example 4

List the odd numbers that are factors of 12.

### Solution

The factors of 12 are 1, 2, 3, 4, 6 and 12.

The odd factors of 12 are 1 and 3.


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**Example 5**

Write two consecutive odd numbers that add to 568.

**Solution**

We split 568 in halves: $568 \div 2 = 284$.

$284 + 284 = 568$ (Both the numbers are even.)

Hence, $283 + 285 = 568$ (Subtract 1 from one of the numbers and add 1 to the other.)

## Exercise 2B

Example 4

**1** **a** List the odd numbers that are factors of 30.

**b** List the even numbers that are factors of 30.

**c** List the even multiples of 3 that are less than 60.

**2** **a** Write down all the odd numbers between 20 and 34.

**b** Write down all the even numbers between 375 and 393.

**c** How many odd whole numbers are less than 100?

**d** How many even whole numbers are less than 21?

Example 5

**3** Write two consecutive odd numbers that add to:

**a** 432 **b** 984 **c** 1028

**4** Write two different even numbers, greater than 1, and one odd number that add to:

**a** 633 **b** 1001 **c** 2397

**5** Complete each sentence by inserting 'odd' or 'even' to make a true statement.

**a** The sum of two even numbers is an _____ number.

**b** The sum of two odd numbers is an _____ number.

**c** The sum of an odd number and an even number is an _____ number.

**6** **a** Write down these products:

**i** $6 \times 8 =$ ___ **ii** $6 \times 5 =$ ___ **iii** $7 \times 5 =$ ___

**b** Make each statement true by adding the word 'odd' or 'even'.

**i** The product of two even numbers is an _____ number.

**ii** The product of two odd numbers is an _____ number.

**iii** The product of an odd number and an even number is an _____ number.


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**7** Olivia notices that writing reflected in one mirror is unreadable, but that writing reflected in two mirrors is readable again. State whether it would be readable or unreadable after:

**a** 5 reflections **b** 8 reflections **c** 15 reflections **d** 2224 reflections

**8** Hosni has lined up all his toy soldiers on a table in front of a mirror. He counts all the soldiers he can see, including those in the mirror. Will the number of soldiers be an odd or an even number?

# 2C Prime and composite numbers

A **prime number** is a whole number greater than 1 that has no factors other than 1 and itself. For example:

- The number 2 is a prime number as the only factors of 2 are 2 and 1.
- The number 5 is a prime number as the only factors of 5 are 5 and 1.
- The number 6 is not a prime number. It has factors 1, 2, 3 and 6.

The first few prime numbers are 2, 3, 5, 7 and 11.

The rectangular arrays of prime numbers must be single rows or single columns. The only prime number that is even is 2.

Two: ● ●  Three: ● ● ●

A whole number greater than 1 that has more than two factors is called a **composite number**. The numbers 1 and 0 are special numbers because they are neither prime nor composite.

### Prime and composite numbers

- A whole number greater than 1 that has only two factors, 1 and itself, is called a **prime number**.
- A whole number greater than 1 that has more than two factors is known as a **composite number**.
- The numbers 1 and 0 are special numbers because they are neither prime nor composite.

### Example 6

List the prime numbers that are factors of:

**a** 30 **b** 27

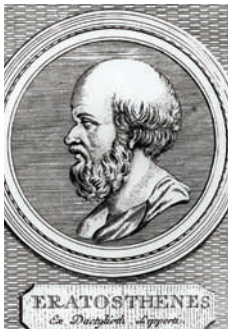
### Solution

**a** The factors of 30 are 1, 2, 3, 5, 6, 10, 15 and 30. The prime factors of 30 are 2, 3 and 5.

**b** The factors of 27 are 1, 3, 9 and 27. The only prime factor of 27 is 3.


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Eratosthenes (278–195 BC) is known as the scholar who found a way to estimate the circumference of the Earth. He is credited with inventing the prime number 'sieve' developed in Exercise 2C.




## Exercise 2C

**Primes less than 100 – the prime number 'sieve'**

- Draw a 100-square chart like the one below.
- Colour the squares that contain even numbers greater than 2.
- Colour the squares that contain numbers greater than 3 that are divisible by 3.
- Colour the squares that contain numbers greater than 5 that are divisible by 5.
- Colour the squares that contain numbers greater than 7 that are divisible by 7.

The 25 numbers that are left are the prime numbers less than 100.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

Example 6

**1** List the prime numbers that are factors of:

**a** 12 **b** 15 **c** 21 **d** 32 **e** 35

**2** What is the first pair of primes that differs by:

**a** 1? **b** 2? **c** 4? **d** 6?

**3** What is the largest prime number less than:

**a** 50? **b** 70? **c** 100?


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**4** Find two prime numbers whose sum is an odd number.

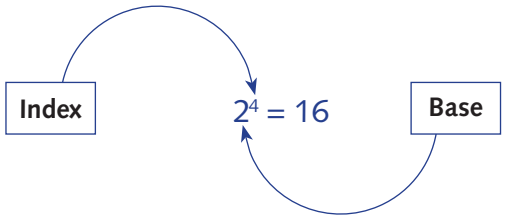
**5** Goldbach's conjecture states that 'every even number greater than 2 is the sum of two primes'. Show that this is the case for even numbers from 32 to 62. (A conjecture is a mathematical statement that is thought to be true but for which no proof is known.)

**6** Find the smallest odd number greater than 3 that is **not** the sum of two primes.

# 2D Powers of numbers

In Chapter 1 we looked at powers of 10. The notation for powers is useful when we are considering factors of a number. For example, we can write 16 as a product of 2s: $16 = 2 \times 2 \times 2 \times 2$.

Written in index notation, $16 = 2^4$, the 4 is the **index**. (The plural of *index* is *indices*.)




The whole expression $2^4$ is called a **power**. It is the fourth power of 2, and we say 'two to the fourth'. The 2 here is called the **base**.

First we will look at powers of the number 3.

$3^1 = 3$

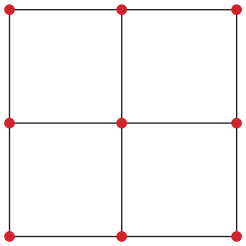
We read this as 'three to the power of one'.



This can be represented using a single row of 3 dots.

$3^2 = 3 \times 3$
$= 9$

We read $3^2$ as 'three to the power of two' or 'three squared'.

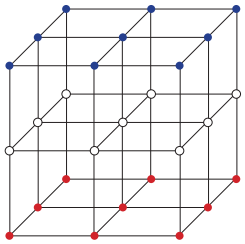


A square number can be illustrated by a square array of rows of dots. The 9 dots shown here form a square. This is an array with 3 rows of 3 dots.

$3^3 = 3 \times 3 \times 3$
$= 27$

We read $3^3$ as 'three to the power of three' or 'three cubed'.


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Cube numbers can be shown by layers and rows of dots.

These 27 dots form a cube. This is a three-dimensional array with 3 layers of 3 rows of 3 dots.

$3^4 = 3 \times 3 \times 3 \times 3$
$= 81$

We read $3^4$ as 'three to the power of four' or 'three to the fourth'.

$3^5 = 3 \times 3 \times 3 \times 3 \times 3$
$= 243$

We read $3^5$ as 'three to the power of five' or 'three to the fifth'.

## A rule for multiplying powers of a given number

There is a simple rule for multiplying two powers with the same base:

$$3^2 \times 3^3 = (3 \times 3) \times (3 \times 3 \times 3) \qquad 5^2 \times 5^1 = (5 \times 5) \times 5$$
$$= 3^5 \qquad = 5^3$$

$$10^3 \times 10^5 = (10 \times 10 \times 10) \times (10 \times 10 \times 10 \times 10 \times 10)$$
$$= 10^8$$

When we multiply two powers with the same base, we add the indices. Here is another example:

$$7^{11} \times 7^6 = 7^{17}$$

## Order of operations

Order of operations was considered in Chapter 1. An important rule is that if there are no brackets, calculate the index first. For example:

$$3 \times 5^2 = 3 \times 25$$
$$= 75$$

If there are brackets, work out the calculation inside the brackets first. For example:

$$(3 \times 5)^2 = 15^2$$
$$= 225$$

### Powers

- The powers of 3 are:

  $3^1 = 3$
  $3^2 = 3 \times 3$
  $3^3 = 3 \times 3 \times 3$
  $3^4 = 3 \times 3 \times 3 \times 3$

  and so on.

- When multiplying two powers of the same number, we add the indices. For example:
  $3^5 \times 3^8 = 3^{13}$


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### Example 7

Write down and evaluate all the powers of 4 up to $4^4$.

#### Solution

$4^1 = 4$
$4^2 = 4 \times 4 = 16$
$4^3 = 4 \times 4 \times 4 = 64$
$4^4 = 4 \times 4 \times 4 \times 4 = 256$

### Example 8

Copy and complete each statement by filling in the index in the box. Use the rule 'To multiply powers of the same number, add the indices'.

**a** $5^2 \times 5^3 = 5^{\square}$

**b** $7 \times 7^3 \times 7^4 = 7^{\square}$

#### Solution

**a** $5^2 \times 5^3 = 5^{2+3}$
$= 5^5$

**b** $7 \times 7^3 \times 7^4 = 7^{1+3+4}$
$= 7^8$

### Example 9

Write $6^3 \times 6^4$ as a power of 6 using the rule.

#### Solution

$6^3 \times 6^4 = 6^{3+4}$
$= 6^7$

## Exercise 2D

Example 7

**1** Write down and evaluate all the powers of 2 up to $2^{10} = 1024$.

**2** Evaluate all the powers of 3 up to $3^5$.

**3** Evaluate all the powers of 5 up to $5^5$.


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**4** Rewrite each expression in index notation.

**a** $4 \times 4 \times 4 \times 4 \times 4$

**b** $12 \times 12 \times 12 \times 12 \times 12 \times 12 \times 12 \times 12 \times 12$

**c** $5 \times 5 \times 6 \times 6 \times 6$

**d** $7 \times 7 \times 7 \times 11 \times 11 \times 11 \times 11$

Example 8

**5** Copy and complete each statement by filling in the index in the box. Use the rule 'To multiply powers of the same number, add the indices'.

**a** $2^2 \times 2^3 = 2^{\square}$ **b** $6^5 \times 6^3 = 6^{\square}$ **c** $4^7 \times 4^3 = 4^{\square}$

**d** $3^8 \times 3^8 = 3^{\square}$ **e** $7^6 \times 7 = 7^{\square}$ **f** $11 \times 11^4 = 11^{\square}$

**g** $5^4 \times 5^4 \times 5^4 = 5^{\square}$ **h** $9 \times 9 \times 9^4 = 9^{\square}$ **i** $12 \times 12^{20} \times 12 \times 12^2 = 12^{\square}$

**6** Copy and complete each statement.

**a** $2^3 \times 2^{\square} = 2^8$ **b** $3 \times 3^{\square} = 3^{10}$

**c** $7^{\square} \times 7^2 = 7^6$ **d** $8^{\square} \times 8^5 = 8^6$

**e** $2^{\square} \times 2^2 \times 2^2 = 2^6$ **f** $6 \times 6 \times 6^{\square} = 6^5$

**7** Rewrite each expression in index notation.

**a** $3 \times 3 \times 5 \times 5$ **b** $6^2 \times 6 \times 7^3 \times 7$

**c** $2 \times 2 \times 2 \times 2^2 \times 3 \times 3 \times 3^3$ **d** $2 \times 3 \times 2 \times 3 \times 2 \times 3$

**e** $2 \times 5 \times 7 \times 5 \times 7 \times 7$ **f** $3^2 \times 7^2 \times 3 \times 7^3$

Example 9

**8** Evaluate:

**a** $6^2 \times 6$ **b** $4 \times 4^2$ **c** $2^3 \times 2$

**d** $5^2 \times 5^3$ **e** $5 \times 5^2$ **f** $7^2 \times 7^3$

**9** Evaluate:

**a** $3^2 \times 2$ **b** $(3 \times 2)^2$ **c** $4 \times 5^2$

**d** $(4 \times 5)^2$ **e** $2 \times 2^3$ **f** $(2 \times 2)^3$

**g** $(3 \times 10^3) \times (2 \times 10^2)$ **h** $(4 \times 10^2) \times (7 \times 10^5)$

**10** **a** How many zeros are there in the number $10^{10} \times 10^{20} \times 10^{30}$?

**b** How many zeros are there in the number $(2 \times 10^5) \times (3 \times 10^6) \times (7 \times 10^7)$?

**11** In question 1, you showed that $2^{10} = 1024$. Use long multiplication to find $2^{20}$.


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# 2E Using mental strategies to multiply and divide

What we know about multiplication and powers can help us perform mental calculations more efficiently. It is easy to double a number or multiply it by 10. Hence, some of these mental strategies are based on the fact that 2 and 5 are factors of many numbers.

## Multiplying by 4 and 8 using powers of 2

Since $4 = 2 \times 2 = 2^2$, you can multiply by 4 by multiplying by 2 twice.

That is, you can double and double again. For example:

$$\begin{aligned} 34 \times 4 &= 34 \times 2 \times 2 \\ &= 68 \times 2 \\ &= 136 \end{aligned}$$

Since $8 = 2 \times 2 \times 2 = 2^3$, you can multiply by 8 by multiplying by 2 three times. That is, you can double, then double again and then double again. For example:

$$\begin{aligned} 13 \times 8 &= 13 \times 2 \times 2 \times 2 \\ &= 26 \times 2 \times 2 \\ &= 52 \times 2 \\ &= 104 \end{aligned}$$

## Multiplying by 5

Since $5 = 10 \div 2$, first multiply by 10 and then divide by 2. For example:

$$\begin{aligned} 36 \times 5 &= 36 \times 10 \div 2 \\ &= 360 \div 2 \\ &= 180 \end{aligned}$$

## Mentally grouping powers of 5 and 2

You can pair powers of 5 and 2 to simplify mental multiplication. For example:

$$\begin{aligned} 25 \times 14 &= 25 \times 2 \times 7 \\ &= 50 \times 7 \\ &= 350 \end{aligned}$$

$$\begin{aligned} 75 \times 6 &= 75 \times 2 \times 3 \\ &= 150 \times 3 \\ &= 450 \end{aligned}$$

## Mentally dividing by 5

To divide an even number by 5, first divide by 10 and then multiply by 2. For example:

$$\begin{aligned} 470 \div 5 &= (470 \div 10) \times 2 \qquad \text{(First divide by 10, then multiply by 2.)} \\ &= 94 \end{aligned}$$

If you prefer, multiply by 2 and then divide by 10. For example:

$$\begin{aligned} 470 \div 5 &= (470 \times 2) \div 10 \qquad \text{(First multiply by 2, then divide by 10.)} \\ &= 94 \end{aligned}$$


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## Mentally dividing by 4

To divide by 4, halve the result and halve it again. For example:

$$\begin{aligned} 428 \div 4 &= 214 \div 2 \\ &= 107 \end{aligned}$$

## Doing several divisions mentally

Sometimes you can make divisions simpler by performing a chain of divisions. For example:

$$\begin{aligned} 288 \div 6 &= (288 \div 2) \div 3 \\ &= 144 \div 3 \\ &= 48 \end{aligned}$$

You do not have to use brackets, but it makes the process clearer. You can often use this strategy in mental calculations.

## Using doubling and halving

Sometimes it is possible to multiply two numbers by doubling one number and halving the other. Keep doubling and halving until the numbers are manageable. For example:

$$\begin{aligned} 34 \times 5 &= 17 \times 10 \\ &= 170 \end{aligned}$$

$$\begin{aligned} 44 \times 3 &= 22 \times 6 \\ &= 11 \times 12 \\ &= 132 \end{aligned}$$

## Multiplying by 9 mentally

To multiply a number by 9, multiply the number by 10 and then take away the original number from the result. For example:

$$\begin{aligned} 128 \times 9 &= 128 \times (10 - 1) \\ &= 128 \times 10 - 128 \\ &= 1280 - 128 \\ &= 1152 \end{aligned}$$

## Multiplying two-digit numbers by 11 mentally

To multiply a number by 11, multiply the number by 10 and add the number. For example:

$$\begin{aligned} 128 \times 11 &= 128 \times 10 + 128 \\ &= 1280 + 128 \\ &= 1408 \end{aligned}$$

### Example 10

Calculate:

**a** $15 \times 26$ **b** $15 \times 28$ **c** $27 \times 8$

### Solution

**a** $\begin{aligned} 15 \times 26 &= 30 \times 13 \\ &= 390 \end{aligned}$

**b** $\begin{aligned} 15 \times 28 &= 15 \times 4 \times 7 \\ &= 60 \times 7 \\ &= 420 \end{aligned}$

**c** $\begin{aligned} 27 \times 8 &= 54 \times 4 \\ &= 108 \times 2 \\ &= 216 \end{aligned}$


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### Example 11

Calculate:

**a** $52 \times 11$ **b** $68 \times 9$

### Solution

**a** $52 \times 11 = 52 \times (10 + 1)$
$= 520 + 52$
$= 572$

**b** $68 \times 9 = 68 \times (10 - 1)$
$= 68 \times 10 - 68$
$= 680 - 68$
$= 612$

### Example 12

Calculate:

**a** $864 \div 16$ **b** $1250 \div 50$

### Solution

**a** $864 \div 16 = 432 \div 8$
$= 216 \div 4$
$= 108 \div 2$
$= 54$

This shows that dividing by 16 can be done by dividing by 2 four times, because $16 = 2 \times 2 \times 2 \times 2$.

**b** $1250 \div 50 = 125 \div 5$
$= 25$

*or*

$1250 \div 50 = 2500 \div 100$
$= 25$

## Exercise 2E

Example 10

**1** Show your working to make clear the steps you would take to do these calculations mentally.

**a** $15 \times 14$ **b** $5 \times 16$ **c** $5 \times 18$

**d** $5 \times 24$ **e** $15 \times 24$ **f** $15 \times 36$

Example 10

**2** Show your working to make clear the steps you would take to do these calculations mentally.

**a** $24 \times 4$ **b** $112 \times 4$ **c** $532 \times 4$

**d** $42 \times 8$ **e** $131 \times 8$ **f** $504 \times 8$


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**3** Show your working to make clear the steps you would take to do these calculations mentally.

**a** $125 \times 6$ **b** $19 \times 6$ **c** $24 \times 37$ **d** $25 \times 24$

**e** $24 \times 28$ **f** $32 \times 32$ **g** $18 \times 6$ **h** $37 \times 8$

Example 11

**4** Show your working to make clear the steps you would take to do these calculations mentally.

**a** $62 \times 11$ **b** $48 \times 11$ **c** $99 \times 11$ **d** $87 \times 9$ **e** $421 \times 9$

Example 12

**5** Show your working to make clear the steps you would take to do these calculations mentally.

**a** $430 \div 5$ **b** $870 \div 5$ **c** $635 \div 5$ **d** $136 \div 4$

**e** $108 \div 4$ **f** $408 \div 8$ **g** $264 \div 8$ **h** $126 \div 6$

**i** $186 \div 6$ **j** $762 \div 6$ **k** $168 \div 4$ **l** $1512 \div 27$

**6** Show your working to make clear the steps you would take to do these calculations mentally.

**a** $125 \times 32$ **b** $135 \times 22$ **c** $2750 \div 50$

**d** $1344 \div 16$ **e** $367 \times 9$ **f** $48 \times 5$

# 2F Using powers in factorisation

Prime numbers are the building blocks from which all whole numbers are made. The building is done by multiplication.

Any number can be expressed as a product of powers of prime numbers. For example:

$$\begin{aligned} 12 &= 3 \times 4 \\ &= 3 \times 2^2 \\ &= 2^2 \times 3 \end{aligned}$$

$$\begin{aligned} 72 &= 8 \times 9 \\ &= 2^3 \times 3^2 \end{aligned}$$

$$\begin{aligned} 100 &= 10 \times 10 \\ &= 5 \times 2 \times 5 \times 2 \\ &= 5^2 \times 2^2 \\ &= 2^2 \times 5^2 \end{aligned}$$

We say that $2^2 \times 5^2$ is **the prime factorisation** of 100 since the primes are in increasing order.

In fact, every whole number greater than 1 is either prime or can be written as a product of prime numbers. Also, the factorisation for each number is unique, apart from the order in which the factors are written. Taken together, these results form an important fact known as the **fundamental theorem of arithmetic**.


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## Finding prime factors

Three different methods for finding the prime factors of numbers are shown below.

### Method 1: Repeated factorisation

$$\begin{aligned} 420 &= 42 \times 10 \\ &= 7 \times 6 \times 5 \times 2 \\ &= 2^2 \times 3 \times 5 \times 7 \end{aligned}$$

### Method 2: Repeated division

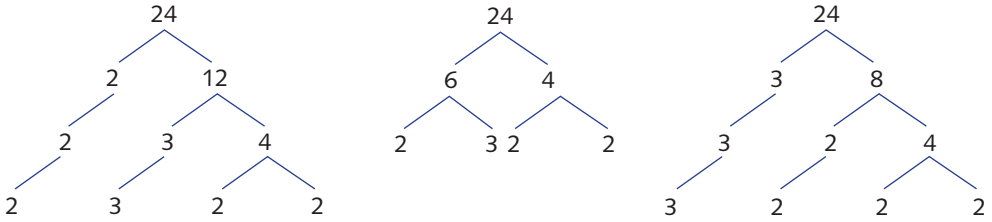
To find the prime factorisation of 420, we divide by the lowest prime factor as many times as we can first, then divide by the next lowest and so on until the quotient is 1.

| | |
|---|---|
| 2 | 420 |
| 2 | 210 |
| 3 | 105 |
| 5 | 35 |
| 7 | 7 |
| | 1 |

So the prime factorisation of $420 = 2^2 \times 3 \times 5 \times 7$.

### Method 3: Factor trees

To find the prime factorisation of 24, we can use a factor tree diagram. In the first example below, 24 is split into its largest and smallest factors (besides 24 and 1), 12 and 2. Then each of these is split into factors until no further splitting is possible.




It is not necessary to start with the smallest and largest factors other than the number itself and 1. It is helpful to start with a factor you know. Although a number can have more than one factor tree, each factor tree will have the same prime factors in the last row.

From the diagrams above, the prime factorisation of 24 is:

$$\begin{aligned} 24 &= 2 \times 2 \times 2 \times 3 \\ &= 2^3 \times 3 \end{aligned}$$

### Fundamental theorem of arithmetic

- Every whole number greater than 1 is either prime or can be written as a product of prime numbers.
- The prime factorisation for each number is unique, apart from the order in which the factors are written.


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### Example 13

Express 72 as a product of prime factors and give your answer in index notation.

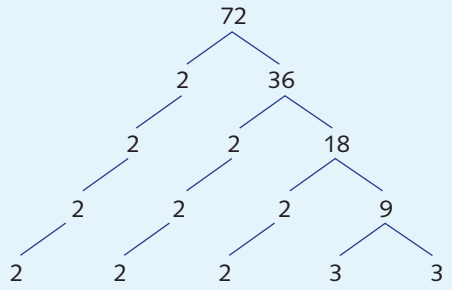
### Solution

Divide by the lowest prime number, then continue dividing by primes until the result is 1.

| | |
|---|---|
| 2 | 72 |
| 2 | 36 |
| 2 | 18 |
| 3 | 9 |
| 3 | 3 |
| | 1 |

$72 = 2^3 \times 3^2$

A second method uses a factor tree to find the prime factors of 72.




$72 = 2^3 \times 3^2$

## Exercise 2F

Example 13

**1** Express each number as a product of prime factors.

**a** 6 **b** 8 **c** 9 **d** 12 **e** 18 **f** 24

**2** Express each number as a product of prime factors.

**a** 15 **b** 75 **c** 36 **d** 96 **e** 256 **f** 841

**3** What is the number given by each of these prime factorisations?

**a** $2^2 \times 3^2 \times 5^2$ **b** $5^2 \times 3^3 \times 2^4$ **c** $2^8 \times 3^2 \times 5^2$

**4** **a** Write 105 and 154 as a product of prime factors.

**b** What is the largest factor of both of these numbers?

**5** Find all two-digit numbers that are the product of three different prime numbers.

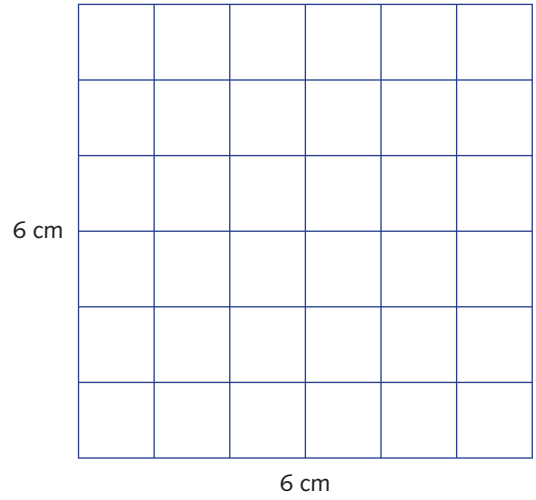

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# 2G Squares and square roots

The **square** of a number is the number multiplied by itself. For example, 6 squared ($6^2$) is the number $6 \times 6 = 36$.

This can also be shown as a diagram. A square of side length 6 cm has an area of:

$$\begin{aligned} 6\text{ cm} \times 6\text{ cm} &= 6^2\text{ cm}^2 \\ &= 36\text{ cm}^2 \end{aligned}$$




The **square root** of a number is the number that when multiplied by itself gives the original number. In this chapter we will only find the square roots of square numbers. For example:

- $36 = 6 \times 6 = 6^2$, so 6 is the square root of 36. We write $\sqrt{36} = 6$.
- $25 = 5 \times 5 = 5^2$, so 5 is the square root of 25. We write $\sqrt{25} = 5$.

Geometrically, the square root of a number is the side length of a square whose area is that number.

## Products of squares and square roots

The product of two or more square numbers is equal to the square of the product of the original numbers. For example:

$$\begin{aligned} 3^2 \times 2^2 &= 3 \times 3 \times 2 \times 2 \\ &= (3 \times 2) \times (3 \times 2) \\ &= 6 \times 6 \\ &= 6^2 \end{aligned}$$

$$\begin{aligned} 8^2 \times 7^2 &= 8 \times 8 \times 7 \times 7 \\ &= (8 \times 7) \times (8 \times 7) \\ &= 56 \times 56 \\ &= 56^2 \end{aligned}$$

The square root of a product is the product of the square roots of the original numbers. For example:

$$\begin{aligned} \sqrt{100} &= \sqrt{25 \times 4} \\ &= \sqrt{25} \times \sqrt{4} \\ &= 5 \times 2 \\ &= 10 \end{aligned}$$

$$\begin{aligned} \sqrt{484} &= \sqrt{4} \times \sqrt{121} \\ &= 2 \times 11 \\ &= 22 \end{aligned}$$


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## Prime factorisation and square roots

Finding the prime factors of a number can help us to determine whether the number is a square number and, if it is, to find its square root.

### Example 14

Find the square root of 2025 by first finding the prime factors of 2025.

#### Solution

| | |
|---|---|
| 5 | 2025 |
| 5 | 405 |
| 3 | 81 |
| 3 | 27 |
| 3 | 9 |
| 3 | 3 |
| | 1 |

We can write

$$\begin{aligned} 2025 &= 5\times5\times3\times3\times3\times3 \\ &= (5\times3\times3)\times(5\times3\times3) \\ &= (5\times3\times3)^2 \end{aligned}$$

so 2025 is a square number and

$$\begin{aligned} \sqrt{2025} &= 3^2\times5 \\ &= 45 \end{aligned}$$

### Squares and square roots

- A number multiplied by itself is called the **square** of the original number.
- The **square root** of a square number is the number that when multiplied by itself gives the original number. For example, $\sqrt{25} = 5$, as $5^2 = 25$.
- The product of the squares is equal to the square of the product. For example: $5^2\times2^2 = (5\times2)^2 = 100$
- The square root of a product of square numbers is the product of the square roots of the original numbers. For example: $\sqrt{49\times25} = 7\times5 = 35$

## Exercise 2G

**1** Evaluate:

**a** $4^2$ **b** $12^2$ **c** $16^2$ **d** $17^2$ **e** $22^2$ **f** $33^2$

Example 14

**2** Find the square root of:

**a** 49 **b** 144 **c** 400 **d** 169 **e** 225 **f** 361

**g** 625 **h** 961 **i** 1444 **j** 5625


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**3** Complete each sentence by filling in the blanks with 'odd' or 'even' to make a true statement.

**a** The square of an even number is an _____ number.

**b** The square of an odd number is an _____ number.

**4** I am thinking of a number that is the sum of two square numbers, each of which is odd. Is my number odd or even?

**5** I am thinking of a number that is the sum of two square numbers. One square number is odd and the other is even. Is my number odd or even?

**6** Write each number as the sum of three, not necessarily different, square numbers.

**a** 14 **b** 38 **c** 45 **d** 59 **e** 70 **f** 230

**7** Write 51 as the difference of two squares.

**8** Show that 85 can be written both as the sum of two squares and the difference of two squares.

**Did you know?**

The number 81 is the only number greater than 1 whose square root equals the sum of its digits.

# 2H Lowest common multiple and highest common factor

## Lowest common multiple

Look at the lists of positive multiples of 24 and 18. Common multiples of 24 and 18 are highlighted by arrows.

The smallest or lowest of the non-zero common multiples of 24 and 18 is 72. We say that 72 is the **lowest common multiple (LCM)** of 24 and 18.

| Positive multiples of 24 | Positive multiples of 18 |
|---|---|
| 24 | 18 |
| 48 | 36 |
| 72 | 54 |
| 96 | 72 |
| 120 | 90 |
| 144 | 108 |
| 168 | 126 |
| 192 | 144 |
| ⋮ | ⋮ |


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**Example 15**

What is the LCM of 180 and 144?

**Solution**

Multiples of 180 are 180, 360, 540, [720], 900, ….

Multiples of 144 are 144, 288, 432, 576, [720], 864, …

So the LCM of 180 and 144 is 720.

## Highest common factor

Look at the list of factors of 24 and 18. The factors common to both 24 and 18 are highlighted with arrows. The largest or highest factor common to both numbers is 6. We say that 6 is the **highest common factor (HCF)** of 24 and 18.

| Factors of 24 | Factors of 18 |
|---|---|
| 1 | 1 |
| 2 | 2 |
| 3 | 3 |
| 4 | 6 |
| 6 | 9 |
| 8 | 18 |
| 12 | |
| 24 | |

**Example 16**

What is the HCF of 180 and 144?

**Solution**

The factors of 180 are 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 30, [36], 45, 60, 90 and 180.

The factors of 144 are 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, [36], 48, 72 and 144.

So 36 is the highest common factor of 180 and 144.

### LCM and HCF

- The **lowest common multiple (LCM)** of two or more numbers is the smallest non-zero whole number that is a multiple of each number.
- The **highest common factor (HCF)** of two or more numbers is the largest number that is a factor of each number.


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## Exercise 2H

Example 15

**1** Find the LCM of:

**a** 8 and 12 **b** 8 and 9 **c** 17 and 1

**d** 12 and 15 **e** 7 and 49 **f** 20 and 90

**g** 3, 4 and 6 **h** 8, 9 and 12 **i** 12, 8, 10 and 30

Example 16

**2** Find the HCF of:

**a** 30 and 24 **b** 15 and 21 **c** 12 and 72

**d** 25 and 16 **e** 36 and 16 **f** 26 and 65

**g** 12, 18 and 30 **h** 15, 6 and 14 **i** 60, 20 and 10

**3** Find the HCF and LCM of:

**a** 224 and 336 **b** 18 and 42 **c** 45 and 150

**4** A bell rings every 15 minutes and a whistle is blown every 18 minutes. The bell is rung and the whistle is blown at 8 a.m. How long will it be before the bell is rung and the whistle blown at the same time again?

**5** Three patients visit the doctor at intervals of 8 days, 15 days and 24 days, respectively. If they all go to the doctor on 1 March, what will be the date when they next all go to the doctor on the same day?

**6** Three swimmers take 28 seconds, 44 seconds and 68 seconds to complete a lap of the pool. If they start together, how long will it be before they are side by side at one end of the pool again?

**7** A rectangular region of dimensions 30 m by 28 m is covered by square tiles. What is the smallest number of tiles required?

# 2I Divisibility tests

A number is **divisible** by a second number if the second number divides into the first with no remainder. This is easy to see with small numbers. For example, 12 is divisible by 2, since $12 \div 2 = 6$.

What about large numbers? It is helpful to know how to test for divisibility without actually having to divide.

You will already know some of these tests.

- A number is divisible by 10 if it ends in 0. For example, 20, 450 and 23 890 are all divisible by 10.
- A number that ends in 0, 2, 4, 6 or 8 is divisible by 2.
- If the last digit of a number is 5 or 0, then the number is divisible by 5. For example, 15, 255 and 439 780 are all divisible by 5.


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There is no simple test for divisibility by 7, but there are simple tests of divisibility by 3 and 9, 4 and 8, and 6.

## Divisibility tests for 3 and 9

A number is:

- divisible by 3 if the sum of its digits is divisible by 3
- divisible by 9 if the sum of its digits is divisible by 9.

For example, 27 is divisible by 3 and 9 as the sum of the digits is 9.

This is because $10 = 9 + 1$, $100 = 99 + 1$, $1000 = 999 + 1$ and so on, *and* 9, 99, 999 and so on are all divisible by both 3 and 9. For example:

$$\begin{aligned} 21 &= 2 \times 10 + 1 \\ &= 2 \times (9 + 1) + 1 \\ &= 2 \times 9 + (2 + 1) \end{aligned}$$

The sum of $2 + 1 = 3$ is divisible by 3, and $2 \times 9 = 18$ is also divisible by 3.

Another example:

$$\begin{aligned} 378 &= 3 \times (99 + 1) + 7 \times (9 + 1) + 8 \\ &= 3 \times 99 + 7 \times 9 + 3 + 7 + 8 \end{aligned}$$

The sum of $3 + 7 + 8 = 18$ is divisible by 9, and $3 \times 99$ and $7 \times 9$ are divisible by 9.
37 is a number that is divisible by *neither* 3 nor 9, as the sum of the digits is 10.

## Divisibility tests for 4 and 8

A number is:

- divisible by 4 if its last two digits are a number that is divisible by 4
- divisible by 8 if its last three digits are a number that is divisible by 8.

## Divisibility test for 6

We know that $2 \times 3 = 6$.

Hence, if a number is both divisible by 3 and 2, then it is divisible by 6.

### Divisibility tests

- A number is divisible by 2 if it ends in 0, 2, 4, 6 or 8.
- A number is divisible by 3 if the sum of its digits is divisible by 3.
- A number is divisible by 4 if its last two digits form a number that is divisible by 4.
- A number is divisible by 5 if it ends in 0 or 5.
- A number is divisible by 6 if it is both even and divisible by 3.
- A number is divisible by 8 if its last three digits form a number that is divisible by 8.
- A number is divisible by 9 if the sum of its digits is divisible by 9.
- A number is divisible by 10 if its last digit is 0.


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### Example 17

Is the number 493 756 divisible by 2, 3, 4, 5, 6, 8, 9 and 10?

### Solution

| Number | Test | Divisible? |
|---|---|---|
| 2 | 493 756 ends in 6, so it is divisible by 2. | Yes |
| 3 | The sum of the digits of 493 756 is 4 + 9 + 3 + 7 + 5 + 6 = 34, which is not divisible by 3, so 493 756 is not divisible by 3. | No |
| 4 | The last two digits of 493 756 form the number 56, which is divisible by 4, so 493 756 is divisible by 4. | Yes |
| 5 | 493 756 does not end in 5 or 0, so it is not divisible by 5. | No |
| 6 | Although 493 756 is even, it is not divisible by 3. Therefore, it is not divisible by 6. | No |
| 8 | The last three digits of 493 756 form the number 756, which is not divisible by 8. Therefore, 493 756 is not divisible by 8. | No |
| 9 | The sum of the digits of 493 756 is 4 + 9 + 3 + 7 + 5 + 6 = 34, which is not divisible by 9. Therefore, 493 756 is not divisible by 9. | No |
| 10 | 493 756 does not end in 0, so it is not divisible by 10. | No |

## Exercise 21

**1** Test each number for divisibility by 2.

**a** 536 **b** 22 253 **c** 1782 **d** 188

**2** Test each number for divisibility by 4.

**a** 72 **b** 236 **c** 682 **d** 36 338 **e** 387 232

**3** Test each number for divisibility by 8.

**a** 96 **b** 1128 **c** 387 480 **d** 22 500

**4** Test each number for divisibility by 3.

**a** 48 **b** 543 **c** 8987 **d** 864 468 **e** 1 001 001

**5** Test each number for divisibility by 9.

**a** 108 **b** 33 543 **c** 5652 **d** 3 000 015 **e** 1 357 911

**6** Which of the five numbers below is divisible by:

**a** 3? **b** 9?

179 792 5838 45 891 19 283 746 556 000 000 001


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Example 17

**7** Which of the numbers below divide the number 584 760?

2, 3, 4, 5, 6, 8, 9, 10

**8** Which of the numbers below divide the number 38 190 306?

2, 3, 4, 5, 6, 8, 9, 10

**9** Find the smallest number that is greater than 1000 and divisible by:

**a** 3 **b** 8 **c** 4 **d** 6 **e** 3 and 4 **f** 3, 4 and 5

**10** Fill in the gaps in the five-digit number 7□42□ to make a number that is divisible by:

**a** 3 **b** 4 **c** 5

**d** 6 **e** 9 **f** 4 and 5

**g** 3 and 4 **h** 4 and 9 **i** 9 and 10

**j** 3 and 5, but not 2 **k** 2, 3, 4 and 5, but not 9 **l** 3 and 5, but not 6

**11** Use the various divisibility tests and short division to help you find all the factors of:

**a** 147 **b** 345 **c** 2688

**12** What are the smallest and largest three-digit numbers that are:

**a** multiples of 9? **b** multiples of 23? **c** multiples of both 9 and 23?

**13** **a** Find the number of whole numbers less than 100 that are divisible by 2 or 3, or both.

**b** Find the number of whole numbers less than 100 that are **not** divisible by 2 or 3.

**c** Find the number of whole numbers less than 500 that are **not** divisible by 2 or 3.

**14** Find the least whole number divisible by 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10.

# Review exercise

**1** List all the factors of the following numbers in order from smallest to largest.

**a** 18 **b** 24 **c** 25 **d** 48 **e** 50 **f** 96

**2** List the multiples of the following numbers that are less than 100.

**a** 9 **b** 17 **c** 33 **d** 18 **e** 15 **f** 21

**3** Write one number that:

**a** is odd, greater than 50 and a multiple of 7

**b** is even, divisible by 5 and between 106 and 150

**c** has 96 and 72 as multiples, is odd and prime.

**4** Write three different three-digit numbers that are odd and sum to:

**a** 871 **b** 903 **c** 479 **d** 347 **e** 1835 **f** 2851


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**5** Write down and evaluate all the powers of 6 up to $6^6$.

**6** Write down and evaluate all the powers of 9 up to $9^5$.

**7** Write each of the following using index notation.

**a** $3 \times 3 \times 3 \times 4 \times 4 \times 4 \times 4$ **b** $8 \times 8 \times 8 \times 8^4$

**c** $100 \times 100 \times 100$ **d** $7 \times 7 \times 7 \times 3 \times 3 \times 3$

**8** Evaluate:

**a** $3 \times 3 \times 2^2$ **b** $(3 \times 3 \times 2)^2$ **c** $3 \times (3 \times 2)^2$

**d** $(3 \times 3)^2 \times 2$ **e** $(2^3 \times 2^3)^2$ **f** $(5 \times 2)^3$

**9** Do each of these calculations mentally.

**a** $17 \times 4$ **b** $17 \times 8$ **c** $17 \times 5$

**d** $17 \times 12$ **e** $17 \times 20$ **f** $17 \times 25$

**g** $436 \div 4$ **h** $620 \div 5$ **i** $312 \div 6$

**10** Evaluate:

**a** $5^2$ **b** $10^2$ **c** $13^2$ **d** $15^2$ **e** $21^2$ **f** $43^2$

**11** Find the square root of:

**a** 400 **b** 1600 **c** 900 **d** 225 **e** 484 **f** 361

**12** Write the number that is divisible by 9 and is closest to:

**a** 50 **b** 100 **c** 150

**d** 200 **e** 250 **f** 300

**13** What are the smallest and largest four-digit numbers that are:

**a** multiples of 9?

**b** multiples of 29?

**c** multiples of both 9 and 29?

**14** **a** How many zeros are there in the number $2^2 \times 5^2$?

**b** How many zeros are there in the number $2^3 \times 5^3$?

**c** How many zeros are there in the number $2^4 \times 5^3$?

**d** How many zeros are there in the number $2^3 \times 5^4$?

**e** How many zeros are there in the number $2^{20} \times 5^{20}$?

**f** How many zeros are there in the number $2^{21} \times 5^{20}$?

**g** How many zeros are there in the number $2^{23} \times 5^{20}$?

**15** Express each of the following as a product of prime factors.
Use index notation where appropriate.

**a** 512 **b** 1024 **c** 125 **d** 500 **e** 1001 **f** 17 017

**g** 19 019 **h** 4000 **i** 5120 **j** 243 **k** 1225 **l** 2666


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**16** What is the number given by these prime factorisations?

**a** $2^3 \times 5^3$ **b** $7 \times 11 \times 13$ **c** $7 \times 11 \times 13 \times 29$

**d** $2^3 \times 3^3 \times 5^3$ **e** $2^4 \times 7 \times 5^3$ **f** $2^3 \times 7^2 \times 5^2$

**g** $5^3 \times 7^3 \times 2^3$ **h** $5^2 \times 7^2$ **i** $2 \times 5^2 \times 7^2$

**17** Find all whole numbers less than 50 that are the product of two different prime numbers.

# Strengthening multiplication tables skills

You need to know the multiplication tables up to 12 to work efficiently with numbers. Think carefully about how well you know each fact. Can you recall it very, very quickly off the top of your head? If not, then you need to work on that fact.

Draw up a chart like the one below and colour in:

- the multiples of 1
- each multiplication fact that is a duplicate of another in the table (For example, $4 \times 3 = 3 \times 4 = 12$, so you have just halved the number of facts that you need to remember.)
- the multiples of 2, because you know your doubles
- the multiples of 10, because they end in 0
- the multiples of 5, because they end in 5 or 0
- the square numbers, for example, $1 \times 1$, $2 \times 2$, $3 \times 3$
- the multiples of 11 up to $10 \times 11$
- the multiples of 3. (Just double the number and add the number again. So for 7: double 7 is 14, plus 7 is 21. For 12: double 12 is 24, plus 12 is 36.)

| × | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 | 48 |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 66 | 72 |
| 7 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 | 77 | 84 |
| 8 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 | 88 | 96 |
| 9 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 | 99 | 108 |
| 10 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| 11 | 11 | 22 | 33 | 44 | 55 | 66 | 77 | 88 | 99 | 110 | 121 | 132 |
| 12 | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | 144 |

Colour in the other facts as you learn them.

The facts that are not coloured in are the ones that you need to work on. There aren't so many of them now, are there?


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# Challenge exercise

**1** The prime numbers 73 and 37 have reversed digits. What are the other pairs of prime numbers less than 100 with the same property?

**2** The 7 button on my calculator does not work. Show how I might use my calculator to work out:

**a** $7 \times 20$ **b** $7 \times 25$ **c** $72 \times 7$ **d** $78 \times 11$

**3** Any number that is divisible by 18 is also divisible by 3 and 6. The opposite is not always true. Write down a number larger than 100 that is divisible by 3 and 6, but not divisible by 18.

**4** **a** A number is called **abundant** if the sum of all of its factors (not including the number itself) is greater than itself. The first abundant number is 12. The sum of its factors, not including 12 itself, is $1 + 2 + 3 + 4 + 6 = 16$. Find five more abundant numbers less than 40.

**b** Show that 945 is abundant. It is the smallest odd abundant number.

**5** The smallest number that can be written as the sum of two non-zero squares in two different ways is 50: $5^2 + 5^2 = 50$ and $7^2 + 1^2 = 50$. Find the other two numbers less than 100 that can be written as the sum of two non-zero squares in two different ways.

**6** The number 36 can be written as the sum of three cubes: $1^3 + 2^3 + 3^3 = 36$. Find the three cubes that sum to:

**a** 73 **b** 92 **c** 99

**7** In each part, continue the pattern, filling in the gaps.

**a** $1^3 + 2^3 = (1 + \square)^2$

**b** $1^3 + \square^3 + 3^3 = (1 + 2 + 3)^2$

**c** $1^3 + 2^3 + 3^3 + 4^3 = (\square + \square + \square + \square)^2$

**d** $\square^3 + 2^3 + \square^3 + 4^3 + 5^3 = (1 + \square + 3 + \square + 5)^2$

**8** Write each number as a sum of no more than 4 cubes.

**a** 10 **b** 1729 **c** 36

**d** 37 **e** 51 **f** 99

**g** 100 **h** 152 **i** 270

**9** Fill in the gaps in the six-digit number 12 $\square$ 94 $\square$ to make a number that is divisible by:

**a** 4 and 10 **b** 3, but not 9 **c** 2 and 5

**d** 5 and 3, but not 4 or 9 **e** 3 and 5, but not 6


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**10** The five-digit number $\square 725 \triangle$ is divisible by 72. Find the values of $\square$ and $\triangle$.

**11** **a** How many factors does $1\,048\,576 = 2^{20}$ have?

**b** How many factors does $729 = 3^6$ have?

**c** How many factors does $2^{20} \times 3^6$ have?

**12** **a** List the primes less than 100. (Hint: There are 25 of them.)

**b** Extend the list to primes less than 200, using the sieve method but including multiples of 11 and 13, of course. (Hint: There should be 21 new primes.)

**c** What is the largest gap between adjacent primes less than 200?

**d** A **prime pair** is a pair of prime numbers that differ by 2. List the 15 prime pairs less than 200.

**e** Show that, with one exception, the sum of a prime pair is divisible by 12, and that the product of a prime pair is one less than a multiple of 36.

**f** Prove that there are no 'prime triples'; that is, that $n$, $n + 2$ and $n + 4$ are never all prime, except for the one case 3, 5, 7.

**13** **a** How many zeros are there in the number obtained from $10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$?

**b** How many zeros are there in the number obtained by multiplying the numbers from 1 to 100 together?

**14** Write each number as the sum of 4 squares.

**a** 30 **b** 71 **c** 130

**d** 216 **e** 630 **f** 654

**15** Write 170 as the sum of the smallest possible number of square numbers without using $1^2 = 1$.

**16** A whole number is called a palindrome if it reads the same backwards as forwards. For example, 12 321 is a palindrome, 12 345 is not. Find the smallest five-digit palindrome divisible by:

**a** 2 **b** 3 **c** 4 **d** 5

**e** 6 **f** 7 **g** 8 **h** 9


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CHAPTER

3

Number and Algebra

# An introduction to algebra

Algebra is an important part of mathematical language. It helps us to state ideas more simply and to make general statements about mathematics in a concise way. It enables us to solve problems that are difficult to do otherwise.

Algebra was developed by Abu Abdullah Muhammad ibn Musa al-Khwarizmi (or simply Al-Khwarizmi) in about 830 CE. His work was also influential in the introduction of algebra into Europe in the early thirteenth century.



Al-Khwarizmi was a scholar at the House of Wisdom in Baghdad. There scholars translated Greek scientific manuscripts and wrote about algebra, geometry and astronomy. Al-Khwarizmi worked under the patronage of the Caliph, to whom he dedicated two of his texts – one on algebra and one on astronomy. The algebra text, *Hisab al-jabr w'al-muqabala*, is the most famous and most important of all of his works. It is the title of this text that gives us the word *algebra*.



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# 3A Using algebra

In algebra, letters are often used to stand for numbers. For example, if a box contains $x$ stones and you put in 5 more stones, then there are $x + 5$ stones in the box. You may or may not know the value of $x$. In algebra, we call such a letter a **pronumeral**.

## Example 1

Joe has a pencil case that contains a number of pencils. He has 3 other pencils. How many pencils does Joe have in total?



### Solution

We do not know how many pencils there are in the pencil case, so let $x$ be the number of pencils in the pencil case.

Joe has a total of $x + 3$ pencils.

## Example 2

Write an algebraic expression for each of the following.

**a** Adding 7 to the number $a$
**b** The sum of $d$ and 7
**c** Subtracting 6 from $m$
**d** 7 less than $x$
**e** The sum of $m$ and 3 and $n$
**f** The difference of $x$ and 6 ($x$ is greater than 6)

### Solution

**a** $a + 7$ **b** $d + 7$ **c** $m - 6$
**d** $x - 7$ **e** $m + 3 + n = m + n + 3$ **f** $x - 6$


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### Example 3

Write an algebraic expression for each of the following.

**a** The product of 3 and $b$ **b** $n$ multiplied by 5

**c** The quotient when $a$ is divided by 6

#### Solution

**a** $3 \times b$ **b** $n \times 5$ **c** $a \div 6$

### Example 4

Write an algebraic expression for each of the following.

**a** 2 times a number, $n$, plus 6 **b** 6 minus 2 times a number, $n$

#### Solution

**a** $2 \times n + 6$ **b** $6 - 2 \times n$

*Note*: The conventions for the order of operations discussed in Chapter 1 also apply in algebra.

### Example 5

Theresa takes 5 chocolates from a box with a large number of chocolates in it. How many chocolates are left in the box?

#### Solution

Let $z$ be the original number of chocolates in the box.

Theresa removes 5 chocolates, so there are $z - 5$ chocolates left in the box.

### Example 6

There are three boxes, each with the same number of marbles in them. If there are $x$ marbles in each box, how many marbles are there in total?

#### Solution



$x$ marbles $x$ marbles $x$ marbles

There are $3 \times x$ marbles.



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**Example 7**

There are $n$ oranges to be divided equally among 5 people.

How many oranges does each person receive?

**Solution**

Each person receives $n \div 5$ oranges.

The following table gives us the meanings of some commonly occurring algebraic expressions.

| | |
|---|---|
| $x + 3$ | • The sum of $x$ and 3<br>• 3 added to $x$, or $x$ added to 3<br>• 3 more than $x$, or $x$ more than 3 |
| $x - 3$ | • The difference of $x$ and 3<br>• 3 subtracted from $x$<br>• 3 less than $x$<br>• $x$ minus 3 |
| $3 \times x$ | • The product of 3 and $x$<br>• $x$ multiplied by 3, or 3 multiplied by $x$ |
| $x \div 3$ | • $x$ divided by 3<br>• The quotient when $x$ is divided by 3 |
| $2 \times x - 3$ | • $x$ is first multiplied by 2 and then 3 is subtracted |
| $x \div 3 + 2$ | • $x$ is first divided by 3 and then 2 is added |

## Expressions with zeros and ones

Zeros and ones can often be eliminated entirely. For example:

$x + 0 = x$ (Adding zero does not change the number.)
$1 \times x = x$ (Multiplying by one does not change the number.)

## Exercise 3A

Example 1

**1** Joan has a pencil case that contains $x$ pencils. She has 3 other pencils. How many pencils does Joan have in total?

Example 2

**2** Write an algebraic expression for each of the following.

**a** Adding 6 to the number $a$

**b** The sum of $p$ and 4

**c** Subtracting 2 from $p$

**d** Subtracting 3 from $y$

**e** The difference of $x$ and 4 (where $x$ is greater than 4)

**f** 8 less than $x$

**g** The sum of $p$ and $q$

**h** The sum of $x$ and 2 and $y$

**i** The sum of $a$, $b$ and $c$


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Example 3

**3** Write an algebraic expression for each of the following.

**a** The product of 5 and $x$

**b** The product of $x$ and $y$

**c** $x$ is multiplied by 3

**d** The quotient when $p$ is divided by $q$

**4** Write an algebraic expression for each of the following.

**a** The product of 7 and $x$

**b** The product of $a$ and $b$

**c** The difference of $y$ and 6 (where $y$ is greater than 6)

Example 4

**d** $x$ is multiplied by 4, and 3 is added to the result

**e** $m$ is multiplied by 5, and 3 is subtracted from the result

**f** 3 is multiplied by $a$, and 2 is added to the result

**5** Write an algebraic expression for each of the following.

**a** $x$ is divided by 3, and 2 is added to it

**b** $x$ is divided by 3, and 2 is subtracted from it

**c** $p$, $q$ and $r$ are added together

**d** The product of $x$, $y$ and $z$

Example 5

**6** An apricot tree has $n$ apricots on it. How many are left on the tree if 20 apricots drop off?

Example 6

**7** There are 5 boxes, each containing $x$ chocolates. What is the total number of chocolates?

Example 7

**8** There are $n$ bananas to be divided equally among 3 people. How many bananas does each person receive?

**9** John has \$$w$ dollars in the bank. He deposits \$1000 in the bank. How much does he have in the bank now?

**10** Xiu is 5 years older than Harry, who is $x$ years old. How old is Xiu?

**11** The perimeter of a square is $x$ cm. How long is each side?

**12** If $m$ boxes each contain 8 toffees, how many toffees are there in total?

**13** The sum of two whole numbers is 10. If one of the numbers is $n$, what is the other number?

**14** Karla has $n$ cards and collects 20 more. How many cards does Karla have?

**15** A triangle has 3 sides of equal length. The sum of the lengths of the sides of the triangle is $s$ cm. What is the length of each side?


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# 3B Algebraic notation

In algebra, there are concise ways of expressing multiplication, division and powers.

## Notation for multiplication

In algebra, the $\times$ sign is usually omitted. For example:

the product $3 \times x$ is written as $3x$ instead of $3 \times x$

If $a$ and $b$ are numbers, then their product is $a \times b$. This is written as $ab$. Similarly, we write $3 \times a = 3a$.

This is done because it looks simpler and because the multiplication sign could be confused with a letter.

Note that the number is written first. In the example above:

$3 \times a$ is written as $3a$ and not $a3$

Similarly:

$x \times y \times z$ is written as $xyz$ and $7 \times x \times y \times z$ is written as $7xyz$

## Notation for division

The division sign, $\div$, is rarely used in algebra. We use the alternative notation for division, which was introduced in Chapter 1.

Recall that $24 \div 6$ can also be written as $\frac{24}{6}$. In a similar way, we use the notation $\frac{x}{5}$ for '$x$ divided by 5'. So:

$x \div 5$ is written as $\frac{x}{5}$

## Notation for powers

The expressions for powers are written as follows:

$x \times x$ is written as $x^2$

$y \times y \times y$ is written as $y^3$

$z \times z \times z \times z$ is written as $z^4$

and so on.

### Example 8

Write these expressions without multiplication signs.

**a** $3 \times b$

**b** $8 \times x \times x \times y \times y$

**c** $4 \times x \times 6 \times x$

**d** $c \times b \times a \times 5$

### Solution

**a** $3 \times b = 3b$

**b** $8 \times x \times x \times y \times y = 8x^2y^2$

**c** $4 \times x \times 6 \times x = 24x^2$

**d** $c \times b \times a \times 5 = 5abc$


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### Example 9

Write each expression using the algebraic way of representing division.

**a** $a \div 5$ **b** $m \div n$

### Solution

**a** $a \div 5 = \frac{a}{5}$ **b** $m \div n = \frac{m}{n}$

### Example 10

Write each statement below in the notation just introduced.

**a** A number, $x$, is multiplied by itself and then doubled.

**b** A number, $x$, is squared and then multiplied by the square of a second number, $y$.

### Solution

**a** $x \times x \times 2 = x^2 \times 2 = 2x^2$ **b** $x^2 \times y^2 = x^2y^2$

### Notation for products, quotients and powers

- $2 \times x$ is written as $2x$.
- $x \times y$ is written as $xy$.
- $x \times y \times z$ is written as $xyz$.
- $x \times x$ is written as $x^2$ and is called '$x$ squared'.
- $x \times x \times x$ is written as $x^3$ and is called '$x$ cubed'.
- $x \times x \times x \times x$ is written as $x^4$ and is called '$x$ to the fourth'.
- $x \div 3$ is written as $\frac{x}{3}$.
- $x \div z$ is written as $\frac{x}{z}$.

Note that $x^1 = x$ (the first power of $x$ is $x$), $1x = x$ and $0x = 0$.

## Exercise 3B

Example 8

**1** Write each expression without multiplication signs.

**a** $5 \times x$ **b** $2 \times a$ **c** $m \times n$

**d** $6 \times x \times y$ **e** $3 \times x \times x$ **f** $7 \times 5 \times x$

**g** $6 \times a \times 3 \times c$ **h** $6 \times x \times x \times y \times y$ **i** $5 \times x \times 7 \times 3$

**j** $7 \times x \times 3 \times x$ **k** $2 \times x \times x \times y \times y \times y$ **l** $3 \times x + 9 \times y$


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**2** Write each expression in simplest form.

**a** $x \times y$ **b** $a \times b$

**c** $6 \times p \times p$ **d** $3 \times 4 \times x$

**e** $5 \times a \times 3$ **f** $6 \times p \times p \times x \times x$

Example 9

**3** Rewrite each expression using the algebraic way of representing division.

**a** $x \div 4$ **b** $x \div 5$

**c** $x \div 7$ **d** $z \div 10$

**e** $w \div z$ **f** $q \div p$

**g** $w \div x$ **h** $x \div y$

**4** Rewrite these expressions using the multiplication sign, $\times$.

**a** $5a$ **b** $a^2$ **c** $abc$ **d** $7a^2$ **e** $7x^2y$

**5** Rewrite these expressions using the division sign, $\div$.

**a** $\frac{x}{3}$ **b** $\frac{f}{3}$ **c** $\frac{x}{y}$ **d** $\frac{mn}{p}$

Example 10

**6** Write an algebraic expression for each of the following. Use the notation for products, quotients and powers introduced in this section.

**a** The product of 7 and $x$ **b** The product of $x$ with itself

**c** The product of $6a$ and $b$ **d** The product of $7a$ and $3a$

**e** The quotient of $x$ divided by 3 **f** The quotient of $p$ divided by $q$

**g** $x$ is multiplied by 7, and 5 is added to the result

**h** $m$ is multiplied by 7, and 2 is subtracted from the result

**7** There are $x$ apples to be divided equally among 7 people. How many apples will each person get?

**8** A jar of jelly beans contains $x$ jelly beans. How many jelly beans are there in 5 jars?




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**9** A small minibus carries 11 passengers and a large minibus carries 18 passengers. If there are $x$ small minibuses and $y$ large minibuses, how many passengers can be carried in total?

**10** Tomatoes are \$5 a kilogram and potatoes are \$6 a kilogram. What is the total cost of $x$ kilograms of tomatoes and $z$ kilograms of potatoes?

**11** Write each of these statements in algebraic notation.

**a** The number $x$ is multiplied by itself.

**b** The number $x$ is multiplied by itself, and 3 is added to the result.

**c** The cube of $a$ is taken.

**d** The cube of $a$ is taken and added to 3.

# 3C Substitution

If we take the algebraic expression $3a^2$ and replace $a$ by the particular value 4, then we get the result:

$$\begin{aligned} 3 \times 4^2 &= 3 \times 16 \\ &= 48 \end{aligned}$$

The process in which we replace a pronumeral in an expression with a particular value is called **substitution**. In the example above, we substituted the value 4 for $a$ in the expression $3a^2$ and got 48.

## Example 11

Find the value of each expression if $x$ is given the value 4.

**a** $x + 3$ **b** $5x$ **c** $\frac{x}{2}$

**d** $x^2$ **e** $2x - 5$ **f** $\frac{x}{4} + 2$

## Solution

**a** $\begin{aligned} x + 3 &= 4 + 3 \\ &= 7 \end{aligned}$

**b** $\begin{aligned} 5x &= 5 \times 4 \\ &= 20 \end{aligned}$

**c** $\begin{aligned} \frac{x}{2} &= \frac{4}{2} \\ &= 2 \end{aligned}$

**d** $\begin{aligned} x^2 &= 4^2 \\ &= 16 \end{aligned}$

**e** $\begin{aligned} 2x - 5 &= 2 \times 4 - 5 \\ &= 3 \end{aligned}$

**f** $\begin{aligned} \frac{x}{4} + 2 &= \frac{4}{4} + 2 \\ &= 3 \end{aligned}$


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### Example 12

If $a = 6$ and $b = 5$, evaluate:

**a** $a - b$ **b** $4a - 3b$

**Solution**

**a** $a - b = 6 - 5$
$= 1$

**b** $4a - 3b = 4 \times 6 - 3 \times 5$
$= 24 - 15$
$= 9$

### Example 13

Evaluate each expression for $x = 3$.

**a** $x^2 + 4$ **b** $2x^2 - 4$

**Solution**

**a** $x^2 + 4 = 3^2 + 4$
$= 13$

**b** $2x^2 - 4 = 2 \times 3^2 - 4$
$= 2 \times 9 - 4$
$= 18 - 4$
$= 14$

## Exercise 3C

Example 11a **1** Find the value of each expression if 3 is substituted for $a$.

**a** $a + 4$ **b** $a + 6$ **c** $7 + a$ **d** $a + 20$
**e** $a + 100$ **f** $1000 + a$ **g** $a + 200$ **h** $a + 10\,000$

**2** Find the value of each expression if 9 is substituted for $m$.

**a** $m - 3$ **b** $m - 6$ **c** $m - 2$ **d** $m - 9$

Example 11b **3** Find the value of each expression if 4 is substituted for $x$.

**a** $5x$ **b** $2x$ **c** $10x$ **d** $87x$

Example 11c **4** Find the value of each expression if 24 is substituted for $x$.

**a** $\frac{x}{2}$ **b** $\frac{x}{4}$ **c** $\frac{x}{6}$ **d** $\frac{x}{12}$

Example 11d **5** Find the value of $x^2$ for these values of $x$.

**a** 2 **b** 3 **c** 9 **d** 10


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Example 11e, f

**6** Find the value of each expression if 12 is substituted for $x$.

**a** $2x + 7$ **b** $3x + 4$ **c** $2x - 1$ **d** $23 + 2x$

**e** $37 - 3x$ **f** $\frac{x}{4} + 3$ **g** $\frac{x}{4} - 2$ **h** $\frac{x}{12} + 5$

Example 12

**7** If $n = 2$ and $m = 6$, evaluate:

**a** $m + 3$ **b** $3m$ **c** $m + n$ **d** $m - n$ **e** $mn$ **f** $2m + 4$

**g** $10 - m$ **h** $20 - 2n$ **i** $3m - 2n$ **j** $3n + 6$ **k** $6m + 2n$

Example 13

**8** Find the value of each expression if 2 is substituted for $x$.

**a** $x^2 + 2$ **b** $x^2 + 3$ **c** $3x^2 + 2$ **d** $3x^2 - 2$

**9** If $v = 12$ and $t = 3$, find the value of:

**a** $\frac{v}{6}$ **b** $\frac{v}{t}$ **c** $\frac{v}{4} - 2$ **d** $\frac{v}{3} - 4$ **e** $\frac{v}{t} + 6$

**10** The cost of $n$ pencils is $2n$ dollars. Find the cost of 20 pencils.

**11** The profit from selling $n$ crates of bananas is $3n$ dollars. What is the profit if 100 crates of bananas are sold?

**12** The cost of hiring an electric saw for $n$ hours is $(3n + 20)$ dollars. How much does it cost to hire the saw for 5 hours?

**13** The number of seats in a small theatre of $x$ rows of 16 seats is $16x$. If there are 8 rows, how many seats are there?

# 3D Addition and subtraction of like terms

If Tim has 3 pencil cases with the same number, $x$, of pencils in each, he has $3x$ pencils in total.



$x$ pencils



$x$ pencils



$x$ pencils

If Yuko gives Tim 2 more pencil cases with $x$ pencils in each, then he has $3x + 2x = 5x$ pencils in total. This is because the number of pencils in each case is the same.


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## Like terms

The terms $3x$ and $2x$ are said to be **like terms** and they have been **collected** together.

Consider another example:

> If Jane has $x$ packets of chocolates, each containing $y$ chocolates, then she has $x \times y = xy$ chocolates. If David has twice as many chocolates as Jane, he has $2 \times xy = 2xy$ chocolates. Together they have $2xy + xy = 3xy$ chocolates.

The terms $2xy$ and $xy$ above are **like terms**. The pronumerals are the same and have the same index. (Remember that $x = x^1$, $y = y^1$ and so on.)

The distributive law can be used to explain the addition and subtraction of like terms.

$$\begin{aligned} 2xy + xy &= 2 \times xy + 1 \times xy \\ &= (2 + 1)xy \\ &= 3xy \end{aligned}$$

The terms $2x$ and $3y$ are *not* like terms because the pronumerals are different. The terms $3x$ and $3x^2$ are *not* like terms because the indices are different. For the sum $6x + 2y + 3x$, the terms $6x$ and $3x$ are like terms and can be added. There are no like terms for $2y$, so the sum is:

$$\begin{aligned} 6x + 2y + 3x &= 6x + 3x + 2y \\ &= 9x + 2y \end{aligned}$$

### Example 14

Which of the following pairs are like terms?

**a** $3x$, $5x$ **b** $4x^2$, $8x$ **c** $4xy$, $12xy$

**d** $ab$, $2ba$ **e** $3mn$, $5mx$

#### Solution

**a** $5x$ and $3x$ are like terms.

**b** These are not like terms. The powers of $x$ are different.

**c** These are like terms because each is a number times $x$ times $y$.

**d** These are like terms because $ab = ba$.

**e** These are not like terms because one is a number multiplied by $m$ and then by $n$, and the other is a number multiplied by $m$ and then by $x$.

## Addition and subtraction of like terms

Like terms can be added and subtracted as shown in the following examples.

### Example 15

Simplify each expression by adding or subtracting like terms.

**a** $3m + 5m$ **b** $8m - 2m$

**c** $7n + 4n + 6n$ **d** $8m + 5m - 2m$

**e** $4m + 6m + 3n + 4n$ **f** $7m - 2m + 5n - n$


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**Solution**

**a** $3m + 5m = 8m$
**b** $8m - 2m = 6m$
**c** $7n + 4n + 6n = 17n$
**d** $8m + 5m - 2m = 11m$
**e** $4m + 6m + 3n + 4n = 10m + 7n$
**f** $7m - 2m + 5n - n = 5m + 4n$

**Example 16**

Simplify each expression by adding or subtracting like terms.

**a** $2x + 3x + 5x$
**b** $3xy + 2xy$
**c** $4x^2 - 3x^2$
**d** $2x^2 + 3x + 4x$
**e** $4xy - 3xy + 5xy$

**Solution**

**a** $2x + 3x + 5x = 10x$
**b** $3xy + 2xy = 5xy$
**c** $4x^2 - 3x^2 = 1x^2 = x^2$
**d** $2x^2 + 3x + 4x = 2x^2 + 7x$
**e** $4xy - 3xy + 5xy = xy + 5xy$
$= 6xy$

## Exercise 3D

Example 14

**1** State whether each of the following pairs contains like or unlike terms.

**a** $6z$, $11z$
**b** $5y$, $6y$
**c** $12xy$, $16yx$
**d** $6x$, $11y$
**e** $7yx$, $11yx$
**f** $6m^2$, $6m$
**g** $6a$, $11a^2$
**h** $6ab$, $11ba$

Example 15

**2** Simplify each expression by adding or subtracting like terms.

**a** $2x + 7x$
**b** $11x - 2x$
**c** $5x + 4x + 7x$
**d** $9x + 4x - 2x$
**e** $5x + 6x + 7y + 4y$
**f** $6x - 2x + 7y - y$
**g** $5x + y + 5x + 3y$
**h** $7x + 6y + 2x - 3y$
**i** $5y + 8x - 2y - 5x$

**3** Write down the sum of the terms in each case and simplify by collecting like terms.

**a** $5x$ and $6x$
**b** $6b$ and $9b$
**c** $9ab$ and $6ab$
**d** $11ac$ and $12ac$
**e** $3xy$ and $5xz$
**f** $3abc$ and $5abc$
**g** $5x^2$ and $2x^2$
**h** $7x^2$ and $2y^2$

Example 16

**4** Simplify each expression by adding or subtracting like terms.

**a** $2x + 5x - 2x$
**b** $2xy - xy + 6xy$
**c** $3xy + 2xy - xy$
**d** $2a + 3a - a$
**e** $2x + 3x + 5y + 6y$
**f** $3x + 5y + 7x + 2y$
**g** $2xy + 3xy + 3xy + 5xy$
**h** $5x^2 + 3x^2$
**i** $5x^2 + 3x^2 + 2x + 3x$
**j** $7x^2 + 11x - 4x^2 - 7x$


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**5** Write down the sum of the terms in each case and simplify by collecting like terms.

**a** $3y$ and $7y$ **b** $6xy$ and $7xy$ **c** $7xz$ and $11xy$

**d** $5yx$ and $6xy$ **e** $7xy$ and $23xy$ **f** $6ab$ and $11ba$

**6** Simplify each expression by adding or subtracting like terms.

**a** $2x + 3x + 5y + 6y$ **b** $5x - 2x + 6y + 2y$

**c** $20x - 18x + 11y + 4y$ **d** $60x + 20y + 30x + 5x$

**e** $40a + 20b + 30a + 10b$ **f** $100w - 80w + 60w$

**g** $3wv + 4vw - wv$ **h** $6xy - 2xy - yx$

**7** Show that the sum of five consecutive whole numbers is always a multiple of 5.

# 3E Brackets

Brackets have the same role in algebra as they do in arithmetic. The order of operations discussed in Chapter 1 also applies in algebra.

Here is an example showing how brackets are used in algebra.

**Example 17**

'Six is added to a number and the result is multiplied by 3.'

Write the answer using brackets and a pronumeral.

**Solution**

Let $x$ be the number. We write $(x + 6) \times 3$, because it is clear that we are meant to multiply the result of the first step $(x + 6)$ by the factor 3.

We then write $(x + 6) \times 3$ as $3(x + 6)$. This uses the convention that the factor 3 is moved to the front of the expression $(x + 6)$ without a multiplication sign.

So the result is $3(x + 6)$.

The multiplication sign is dropped when using algebra. For example:

$6 \times (n + 2)$ is written as $6(n + 2)$

$5 \times (x - 2)$ is written as $5(x - 2)$


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### Example 18

'Ten is subtracted from a number and the result is multiplied by 5.'

Write the answer using brackets and a pronumeral.

#### Solution

Let $n$ be the number. We write $(n - 10) \times 5$.

Following the convention that 5 is moved to the front of the expression, we write $5(n - 10)$.

### Example 19

Evaluate each expression by substituting $x = 8$.

**a** $5(x + 2)$

**b** $5x + 2$

**c** $10 + 4(x - 3)$

**d** $2(3x + 4) + 6$

#### Solution

**a** $5(x + 2) = 5(8 + 2)$
$= 5 \times 10$
$= 50$

**b** $5x + 2 = 5 \times 8 + 2$
$= 42$

**c** $10 + 4(x - 3) = 10 + 4 \times 5$
$= 10 + 20$
$= 30$

**d** $2(3x + 4) + 6 = 2(3 \times 8 + 4) + 6$
$= 2 \times 28 + 6$
$= 62$

## Use of brackets and powers

The following example shows how important it is to follow the order of operations when working with powers and brackets. Notice how different the two answers are.

### Example 20

Write each statement using algebra.

**a** A number, $a$, is squared and the result is multiplied by 3.

**b** A number, $a$, is multiplied by 3 and the result is squared.

#### Solution

**a** $3a^2$

**b** $(3a)^2 = 3a \times 3a = 9a^2$


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## Example 21

Evaluate each expression for $x = 3$.

**a** $2x^2$ **b** $(2x)^2$ **c** $2x^3$ **d** $(2x)^3$

### Solution

**a** $2x^2 = 2 \times x \times x$
$= 2 \times 3 \times 3$
$= 18$

**b** $(2x)^2 = (2 \times x)^2$
$= (2 \times 3)^2$
$= 36$

**c** $2x^3 = 2 \times x \times x \times x$
$= 2 \times 3 \times 3 \times 3$
$= 54$

**d** $(2x)^3 = (2 \times x)^3$
$= (2 \times 3)^3$
$= 216$

## Example 22

For a party, the host has prepared 6 baskets of chocolates, each containing $n$ chocolates. Two more chocolates are placed in each basket.



**a** How many chocolates are there in total?

**b** If $n = 12$ (that is, there were initially 12 chocolates in each basket), how many chocolates are there in total?

### Solution

**a** The number of chocolates in each basket is $n + 2$. There are 6 baskets and therefore there are $6 \times (n + 2) = 6(n + 2)$ chocolates in total.

**b** If $n = 12$, the total number of chocolates is
$6 \times (n + 2) = 6 \times (12 + 2)$
$= 6 \times 14$
$= 84$


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## Exercise 3E

Example 17, 18

**1** Write each statement using brackets and algebra.

**a** 6 is added to $x$ and the result is multiplied by 3.

**b** 7 is subtracted from $x$ and the result is multiplied by 5.

**c** 10 is added to $x$ and the result is multiplied by 4.

**d** 11 is subtracted from $x$ and the result is multiplied by 7.

**2** Write each statement using algebra, including brackets where appropriate.

**a** A number, $x$, is multiplied by 3 and 2 is added to the result.

**b** 2 is added to $x$ and the result is multiplied by 3.

**c** A number, $x$, is multiplied by 5 and 3 is subtracted from the result.

**d** 3 is subtracted from a number, $x$, and the result is multiplied by 5.

Example 19

**3** Evaluate each expression by substituting $x = 5$.

**a** $2(x + 3)$ **b** $2x + 3$ **c** $2(x - 2)$

**d** $3 + 2(x - 2)$ **e** $3 + 2(x + 2)$ **f** $(4 + 5x) - 1$

**4** Evaluate each expression by substituting $x = 3$.

**a** $2(x + 1)$ **b** $5(2x + 1)$ **c** $4 + 2(x + 6)$

**d** $6 + 2(x - 1)$ **e** $5 + 2(x - 1)$ **f** $8 + 3(x - 1)$

**5** Evaluate each expression for $a = 6$ and $b = 3$.

**a** $a(3 + 2)$ **b** $4(a - b)$ **c** $3 + 2(a - b)$

**d** $3(a - b) + 4$ **e** $b(a - 3) + 1$ **f** $2(5a - 4b) + 3b$

Example 20

**6** Write each statement using algebra.

**a** A number, $m$, is squared and the result is multiplied by 5.

**b** A number, $x$, is multiplied by 5 and the result is squared.

**c** A number, $z$, is multiplied by 2 and the result is cubed.

**d** A number, $a$, is cubed and then multiplied by 3.

Example 21a, b

**7** Evaluate each expression for $x = 4$.

**a** $5x^2$ **b** $(5x)^2$

Example 21c, d

**8** Evaluate each expression for $x = 2$.

**a** $3x^3$ **b** $(3x)^3$

**9** Evaluate each expression for $x = 3$.

**a** $(3x)^2$ **b** $(2x)^2 + 2$ **c** $6x^2 + 1$


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Example 22

**10** Each crate of bananas contains $n$ bananas. Two extra bananas are placed in each crate.

**a** How many bananas are now in each crate?

**b** If there are 5 crates, how many bananas are there in total?

**11** Four extra seats are added to each row of seats in a theatre. There were originally $x$ seats in each row and there are 20 rows of seats. How many seats are there now, in total?

# 3F Multiplying terms

Multiplying algebraic terms involves the **any-order property of multiplication** discussed in Chapter 1.

In this section you will use the standard way of writing powers:

$$x \times x = x^2$$
$$y \times y \times y = y^3$$

Remember that you can use other letters besides $x$ and $y$.

The following example shows how the any-order property of multiplication can be used.

$$\begin{aligned} 3x \times 2y \times 2xy &= 3 \times x \times 2 \times y \times 2 \times x \times y \\ &= 3 \times 2 \times 2 \times x \times x \times y \times y \\ &= 12x^2y^2 \end{aligned}$$

## Example 23

Simplify each of the following.

**a** $5 \times 2a$ **b** $3a \times 2a$ **c** $5xy \times 2xy$

**d** $(2x)^2$ **e** $(4x)^3$ **f** $(2x)^2 \times 3x$

### Solution

**a** $5 \times 2a = 10a$

**b** $\begin{aligned} 3a \times 2a &= 3 \times a \times 2 \times a \\ &= 6a^2 \end{aligned}$

**c** $\begin{aligned} 5xy \times 2xy &= 5 \times 2 \times x \times x \times y \times y \\ &= 10x^2y^2 \end{aligned}$

**d** $\begin{aligned} (2x)^2 &= 2x \times 2x \\ &= 2 \times 2 \times x \times x \\ &= 4x^2 \end{aligned}$

**e** $\begin{aligned} (4x)^3 &= 4x \times 4x \times 4x \\ &= 4 \times 4 \times 4 \times x \times x \times x \\ &= 64x^3 \end{aligned}$

**f** $\begin{aligned} (2x)^2 \times 3x &= 4x^2 \times 3x \\ &= 12x^3 \end{aligned}$

*Note:* Intermediate steps are not required.


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## Exercise 3F

Example 23a, b, c

**1** Simplify:

**a** $3 \times 2a$ **b** $6 \times 2x$ **c** $4 \times 3m$ **d** $2x \times 4x$

**e** $7x \times 3x$ **f** $3x \times 2y$ **g** $2xy \times 3x$ **h** $4xy \times 2xy$

Example 23d, e

**2** Rewrite each expression without brackets.

**a** $(5n)^2$ **b** $(4z)^2$ **c** $(16z)^2$ **d** $(13z)^2$

**3** Find the product of each pair of terms in simplified form.

**a** $2a$ and $6b$ **b** $3x$ and $4x$ **c** $7ab$ and 11 **d** $6c$ and $11c$

**e** $4m$ and $6n$ **f** $7mn$ and $11mn$ **g** $4n^2$ and 3 **h** $7m^2$ and $6m$

**4** Simplify:

**a** $(2x)^2 \times x^3$ **b** $3x \times (4x)^2$ **c** $6x^2 \times 3x$

**d** $2xy \times 3xy$ **e** $(2x)^2 \times 2xy$ **f** $5z^2y \times 6zy$

**5** Simplify:

**a** $(3x)^2 \times 2x \times x$ **b** $2xy \times y \times x$ **c** $(5a)^2 \times (2a)^2$

**d** $4xy^2 \times (5x)^2$ **e** $3x^2 \times 6y \times 2x$ **f** $5w^2 \times (2w)^2 \times w$

# 3G Describing arrays, areas and number patterns

In Chapter 2 we used arrays of dots to represent products of numbers. For example:



represents $2 \times 6 = 12$

Note that 12 is the sixth non-zero even number. It can be represented by 2 rows of 6 dots.

Any non-zero even number can be represented by an array of 2 rows, each with the same number of dots.

The tenth non-zero even number can be represented by 2 rows of 10 dots. The tenth non-zero even number is $2 \times 10 = 20$.



For the $n$th non-zero even number, there are two rows each containing $n$ dots. The $n$th non-zero even number is $2n$.


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**Example 24**

The diagram shows squares formed by dots. The pattern goes on forever. How many dots are there in the $n$th square?



**Solution**

In the 1st diagram, there is $1 \times 1 = 1^2$ dot.

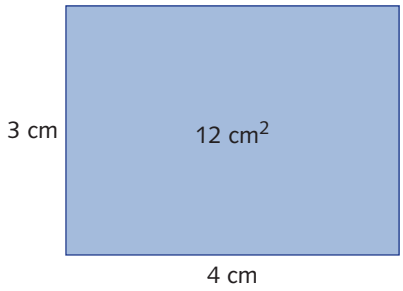
In the 2nd diagram, there are $2 \times 2 = 2^2$ dots.

In the 3rd diagram, there are $3 \times 3 = 3^2$ dots.

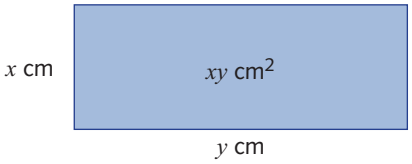
In the $n$th diagram, there are $n \times n = n^2$ dots.

## Area

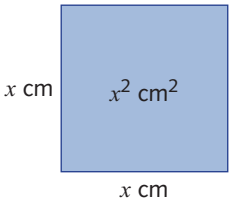
The area of a rectangle with side lengths 3 cm and 4 cm is $3 \times 4 = 12$ cm$^2$.




The area of a rectangle with side lengths $x$ cm and $y$ cm is $x \times y = xy$ cm$^2$.




The area of a square with side length $x$ cm is $x^2$ cm$^2$.

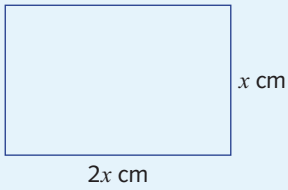




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## Example 25

Find the area of the following rectangle in terms of $x$.




### Solution

The area is $2x \times x = 2x^2$ cm$^2$.

## Example 26

**a** Find the area of a square with each side having length $a$ cm.

**b** If $a = 7$, find the area of the square.

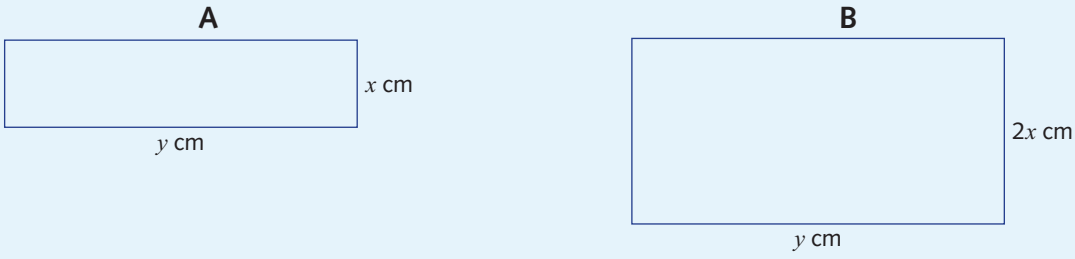
### Solution

**a** The area is $a \times a = a^2$ cm$^2$

**b** If a = 7, then
area $= 7^2$
$= 49$ cm$^2$

## Example 27

Find the total area of the two rectangles in terms of $x$ and $y$.




### Solution

The area of rectangle A is $xy$ cm$^2$ and the area of rectangle B is $2xy$ cm$^2$. Hence, the total area is $xy + 2xy = 3xy$ cm$^2$.


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## Describing number patterns

Algebra is often used to describe patterns that occur among numbers.$x + 0 = x$

### Example 28

The $n$th non-zero even number is $2n$.

**a** What is the square of the $n$th non-zero even number?

**b** If the $n$th non-zero even number is doubled, what is the result?

#### Solution

**a** The $n$th non-zero even number is $2n$, so its square is:

$$\begin{aligned}(2n)^2 &= 2n \times 2n \\ &= 2 \times 2 \times n \times n \\ &= 4n^2\end{aligned}$$

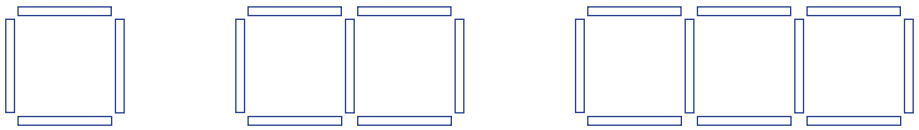
**b** The $n$th non-zero even number is $2n$. Doubling it means multiplying by 2.
So the result is:

$2 \times 2n = 4n$

## Exercise 3G

Example 24

**1** The following diagrams show arrangements of sticks. The first diagram has 4 sticks in it, the second diagram has 7 sticks in it, and the third has 10 in it. The $n$th diagram has $3n + 1$ sticks in it.

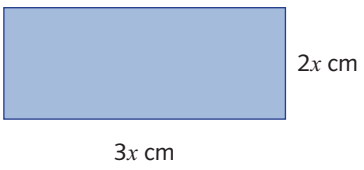


**a** How many sticks are there in the fourth diagram?

**b** How many sticks are there in the 100th diagram?

Example 25, 26

**2** **a** Write down the area of this rectangle in terms of $x$.



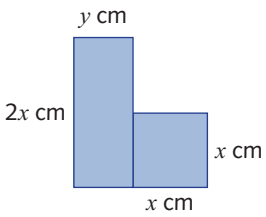

**b** If $x = 5$, what is the area?

**c** If $x = 10$, what is the area?


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Example 27

**3** Find the total area of the shaded region below in terms of $x$ and $y$.




Example 28

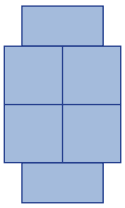
**4** The $n$th non-zero even number is $2n$.

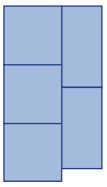
**a** What is the whole number immediately after the $n$th non-zero even number?

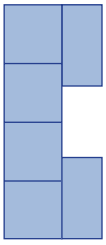
**b** What is the next even number after $2n$?

**5** In each of the following, the area of the square is $m$ cm$^2$ and the area of the rectangle is $n$ cm$^2$. Give the area of each shaded region in terms of $m$ and $n$.

**a** 

**b** 

**c** 

**d** 

**6** If $x$ is a multiple of 5, other multiples of 5 can be generated by adding 5, 10, 15 and so on to it. Which of the following would be multiples of 5?

| | | | |
|---|---|---|---|
| **a** $x + 25$ | **b** $x + 200$ | **c** $50 + x$ | **d** $x - 15$ |
| **e** $3x$ | **f** $3x + 20$ | **g** $3(x + 20)$ | **h** $35 - x$ |
| **i** $53 - 5x$ | **j** $5(x^2 - 1)$ | **k** $5x^2 - 1$ | **l** $5n - 1$ |

**7** **a** Assuming that $b$ is even, what is the next even number?

**b** Assuming that $a$ is a multiple of 3, what are the next two multiples of 3?

**c** Assuming that $n$ is odd and $n > 1$, what is the previous odd number?

**8** If $n$ is an even number, which of the following will be even numbers?

| | | | |
|---|---|---|---|
| **a** $2n$ | **b** $2n + 1$ | **c** $2n + 2$ | **d** $2n + 3$ |
| **e** $3n + 1$ | **f** $3n + 2$ | **g** $3n + 4$ | **h** $4n + 1$ |

**9** A rectangle has side lengths $4b$ cm and $2a$ cm. Find:

**a** the perimeter of the rectangle **b** the area of the rectangle

**10** A square has side length $\ell$ cm. Find:

**a** the perimeter of the square **b** the area of the square

**11** A rectangle has side lengths $x + 2$ cm and 6 cm. Find:

**a** the perimeter of the rectangle **b** the area of the rectangle


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# Review exercise

**1** Write each of the following using algebra.

**a** 3 is added to $x$

**b** 6 is subtracted from $z$

**c** $x$ is multiplied by 4

**d** $t$ is subtracted from 11

**e** 2 is multiplied by $b$, and 3 is added to the result

**f** 6 is added to $z$, and the result is multiplied by 4

**g** $x$ is multiplied by $x$, and 3 is added to the result

**h** $x$ is divided by a number, $t$, and then 3 is added to it

**2** Evaluate each of the following expressions, given that $m = 4$ and $n = 7$.

**a** $n - 4$ **b** $m + n$ **c** $nm$ **d** $2n - m$

**e** $m^2 - 4$ **f** $3m + 2n$ **g** $(2m)^2 - 4$ **h** $2m^2 - 3$

**3** The square numbers are the numbers 1, 4, 9, 16, ... The $n$th square number is $n^2$. What is the 12th square number?

**4** The $n$th odd number is $2n - 1$. What is the 21st odd number?

**5** The sum of the lengths of the sides of a square (the perimeter) is $z$ m. How long is each side?

**6** Kris has \$$n$ dollars, Jenny has \$$m$ dollars and Anna has \$$p$ dollars. How much do they have in total?

**7** The sum of two whole numbers is 24. If one of the numbers is $n$, what is the other number?

**8** Simplify:

**a** $m \times n$ **b** $6 \times m \times m \times m$ **c** $7 \times y \times y \times y$

**d** $6 \times p \times p \times 4 \times p$ **e** $3 \times x \times x \times y \times 10$ **f** $5 \times z \times z \times z \times 8$

**9** Rewrite each expression using the algebraic way of representing division.

**a** $x \div 5$ **b** $20 \div y$ **c** $m \div n$

**10** Write each of the following in simplified form.

**a** The product of 11 and $z$

**b** The quotient of $x$ divided by 3

**c** The cube of $a$

**d** The fourth power of $x$

**e** The product of $8b$ and $3b$

**f** The cube of $3x$

**11** If $x = 5$, evaluate:

**a** $10x + 3$ **b** $15 - x$ **c** $3x^2 + 4$ **d** $x^3$

**e** $50 - 6x$ **f** $10x^2 + 4$ **g** $x^4$ **h** $x^5$


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**12** Simplify each expression by adding or subtracting like terms.

**a** $7x + 4x - 2x$ **b** $6xy - 3xy + 2xy$ **c** $6x^2 + 7x^2 - 2x^2$

**d** $3xy - xy$ **e** $2xy + 3xy - xy$ **f** $20x + 30y + 40x + 20y$

**13** Rewrite each statement using brackets and algebra.

**a** 8 is added to $m$ and the result is multiplied by 6.

**b** 7 is subtracted from a number, $d$, and the result is multiplied by 3.

**14** Rewrite each expression without brackets.

**a** $(3n)^2$ **b** $(2n)^3$ **c** $(2n)^2 \times 4n$ **d** $(2x)^2 \times 2x^2$

**15** Evaluate each expression by substituting $x = 6$.

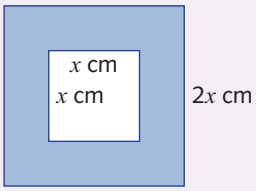
**a** $2(x + 4)$ **b** $2x + 4$ **c** $x^2 + 3$ **d** $6 + 2(x - 2)$

**16** Find the area of each shaded region in terms of $x$.

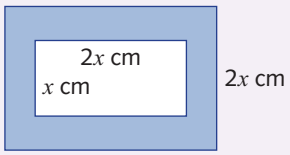
**a**




**b**




**c**




**17** Evaluate each expression by substituting $x = 7$.

**a** $2(10 - x)$ **b** $3(2x - 4)$ **c** $(2x)^2$ **d** $3x^2$

**18** Evaluate each expression by substituting $n = 10$.

**a** $10n + 3$ **b** $(10n)^2$ **c** $3(20 - n)$ **d** $\frac{2n}{5}$

**19** If $a = 20$ and $b = 4$, find the value of:

**a** $a - b$ **b** $\frac{a + 5}{5}$ **c** $\frac{a}{5} + 5$ **d** $a^2 - b^2$

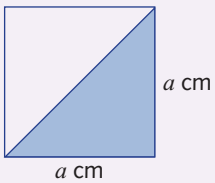
**20** Simplify each expression by collecting like terms.

**a** $10a - a + 2b - b$ **b** $20a^2 + 10a^2 + 16a^2 - 4a^2$

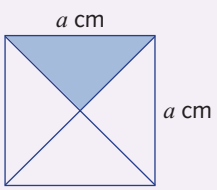
**c** $4a^2b + 6a^2b + 11a^2b + a^2b$ **d** $2c^2d + 2cd + 2dc^2 + 3cd$

**21** Find the area of each shaded region in terms of $a$.

**a**




**b**



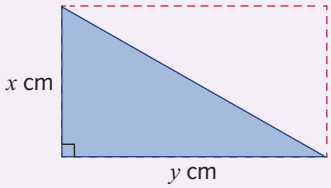


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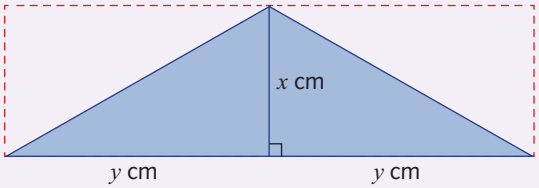
# Challenge exercise

**1** Find the area of each of these shapes in terms of $x$ and $y$.

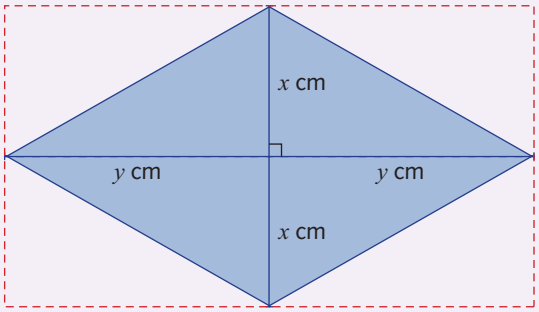
**a**




**b**




**c**




**2** Think of a number. Let this number be $x$.
Write the following using algebra to see what you get.

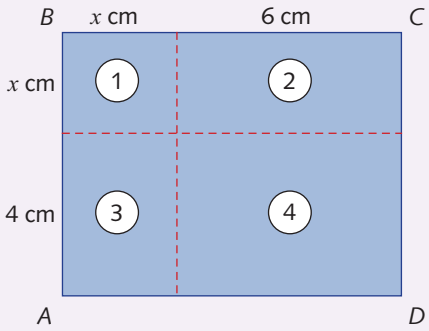
**a** Multiply the number you thought of by 2 and subtract 5.

**b** Multiply the result by 3.

**c** Add 15.

**d** Subtract 5 times the number you first thought of. What is the final answer? Why is it so?

**3**




**a** Find the area of each of the rectangles 1 , 2 , 3 and 4 .

**b** Find the area of rectangle $ABCD$ (shaded) as a product of the lengths of its sides.

**c** Deduce that $(x + 6)(x + 4) = x^2 + 10x + 24$ for all values of $x$.

**d** Find the perimeter of each of the rectangles 1 , 2 , 3 and 4 .

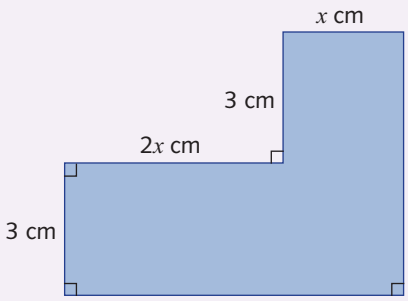
**e** Find the perimeter of rectangle $ABCD$ (shaded).


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**4** For the shape below, find:

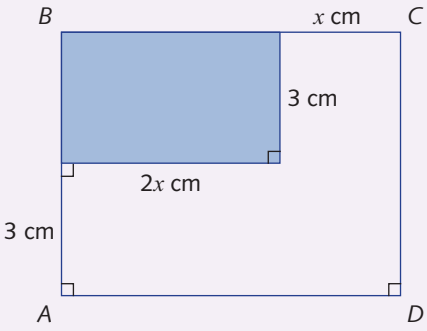
**a** the perimeter of the shaded region

**b** the area of the shaded region




**5** **a** For the diagram below, find the areas of rectangle $ABCD$ and the shaded rectangle.

**b** Find the difference of the two areas found in part **a**.




**6** **a** If $a$ is a whole number, what are the possible last digits of $a^2$? (For example, if $a = 87$, the last digit is 9.)

**b** If $a$ is an even number, what are the possible values of the last digit of $a^2 + 1$? (For example, if $a = 86$, the last digit of $a^2 + 1$ is 7.)

**c** If $b$ is an even whole number divisible by 3, what are the possible last digits of $b$?

**d** Is it true that for any even whole number $a$, $a^2 + 1$ is not divisible by 3?

**7** **a** Copy and fill in this table.

| $n$ | $n + 2$ | $n - 2$ | $(n + 2)(n - 2)$ | $n^2$ |
|---|---|---|---|---|
| 2 | 4 | 0 | 0 | 4 |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |
| 6 | | | | |
| 7 | | | | |

**b** Deduce that $(n + 2)(n - 2) = n^2 - 4$ for $n = 2, 3, 4, 5, 6$ and 7.

**c** Why does $n^2 - 4 = (n + 2)(n - 2)$ for all whole numbers $n$?


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CHAPTER

4

Number and Algebra

# Fractions – part 1

In the preceding chapters of this book, we have looked in detail at whole numbers. Fractions are a very powerful and convenient way to represent numbers that are not whole numbers.

The word *fraction* comes from the Latin word *fractus*, meaning 'broken into pieces'. Fractions have been the standard method of representing *parts of a whole* for thousands of years, and remain of great importance in modern life and in modern mathematics.

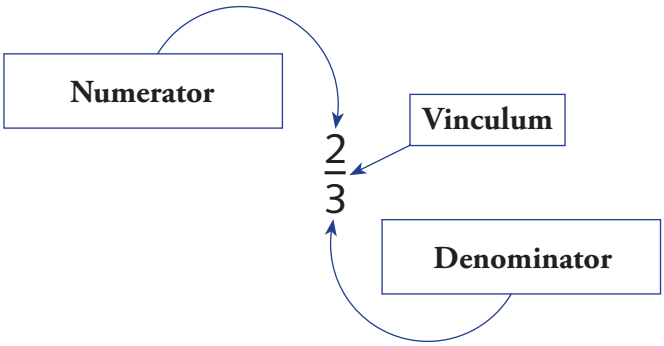


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# 4A What is a fraction?

Fractions are used in everyday language. For example, we refer to half a loaf of bread or one quarter of a kilogram of flour.

Fractions are written as one whole number over another whole number, for example:




- The whole number on the top is called the **numerator**.
- The whole number on the bottom is called the **denominator**.
- The line that separates the two is called the **vinculum**. The vinculum must be a horizontal line.

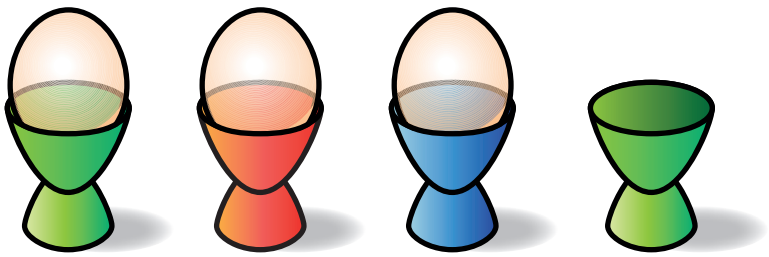


The denominator tells us how many parts the whole is broken up into, and the numerator tells us how many of those parts are taken.

To help you remember, the 'u' in n**u**merator reminds you of the **u**ppermost number, and 'd' for **d**enominator reminds you of the number that is **d**own below the vinculum.

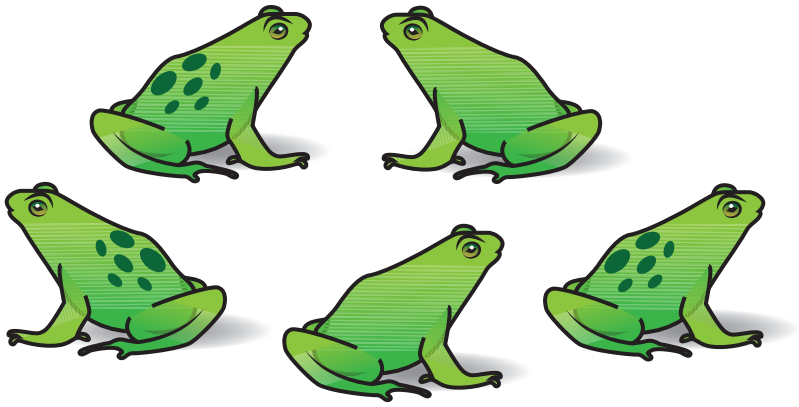
## Part of a whole

Fractions can be used to describe the number of objects out of a group of objects. In the picture below, we think of the 4 cups as 'the whole', and one of them as $\frac{1}{4}$ of the whole, so we say that $\frac{3}{4}$ of the cups have eggs in them.




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Here is another example. We think of the 5 frogs below as the whole. One frog is then $\frac{1}{5}$ of the whole. In the picture, three of the frogs have spots, so we say $\frac{3}{5}$ of the frogs have spots.



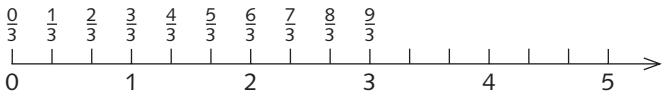
Fractions can be represented using different models. In this chapter we will use number lines or shapes divided into pieces of equal area.

## Using the number line

Here is how we use the number line to represent $\frac{0}{3}, \frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}$ and so on. Draw a number line and divide it into intervals of equal length. Mark the dividing points 0, 1, 2, 3, 4, ..., as shown. Each of the intervals has unit length.




Divide each of the intervals of unit length into three equal subintervals, as shown. There is now a new set of equally spaced markers on the number line. Name these $\frac{0}{3}, \frac{1}{3}, \frac{2}{3}, \frac{3}{3}$ and so on.




Starting at 0, the marker $\frac{4}{3}$ is reached by taking 4 steps, each of length $\frac{1}{3}$.
This gives us all the fractions with denominator 3.

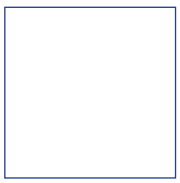
## Whole numbers as fractions

Look again at the diagram above showing the fractions with denominator 3. We see that the fraction $\frac{3}{3}$ is the same as the number 1. We can also see that the fraction $\frac{6}{3}$ is the same as the number 2, the fraction $\frac{9}{3}$ is the same as 3 and so on.

We could draw similar diagrams for fractions with other denominators. That way, we see that a whole number can be written as a fraction in infinitely many ways.


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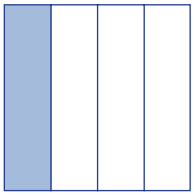
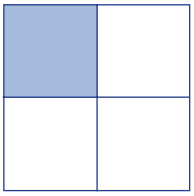
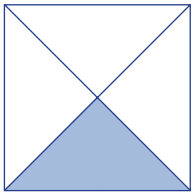
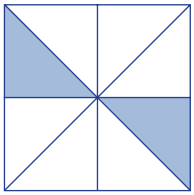
## Using areas



We can also represent fractions by shading parts of a square. We regard the area of the square as 1.

We can shade part of the square to represent a fraction of the whole.

In each of these cases, the same square is divided into regions of equal area in different ways. Each shaded region represents $\frac{1}{4}$. There are other ways to represent $\frac{1}{4}$. Can you suggest some?

## Proper fractions and improper fractions

We call a fraction a **proper fraction** if the numerator is less than the denominator. For example:

$\frac{1}{3}$ and $\frac{2}{3}$ are proper fractions

If the numerator is greater than or equal to the denominator, the fraction is said to be **improper**. For example:

$\frac{4}{3}$ and $\frac{22}{7}$ are improper fractions, and so is $\frac{4}{4}$

### Fractions

- A **fraction** is a number that is written as one whole number over another. The top number is called the **numerator** and the bottom number is called the **denominator**.
- **Proper fractions** are fractions in which the numerator is smaller than the denominator.
- **Improper fractions** are fractions in which the numerator is greater than or equal to the denominator.
- Fractions can be represented on a number line.
- Fractions can also be represented as shaded parts of a square (or rectangle).
- Every whole number can be written as a fraction in many ways. For example:
  $1 = \frac{4}{4}$, $2 = \frac{20}{10} = \frac{6}{3}$, $3 = \frac{9}{3} = \frac{30}{10} = \frac{21}{7}$


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## Example 1

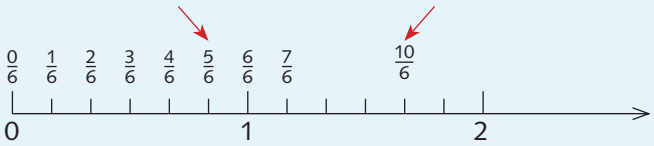
Represent the numbers $\frac{5}{6}$ and $\frac{10}{6}$ on a number line.

### Solution

Draw a number line with intervals of unit length.




Divide each unit interval (from 0 to 1 and 1 to 2) into 6 subintervals of equal length. Each of these subintervals has length $\frac{1}{6}$. Mark $\frac{5}{6}$ on the fifth marker after 0, and $\frac{10}{6}$ on the tenth marker.

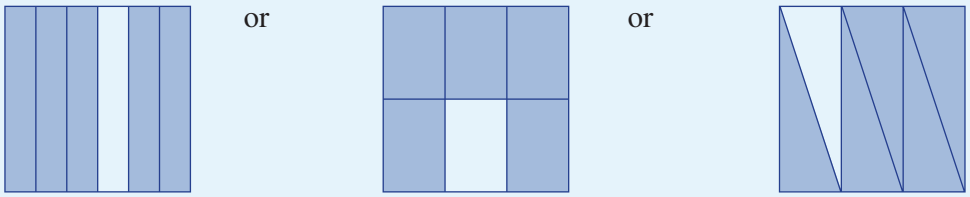



## Example 2

Represent the number $\frac{5}{6}$ using a square with area equal to 1.

### Solution

Divide the square into 6 equal regions, each having area $\frac{1}{6}$. Shade any 5 of the 6 regions.




There are other possibilities. Can you draw some?

## Exercise 4A

Example 1

**1** Draw a number line with 0, 1, 2, 3 and 4 marked on it, and indicate where the markers for $\frac{1}{3}$, $\frac{4}{3}$ and $\frac{9}{3}$ are located.

**2** Draw a number line with 0, 1, 2, 3 and 4 marked on it, and then place the markers for $\frac{1}{7}$, $\frac{9}{7}$ and $\frac{11}{7}$ on it.


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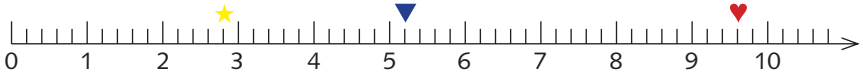
**3** Start with a number line marked in units from 0 to 4, and indicate where the markers for $\frac{2}{6}$, $\frac{5}{6}$ and $\frac{8}{6}$ are located.

**4** What fractions correspond to the positions of the star, triangle and heart for each of the following number lines?

**a**




**b**

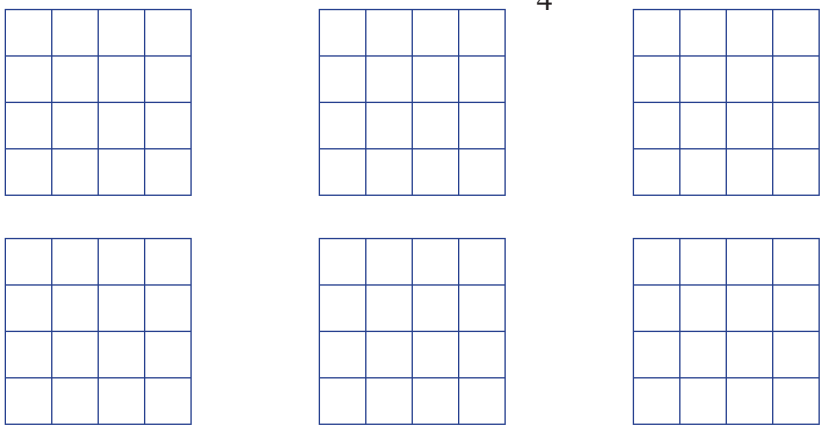



**c**



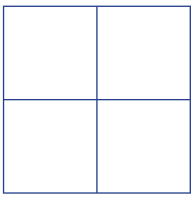

Example 2

**5** The large squares below have each been divided into 16 regions of equal area. Shade the squares in six different ways to represent $\frac{1}{4}$.

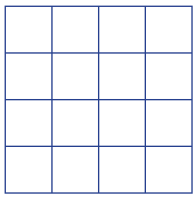


**6** In each part, represent the fraction by shading an appropriate region. Each large square has an area equal to 1. All questions have more than one correct answer. Give two correct answers for each.

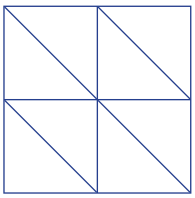
**a** $\frac{3}{4}$



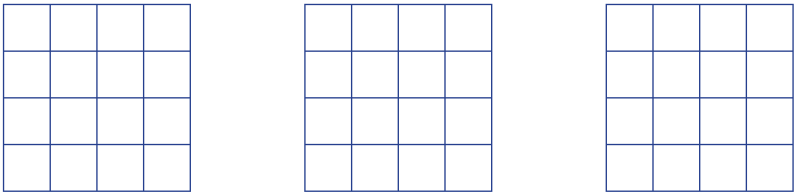
**b** $\frac{7}{16}$



**c** $\frac{3}{8}$

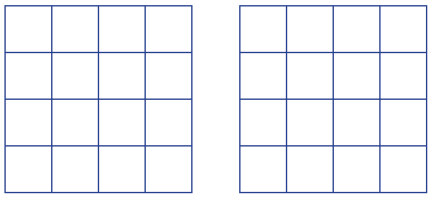


**d** $\frac{18}{8}$

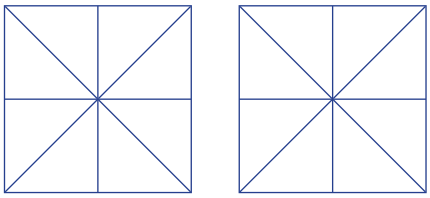



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**e** $\frac{5}{4}$

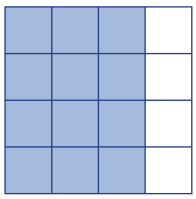


**f** $\frac{7}{4}$

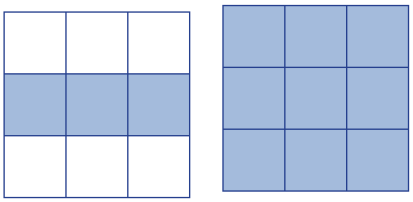


**7** What fraction does each of the shaded areas represent, given that the area of the large square is 1 in each case?

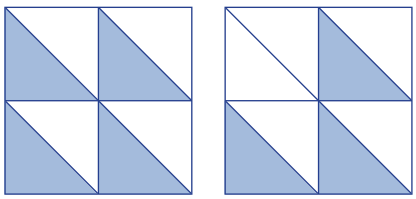
**a**



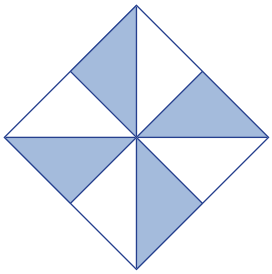
**b**



**c**



**d**



**8** In each case, write the fraction that describes the part of the whole.

**a** 11 errors on a 20-question spelling test

**b** 1 leg of a chair broken

**c** 6 days of 1 week

**d** 3 tyres need replacing on a car

**e** a 14-day holiday in January

**f** 4 broken eggs in a dozen

**g** 43 minutes taken to complete an exam for which 1 hour was allocated

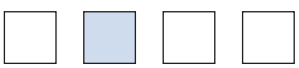
**h** 7 players injured out of a team of 18 AFL players

**i** 1 digit in an 8-digit phone number incorrect

**j** of 23 competitors, 19 finish the race

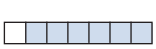
**9** Choose the diagram that correctly shows:

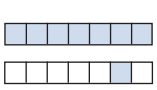
**a** $\frac{1}{4}$ if the whole is 

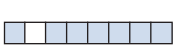
**A** 

**B** 

**b** $\frac{8}{7}$ if the whole is 

**A** 

**B** 

**C** 


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# 4B Equivalent fractions and simplest form

## Equivalent fractions

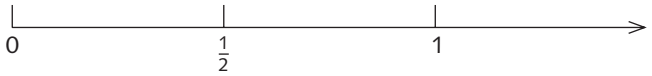


If 2 kilometres of a 4-kilometre road is sealed, we say that $\frac{2}{4}$ (two quarters) of the road is sealed. We also know this means that $\frac{1}{2}$ (half) the road is sealed. The fractions $\frac{2}{4}$ and $\frac{1}{2}$ are called equivalent fractions and we write $\frac{2}{4} = \frac{1}{2}$.

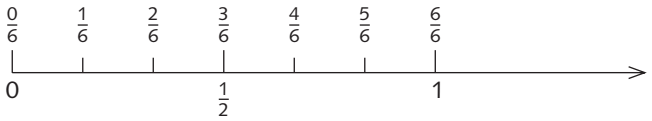
## Using a number line

We say that two fractions are **equivalent** if they are represented by the same marker on a number line.

To show that, for example, $\frac{1}{2} = \frac{3}{6}$, draw a number line and mark in 0 and 1. Form two intervals of equal length from 0 to $\frac{1}{2}$ and $\frac{1}{2}$ to 1. Each interval has length $\frac{1}{2}$.




Now divide each of these intervals into three equal subintervals to form six new intervals in total. Each of these intervals has length $\frac{1}{6}$.




The markers $\frac{1}{2}$ and $\frac{3}{6}$ are at the same point on the number line, so $\frac{1}{2}$ and $\frac{3}{6}$ are equivalent.

## Calculating equivalent fractions

As we saw in the above example, $\frac{1}{2}$ is equivalent to $\frac{3}{6}$.

There are **2** steps of length $\frac{1}{2}$ to go from 0 to 1.

There are **6** steps of length $\frac{1}{6}$ to go from 0 to 1.

There is **1** step of length $\frac{1}{2}$ to go from 0 to $\frac{1}{2}$.

There are **3** steps of length $\frac{1}{6}$ to go from 0 to $\frac{1}{2}$.

We can see that the numerator of $\frac{1}{2}$ is multiplied by 3 and the denominator is multiplied by 3.

$$\frac{1}{2} = \frac{1 \times 3}{2 \times 3}$$
$$= \frac{3}{6}$$


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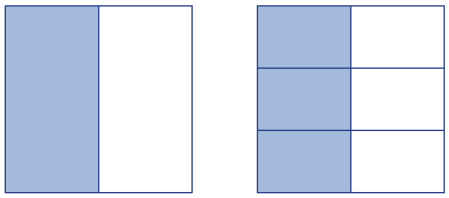
We can also see that the numerator of $\frac{3}{6}$ is divided by 3 and the denominator is divided by 3.

$$\frac{3}{6} = \frac{3 \div 3}{6 \div 3}$$
$$= \frac{1}{2}$$

In general, to form equivalent fractions from some given fraction, multiply or divide the numerator and the denominator of the given fraction by the same (non-zero) whole number.

## Using squares

We can also use squares to see that the area representing $\frac{1}{2}$ equals the area representing $\frac{3}{6}$.



## Simplest form

A fraction is said to be in **simplest form** or **lowest terms** if the only common factor of the numerator and the denominator is 1.

For example, $\frac{2}{15}$ is in simplest form since the highest common factor of 2 and 15 is 1.

However, $\frac{6}{15}$ is not in simplest form since the highest common factor of 6 and 15 is 3.

We can use our knowledge of factors and multiples to help us reduce a fraction to simplest form.

## Cancelling

When reducing $\frac{6}{15}$ to its simplest form, we consider all factors of 6 and 15. The highest common factor of 6 and 15 is 3.

We divide the numerator and the denominator by 3.

$$\frac{6}{15} = \frac{6 \div 3}{15 \div 3} = \frac{2}{5}$$

We can use cancelling notation to write this process more efficiently.

$$\frac{6}{15} = \frac{\cancel{6}^{2}}{\cancel{15}_{5}} = \frac{2}{5}$$

Finding a common factor can be tricky. For example:

$$\frac{105}{49} = \frac{105 \div 7}{49 \div 7} = \frac{15}{7}$$


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Sometimes it is more convenient to cancel in steps, rather than finding the highest common factor. If we do it in steps, we keep going until the highest common factor of the numerator and the denominator is 1.

$$\frac{\cancel{30}^{15}}{\cancel{84}^{42}} = \frac{\cancel{15}^{5}}{\cancel{42}^{14}} = \frac{5}{14}$$

## Equivalent fractions

- Two fractions are said to be **equivalent** if they mark the same place on the number line.
- Starting with a fraction, the fractions obtained by multiplying its numerator and its denominator by the same whole number are equivalent to it. For example:
$$\frac{2}{3} = \frac{2 \times 7}{3 \times 7} = \frac{14}{21}$$
- Starting with a fraction, the fractions obtained by dividing its numerator and its denominator by a common factor are equivalent to it. For example:
$$\frac{12}{36} = \frac{12 \div 12}{36 \div 12} = \frac{1}{3}$$
- The **simplest form** of a fraction is the fraction obtained by dividing the numerator and denominator by their highest common factor.

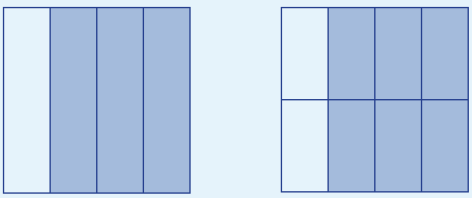
### Example 3

Show that $\frac{3}{4}$ and $\frac{6}{8}$ are equivalent fractions, using:

**a** squares **b** a number line

### Solution

**a** Draw two equal squares and divide the first into 4 equal parts and the second into 8 equal parts. Shade 3 of the 4 regions of the first square to show $\frac{3}{4}$, and shade 6 of the 8 regions to show $\frac{6}{8}$.

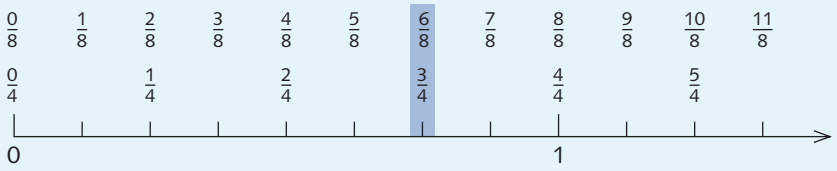


When we compare shaded areas, we see that $\frac{3}{4} = \frac{6}{8}$.

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**b** Draw a number line, marking 0 and 1. Divide the interval from 0 to 1 into four equal subintervals. Each subinterval has $\frac{1}{4}$. Then divide the interval from 0 to 1 into eight equal subintervals. Each of these has length $\frac{1}{8}$. The markers for $\frac{3}{4}$ and $\frac{6}{8}$ are the same.




## Example 4

Fill in the boxes to complete each set of equivalent fractions.

**a** $\frac{1}{5} = \frac{\square}{20} = \frac{\square}{25} = \frac{\square}{60}$

**b** $\frac{3}{5} = \frac{18}{\square} = \frac{\square}{55} = \frac{21}{\square}$

### Solution

**a** $\frac{1}{5} = \frac{1 \times 4}{5 \times 4} = \frac{4}{20}$

$\frac{1}{5} = \frac{1 \times 5}{5 \times 5} = \frac{5}{25}$

$\frac{1}{5} = \frac{1 \times 12}{5 \times 12} = \frac{12}{60}$

$\frac{1}{5} = \frac{4}{20} = \frac{5}{25} = \frac{12}{60}$

**b** $\frac{3}{5} = \frac{3 \times 6}{5 \times 6} = \frac{18}{30}$

$\frac{3}{5} = \frac{3 \times 11}{5 \times 11} = \frac{33}{55}$

$\frac{3}{5} = \frac{3 \times 7}{5 \times 7} = \frac{21}{35}$

$\frac{3}{5} = \frac{18}{30} = \frac{33}{55} = \frac{21}{35}$

## Example 5

Simplify $\frac{21}{24}$.

### Solution

The highest common factor of 21 and 24 is 3. Divide both the numerator and the denominator by 3.

$\frac{21}{24} = \frac{\cancel{21}^{7}}{\cancel{24}^{8}} = \frac{7}{8}$

So the simplest form of $\frac{21}{24}$ is $\frac{7}{8}$.


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## Exercise 4B

Example 3a

**1** Use squares to show that the fractions in each pair are equivalent.

**a** $\frac{4}{8} = \frac{1}{2}$ **b** $\frac{10}{15} = \frac{2}{3}$ **c** $\frac{3}{5} = \frac{6}{10}$

**d** $\frac{2}{12} = \frac{1}{6}$ **e** $\frac{3}{4} = \frac{15}{20}$ **f** $\frac{4}{3} = \frac{12}{9}$

Example 3b

**2** Use number lines to show that the fractions in each pair are equivalent.

**a** $\frac{2}{6} = \frac{1}{3}$ **b** $\frac{1}{8} = \frac{2}{16}$ **c** $\frac{2}{4} = \frac{3}{6}$ **d** $\frac{14}{8} = \frac{7}{4}$

**3** Fill in the boxes to show the number by which the numerator and the denominator were multiplied to arrive at the equivalent fraction. The first one has been done for you.

**a** $\frac{1}{4} \overset{\times 6}{=} \frac{6}{24}$ (×6 above, ×6 below) **b** $\frac{3}{8} = \frac{9}{24}$ **c** $\frac{17}{19} = \frac{34}{38}$ **d** $\frac{8}{25} = \frac{32}{100}$

Example 4

**4** Fill in the boxes to complete each set of equivalent fractions.

**a** $\frac{1}{3} = \frac{\square}{15} = \frac{\square}{75} = \frac{\square}{120}$ **b** $\frac{3}{4} = \frac{6}{\square} = \frac{\square}{68} = \frac{\square}{100}$ **c** $\frac{\square}{3} = \frac{10}{15} = \frac{\square}{96} = \frac{\square}{108}$

**d** $\frac{1}{\square} = \frac{6}{\square} = \frac{\square}{34} = \frac{51}{102}$ **e** $\frac{\square}{8} = \frac{125}{\square} = \frac{\square}{64} = \frac{2}{16}$ **f** $\frac{4}{5} = \frac{24}{\square} = \frac{\square}{50} = \frac{\square}{100}$

**5** In each part, find the value of $n$ that makes the statement true.

**a** $\frac{n}{100} = \frac{25}{50}$ **b** $\frac{3}{n} = \frac{25}{75}$ **c** $\frac{3}{8} = \frac{45}{n}$

**d** $\frac{66}{99} = \frac{10}{n}$ **e** $\frac{3}{n} = \frac{33}{121}$ **f** $\frac{n}{108} = \frac{3}{9}$

Example 5

**6** Reduce each of these fractions to its simplest form.

**a** $\frac{2}{4}$ **b** $\frac{5}{10}$ **c** $\frac{6}{8}$ **d** $\frac{4}{12}$ **e** $\frac{2}{6}$ **f** $\frac{3}{12}$

**g** $\frac{3}{9}$ **h** $\frac{16}{20}$ **i** $\frac{75}{100}$ **j** $\frac{18}{27}$ **k** $\frac{17}{34}$ **l** $\frac{12}{96}$

**m** $\frac{25}{125}$ **n** $\frac{144}{9}$ **o** $\frac{225}{150}$ **p** $\frac{364}{104}$ **q** $\frac{504}{72}$ **r** $\frac{171}{285}$

**s** $\frac{27}{126}$ **t** $\frac{42}{91}$ **u** $\frac{72}{48}$


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# 4C Mixed numerals and division by whole numbers

## Division by whole numbers

Have you ever shared a cake, a block of chocolate or a pie equally with your family? If the answer is 'yes', then you have used division by whole numbers. If you had a whole apple pie to share equally among 3 people, each person would get $\frac{1}{3}$ of the pie. You would have divided a whole number, 1, by another whole number, 3, resulting in a fraction, $\frac{1}{3}$.

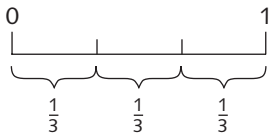
I had four friends over to dinner and ordered three family-sized pizzas. How much did each person get if I shared the pizzas equally between the five of us? Three pizzas were shared equally among five people, so each person received $\frac{3}{5}$ of a pizza.

Division of a whole number by another non-zero whole number always gives a fraction. If we can regard a whole number as a fraction. For example, $2 = \frac{2}{1}$. This partly explains why the alternative notation for division was introduced in Chapter 1.

## Using the number line

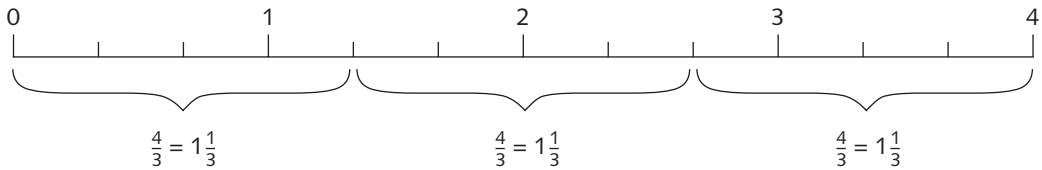
When an interval of unit length is divided into 3 equal subintervals, each subinterval has length

$1 \div 3 = \frac{1}{3}$.




When an interval of length equal to 4 is divided into 3 equal subintervals, each subinterval has length

$4 \div 3 = \frac{4}{3} = 1\frac{1}{3}$.







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Dividing 4 identical cakes equally among 3 people also illustrates the fact that $4 \div 3 = \frac{4}{3}$.

Each person gets a whole cake and the remaining cake has to be divided into three pieces. Each person gets $\frac{4}{3} = 1\frac{1}{3}$ cakes.

Dividing the whole number $m$ by the non-zero whole number $n$ results in the fraction $\frac{m}{n}$.
On the number line, the line segment from 0 to $m$ is divided into $n$ equal parts, and each part has length $\frac{m}{n}$.

$$m \div n = \frac{m}{n}$$

## Mixed numerals

A **mixed numeral**, sometimes called a mixed number, is a whole number plus a proper fraction. If a fraction is **improper**, then it can be written as a mixed numeral. As we saw in the previous section, $\frac{4}{3} = 1\frac{1}{3}$. Another example:

$$\begin{aligned}\frac{32}{3} &= \frac{30+2}{3} \\ &= \frac{30}{3} + \frac{2}{3} \\ &= 10 + \frac{2}{3} \\ &= 10\tfrac{2}{3}\end{aligned}$$

We saw in Chapter 1 that another way of thinking about this is $32 \div 3 = 10$ with remainder 2.

Now that we have fractions, we can use them to write the result of dividing a whole number by another non-zero whole number. So $32 \div 3 = \frac{32}{3} = 10\frac{2}{3}$.

Both $32 \div 3 = 10\frac{2}{3}$ and $32 \div 3 = 10$ remainder 2 are correct, depending on the context.

Note that we can also convert mixed numerals to improper fractions. For example:

$$\begin{aligned}6\tfrac{3}{4} &= 6 + \frac{3}{4} \\ &= \frac{24}{4} + \frac{3}{4} \\ &= \frac{27}{4}\end{aligned}$$


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## Mixed numerals and improper fractions

- Division of a whole number by a non-zero whole number results in a fraction:
  $11 \div 3 = \frac{11}{3}$ or $3\frac{2}{3}$
- A **mixed numeral** is a whole number plus a proper fraction.
- An **improper fraction** can be written as a mixed numeral or a whole number.
- It is usually preferable to write fractions in simplest form.

### Example 6

Write $\frac{17}{5}$ as a mixed numeral.

#### Solution

$\frac{17}{5} = 3\frac{2}{5}$

The division algorithm can be used to convert an improper fraction to a mixed numeral.

### Example 7

Write $\frac{107}{7}$ as a mixed numeral.

#### Solution

$$7\overline{)1\,0^{3}7}\quad \begin{matrix}1\ 5\end{matrix}$$ remainder 2,

so $\frac{107}{7} = 15\frac{2}{7}$

### Example 8

Convert each of these mixed numerals to an improper fraction.

**a** $2\frac{1}{2}$ **b** $27\frac{3}{17}$

#### Solution

**a** $2\frac{1}{2} = \frac{2 \times 2 + 1}{2}$

$= \frac{5}{2}$

**b**
$$\begin{array}{r} 27 \\ \times\ 17 \\ \hline 189 \\ +\ 270 \\ \hline 459 \end{array}$$

$27\frac{3}{17} = \frac{27 \times 17 + 3}{17}$

$= \frac{462}{17}$


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## Exercise 4C

**1** Use a number line to illustrate each of these divisions.

**a** $1 \div 3 = \frac{1}{3}$ **b** $3 \div 2 = 1\frac{1}{2}$ **c** $5 \div 3 = 1\frac{2}{3}$ **d** $7 \div 4 = 1\frac{3}{4}$

**2** Represent each of these numbers on a number line.

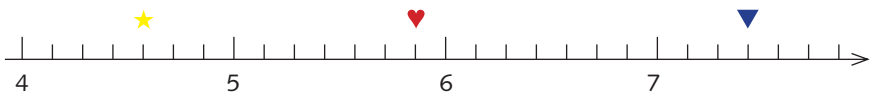
**a** $2\frac{2}{3}$ **b** $1\frac{5}{8}$ **c** $4\frac{2}{5}$ **d** $3\frac{4}{9}$

**3** Draw a number line marked from 5 to 10 and then indicate on it where the markers for $\frac{22}{4}$, $6\frac{3}{4}$ and $\frac{68}{8}$ are located.

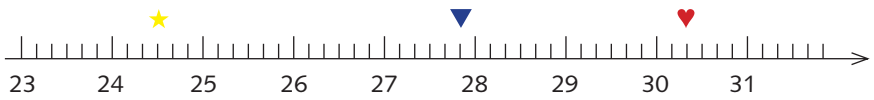
**4** Draw a number line marked from 0 to 6, and then indicate on it where the markers for $\frac{2}{3}$, $\frac{10}{3}$, $5\frac{1}{3}$, $\frac{16}{9}$, $\frac{18}{6}$ and $4\frac{2}{6}$ are located.

**5** Which numbers correspond to the star, the triangle and the heart on each number line?

**a**




**b**




Example 6, 7

**6** Convert each of these improper fractions to a mixed numeral and simplify where possible.

**a** $\frac{11}{3}$ **b** $\frac{48}{9}$ **c** $\frac{54}{12}$ **d** $\frac{44}{12}$ **e** $\frac{13}{3}$ **f** $\frac{175}{100}$

**g** $\frac{109}{27}$ **h** $\frac{475}{100}$ **i** $\frac{308}{56}$ **j** $\frac{312}{35}$ **k** $\frac{297}{25}$ **l** $\frac{412}{50}$

Example 8a

**7** Convert each mixed numeral to an improper fraction.

**a** $6\frac{3}{7}$ **b** $4\frac{3}{4}$ **c** $8\frac{3}{5}$ **d** $2\frac{9}{11}$ **e** $5\frac{3}{5}$ **f** $11\frac{5}{9}$

**g** $10\frac{3}{10}$ **h** $21\frac{1}{2}$ **i** $33\frac{2}{3}$ **j** $5\frac{3}{4}$ **k** $3\frac{6}{7}$ **l** $2\frac{5}{8}$

Example 8b

**8** Convert each mixed numeral to an improper fraction.

**a** $17\frac{3}{5}$ **b** $29\frac{11}{30}$ **c** $52\frac{13}{21}$ **d** $46\frac{11}{13}$

**9** Find the next five numbers in each sequence. (For each one, to go from one term to the next you add the same fraction.)

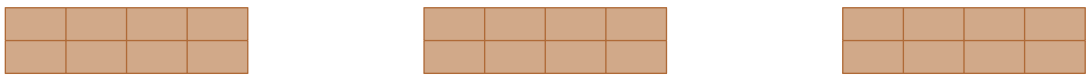
**a** $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1\frac{1}{4}, 1\frac{1}{2}$, __, __, __, __, __

**b** $\frac{1}{3}, \frac{2}{3}, 1, 1\frac{1}{3}, 1\frac{2}{3}, 2, 2\frac{1}{3}$, __, __, __, __, __

**c** $\frac{3}{8}, \frac{3}{4}, 1\frac{1}{8}, 1\frac{1}{2}, 1\frac{7}{8}$, __, __, __, __, __

**d** $\frac{4}{9}, \frac{7}{9}, 1\frac{1}{9}, 1\frac{4}{9}, 1\frac{7}{9}$, __, __, __, __, __

**10** Share these 3 chocolate bars equally among 4 friends. What fraction of a whole chocolate bar will each person get?

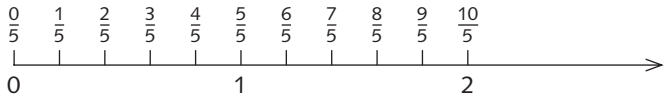



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# 4D Comparison of fractions

A number is **greater** than another if it lies to the right of that number on the number line.

If two fractions have the same denominator, then it is easy to decide which is the larger of the two. In the following diagram, we can see that $\frac{3}{5} > \frac{1}{5}$, and $\frac{7}{5} > \frac{4}{5}$.




## Using common denominators

When the denominators are not the same, it is more difficult to see which of two fractions is larger.

For example, it is not easy to see whether $\frac{5}{7}$ is larger than $\frac{3}{4}$. We need a general method.

To compare $\frac{2}{3}$ and $\frac{4}{5}$, we first express both fractions with a **common denominator**. Usually the best common denominator is the **lowest common denominator**, which is the lowest common multiple (LCM) of the two denominators.

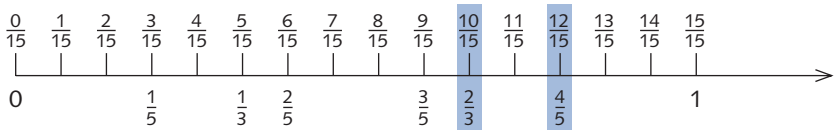
In this example, the lowest common denominator is 15.

We then find equivalent fractions for $\frac{2}{3}$ and $\frac{4}{5}$ with a denominator of 15.

$$\frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15} \qquad \frac{4}{5} = \frac{4 \times 3}{5 \times 3} = \frac{12}{15}$$

We can see that $\frac{12}{15}$ is larger than $\frac{10}{15}$, so $\frac{4}{5}$ is larger than $\frac{2}{3}$.

We can illustrate what we have just discovered on a number line. Consider the equivalent fractions with a denominator of 15.




### Comparing fractions

- If two fractions have the *same* denominator, then the one with the larger numerator is the larger fraction.
- If two fractions have *different* denominators, then to compare them we find equivalent fractions with a common denominator and compare the numerators.


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### Example 9

Which is larger, $\frac{5}{9}$ or $\frac{6}{11}$?

#### Solution

The lowest common multiple of 9 and 11 is 99.

$\frac{5}{9} = \frac{55}{99}$ and $\frac{6}{11} = \frac{54}{99}$

Clearly $\frac{55}{99} > \frac{54}{99}$, so $\frac{5}{9} > \frac{6}{11}$.

### Example 10

Order these fractions from smallest to largest.

$\frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{5}{12}, \frac{7}{24}, \frac{7}{12}$

#### Solution

The lowest common multiple of 2, 4, 8 and 24 is 24.

$\frac{1}{2} = \frac{12}{24}, \frac{3}{4} = \frac{18}{24}, \frac{5}{8} = \frac{15}{24}, \frac{5}{12} = \frac{10}{24}, \frac{7}{24} = \frac{7}{24}, \frac{7}{12} = \frac{14}{24}$

Arranging from smallest to largest gives

$\frac{7}{24}, \frac{10}{24}, \frac{12}{24}, \frac{14}{24}, \frac{15}{24}, \frac{18}{24}$

so the order of the original fractions is $\frac{7}{24}, \frac{5}{12}, \frac{1}{2}, \frac{7}{12}, \frac{5}{8}, \frac{3}{4}$.

## Exercise 4D

**1** Find a fraction that is:

**a** nearly 8 **b** close to one half

**c** a bit more than one quarter but less than one half

Example 9

**2** Identify the fraction that is larger in each case.

**a** $\frac{5}{7}$ or $\frac{6}{8}$ **b** $\frac{13}{5}$ or $\frac{5}{2}$ **c** $\frac{11}{7}$ or $\frac{13}{6}$

**d** $\frac{5}{2}$ or $\frac{12}{5}$ **e** $\frac{7}{8}$ or $\frac{5}{6}$ **f** $\frac{9}{6}$ or $\frac{11}{7}$


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**3** Express the following improper fractions as mixed numerals, and then decide which of each pair is larger.

**a** $\frac{13}{6}$ and $\frac{17}{8}$ **b** $\frac{21}{4}$ and $\frac{47}{8}$ **c** $\frac{12}{11}$ and $\frac{3}{2}$

**d** $\frac{107}{25}$ and $\frac{57}{14}$ **e** $\frac{130}{64}$ and $\frac{101}{50}$ **f** $\frac{143}{12}$ and $\frac{143}{13}$

**4** Compare these fractions by finding a common denominator. Which is smaller?

**a** $\frac{2}{3}$ or $\frac{5}{6}$ **b** $\frac{9}{8}$ or $\frac{8}{7}$ **c** $\frac{8}{17}$ or $\frac{1}{2}$

**d** $\frac{1}{8}$ or $\frac{4}{24}$ **e** $\frac{4}{6}$ or $\frac{17}{27}$

Example 10

**5** Order each set of fractions from smallest to largest.

**a** $\frac{3}{4}, \frac{9}{8}, \frac{10}{8}, \frac{1}{4}, \frac{3}{8}, \frac{1}{2}$ **b** $\frac{4}{3}, \frac{8}{9}, \frac{9}{6}, \frac{3}{6}, \frac{5}{3}, \frac{5}{6}$ **c** $\frac{4}{5}, \frac{1}{3}, \frac{19}{10}, \frac{21}{15}, \frac{4}{3}, \frac{11}{6}$

**d** $\frac{7}{8}, \frac{3}{4}, \frac{3}{5}, \frac{3}{7}, \frac{5}{6}, \frac{5}{9}$ **e** $\frac{1}{3}, \frac{1}{4}, \frac{3}{5}, \frac{4}{11}, \frac{7}{12}, \frac{8}{16}$ **f** $\frac{3}{4}, \frac{2}{5}, \frac{3}{8}, \frac{7}{10}, \frac{11}{20}, \frac{23}{40}$

**6** Find the fraction that is exactly halfway between:

**a** $\frac{1}{4}$ and $\frac{5}{8}$ **b** $\frac{2}{3}$ and 1 **c** $\frac{3}{5}$ and $\frac{5}{3}$ **d** $\frac{81}{10}$ and $\frac{38}{5}$

**7** Which is larger? $\frac{99}{100}$ or $\frac{98}{99}$. Give reasons.

# 4E Addition and subtraction of fractions

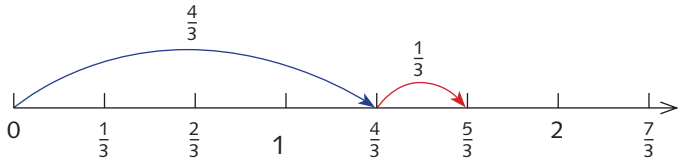
## Addition of fractions

Fractions that are to be added may have the same or different denominators. Just as with whole numbers, we can represent addition of fractions by taking steps to the right on the number line.

### Same denominators

Addition is easy if the denominators are the same. For example:

to find $\frac{4}{3} + \frac{1}{3}$, you take a step of $\frac{4}{3}$ followed by a step of $\frac{1}{3}$ to arrive at $\frac{5}{3}$




When the denominators of the fractions to be added are the same, simply add the numerators.

$$\frac{4}{3} + \frac{1}{3} = \frac{5}{3}$$


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### Different denominators

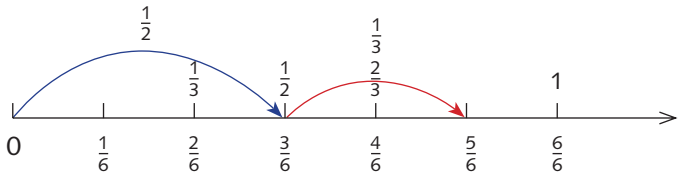
When the denominators are different, a little preliminary work needs to be done.

First, you need to use equivalent fractions with a common denominator to choose the right step size.

For example, to find $\frac{1}{2}+\frac{1}{3}$, first find a common denominator. Look at the multiples of 2 and the multiples of 3 until you find the lowest common multiple. In this case it is 6. Convert $\frac{1}{2}$ and $\frac{1}{3}$ into equivalent fractions with 6 as the lowest common denominator. Now that the fractions have the same denominator you can add, as before.

$$\begin{aligned}\frac{1}{2}+\frac{1}{3} &= \frac{1\times 3}{2\times 3}+\frac{1\times 2}{3\times 2}\\ &= \frac{3}{6}+\frac{2}{6}\\ &= \frac{5}{6}\end{aligned}$$

This can be shown on the number line.




Here we have used the LCM of 2 and 3 to form the two equivalent fractions $\frac{3}{6}$ and $\frac{2}{6}$.

You should always use a common denominator when adding fractions with different denominators.

## Subtraction of fractions

As with addition, if we want to subtract one fraction from a larger fraction, we should first ensure that we have common denominators.

### Addition and subtraction of fractions

- To add or subtract two fractions when the denominators are the same, we use the common denominator and add the numerators. For example:

$\frac{2}{7}+\frac{4}{7}=\frac{6}{7}$ $\qquad$ $\frac{4}{7}-\frac{2}{7}=\frac{2}{7}$

- The lowest common denominator of two fractions is the lowest common multiple (LCM) of the denominators of the two fractions.
- To add or subtract two fractions when the denominators are not the same, we first find equivalent fractions with the lowest common denominator. We then proceed as for fractions with the same denominator. For example:

$$\begin{aligned}\frac{2}{3}+\frac{1}{2} &= \frac{4}{6}+\frac{3}{6}\\ &= \frac{7}{6}\\ &= 1\frac{1}{6}\end{aligned}$$

$$\begin{aligned}\frac{2}{3}-\frac{1}{2} &= \frac{4}{6}-\frac{3}{6}\\ &= \frac{1}{6}\end{aligned}$$


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## Example 11

**a** Find $\frac{7}{15}+\frac{3}{15}$.

**b** Find $\frac{1}{6}+\frac{3}{4}$.

**c** Find $\frac{7}{15}+\frac{2}{3}$.

### Solution

**a** $\frac{7}{15}+\frac{3}{15}=\frac{10}{15}$

$=\frac{2}{3}$ (The fraction $\frac{10}{15}$ must be written in its simplest form.)

**b** $\frac{1}{6}+\frac{3}{4}=\frac{2}{12}+\frac{9}{12}$ (The LCM of 6 and 4 is 12.)

$=\frac{11}{12}$

**c** $\frac{7}{15}+\frac{2}{3}=\frac{7}{15}+\frac{10}{15}$ (The LCM of 15 and 3 is 15.)

$=\frac{17}{15}$

$=1\frac{2}{15}$

## Example 12

**a** Find $\frac{3}{8}-\frac{1}{8}$.

**b** Find $\frac{11}{12}-\frac{3}{8}$.

**c** Find $\frac{4}{5}-\frac{17}{65}$.

**d** Find $\frac{2}{3}+\frac{1}{6}-\frac{7}{9}$.

### Solution

**a** $\frac{3}{8}-\frac{1}{8}=\frac{2}{8}$

$=\frac{1}{4}$

**b** $\frac{11}{12}-\frac{3}{8}=\frac{22}{24}-\frac{9}{24}$

$=\frac{13}{24}$

**c** $\frac{4}{5}-\frac{17}{65}=\frac{52}{65}-\frac{17}{65}$

$=\frac{35}{65}$

$=\frac{7}{13}$

**d** $\frac{2}{3}+\frac{1}{6}-\frac{7}{9}=\frac{12}{18}+\frac{3}{18}-\frac{14}{18}$

$=\frac{1}{18}$


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### Example 13

Find $2\frac{3}{4} + 1\frac{7}{8}$.

### Solution

Deal with the whole numbers first, then find the lowest common denominator.

$$\begin{aligned} 2\tfrac{3}{4} + 1\tfrac{7}{8} &= 2 + 1 + \frac{3}{4} + \frac{7}{8} \\ &= 3 + \frac{3}{4} + \frac{7}{8} \qquad \text{(Now add } \tfrac{3}{4} \text{ and } \tfrac{7}{8}\text{.)} \\ &= 3 + \frac{6}{8} + \frac{7}{8} \\ &= 3 + \frac{13}{8} \\ &= 3 + 1\tfrac{5}{8} \\ &= 4\tfrac{5}{8} \end{aligned}$$

Subtraction of mixed numerals will be considered in section 4F of this chapter.

## Exercise 4E

**1** Use a number line to illustrate what is meant by:

**a** $\frac{3}{4} + \frac{1}{2}$ **b** $\frac{4}{5} + \frac{1}{10}$ **c** $\frac{7}{8} + \frac{3}{4}$

Example 11a

**2** Evaluate:

**a** $\frac{2}{4} + \frac{1}{4}$ **b** $\frac{5}{16} + \frac{8}{16}$ **c** $\frac{32}{100} + \frac{41}{100}$

Example 11b, c

**3** Find a common denominator and then perform each addition.

**a** $\frac{2}{5} + \frac{3}{10}$ **b** $\frac{1}{3} + \frac{3}{6}$ **c** $\frac{3}{4} + \frac{1}{8}$ **d** $\frac{5}{6} + \frac{1}{12}$

**e** $\frac{17}{25} + \frac{3}{5}$ **f** $\frac{11}{30} + \frac{4}{6}$ **g** $\frac{13}{21} + \frac{4}{7}$ **h** $\frac{43}{75} + \frac{14}{25}$

**4** Perform these additions, writing your answers as mixed numerals.

**a** $\frac{3}{4} + \frac{7}{8}$ **b** $\frac{2}{3} + \frac{5}{6}$ **c** $\frac{3}{5} + \frac{8}{10}$ **d** $\frac{5}{6} + \frac{7}{12}$

**e** $\frac{12}{15} + \frac{3}{5}$ **f** $\frac{11}{12} + \frac{1}{3}$ **g** $\frac{11}{30} + \frac{8}{12}$ **h** $\frac{20}{21} + \frac{5}{7}$

**i** $\frac{62}{75} + \frac{19}{25}$ **j** $\frac{3}{5} + \frac{7}{8}$ **k** $\frac{2}{3} + \frac{7}{10}$ **l** $\frac{13}{17} + \frac{7}{8}$


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Example 12a

**5** Calculate:

**a** $\frac{6}{8} - \frac{5}{8}$ **b** $\frac{8}{12} - \frac{7}{12}$ **c** $\frac{43}{100} - \frac{27}{100}$ **d** $\frac{1}{4} - \frac{1}{8}$

Example 12b, c

**e** $\frac{3}{4} - \frac{1}{8}$ **f** $\frac{8}{3} - \frac{7}{6}$ **g** $\frac{3}{5} - \frac{3}{10}$ **h** $\frac{5}{6} - \frac{13}{18}$

**i** $\frac{4}{5} - \frac{8}{15}$ **j** $\frac{4}{5} - \frac{1}{7}$ **k** $\frac{17}{20} - \frac{8}{15}$ **l** $\frac{7}{8} - \frac{2}{9}$

Example 12d

**6** **a** $\frac{1}{5} + \frac{2}{5} + \frac{3}{5}$ **b** $\frac{8}{17} + \frac{3}{17} + \frac{4}{17}$ **c** $\frac{3}{4} + \frac{3}{4} + \frac{3}{4}$ **d** $\frac{1}{7} + \frac{3}{7} + \frac{2}{7}$

**e** $\frac{7}{8} - \frac{1}{8} - \frac{3}{8}$ **f** $\frac{8}{9} - \frac{4}{9} - \frac{2}{9}$ **g** $\frac{3}{8} + \frac{3}{4} + \frac{3}{2}$ **h** $\frac{7}{12} - \frac{1}{4} + \frac{1}{6}$

**i** $\frac{1}{4} + \frac{1}{5} + \frac{1}{6}$ **j** $\frac{5}{9} - \frac{4}{8} + \frac{3}{7}$ **k** $\frac{7}{6} + \frac{1}{3} - \frac{17}{18} + \frac{2}{9}$ **l** $\frac{7}{8} + \frac{2}{3} - \frac{7}{11} + 2$

**7** **a** Find two different fractions that add to give 1.

**b** Find two fractions that add to give $\frac{1}{3}$.

**c** Find three fractions that add to give an answer between 2 and 3.

**8** What do you have to add to each of these fractions to make 1?

**a** $\frac{3}{7}$ **b** $\frac{12}{16}$ **c** $\frac{19}{32}$ **d** $\frac{43}{144}$

Example 13

**9** Perform these additions. Give your answers as mixed numerals.

**a** $2\frac{1}{4} + 1\frac{1}{4}$ **b** $2\frac{2}{5} + 3\frac{3}{5}$ **c** $3\frac{2}{12} + 4\frac{1}{3}$ **d** $4\frac{5}{6} + 1\frac{2}{3}$

**e** $8\frac{4}{21} + 3\frac{3}{7}$ **f** $3\frac{12}{25} + 4\frac{3}{5}$ **g** $2\frac{2}{3} + 2\frac{4}{5}$ **h** $8\frac{2}{5} + 5\frac{2}{8}$

# 4F Word problems involving addition and subtraction of fractions

As with any kind of word problem, ask yourself: 'What am I being asked to do here? What mathematics should I use to solve this?'

Here are some examples of word problems.

## Example 14

**a** Jane has $\frac{2}{3}$ of a litre of water in a jug and then pours in $\frac{1}{5}$ of a litre. How much water is in the jug now?

**b** Fran devotes $\frac{1}{3}$ of each day to schoolwork and she spends $\frac{1}{5}$ of each day watching television. As a fraction of the day, how much more time is spent on schoolwork than on television?


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**Solution**

**a** This is an addition.

$$\frac{1}{5}+\frac{2}{3}=\frac{3}{15}+\frac{10}{15}$$
$$=\frac{13}{15}$$

There is now $\frac{13}{15}$ of a litre of water in the jug.

**b** This is a difference question about fractions of each day.

$$\frac{1}{3}-\frac{1}{5}=\frac{5}{15}-\frac{3}{15}$$
$$=\frac{2}{15}$$

Fran spends $\frac{2}{15}$ of each day longer doing schoolwork than watching television.

## Exercise 4F

Example 14

**1** A jug has $\frac{2}{5}$ of a litre of water in it. Bao pours $\frac{1}{3}$ of a litre of water into the jug. How much water is in the jug now?

**2** A family travelling to Sale covers one-third of the journey before 1 p.m. and a further one-quarter of the journey between 1 p.m. and 2 p.m. What fraction of the journey have they travelled by 2 p.m.?

**3** Two-thirds of the tiles on a veranda are brown and one-quarter of the tiles are black. What fraction of the tiles are either black or brown?

**4** In a class, $\frac{1}{3}$ of the students are 12 years old and $\frac{1}{5}$ of the students are 11 years old. What fraction of the students in the class are either 11 or 12 years old?

**5** A girl has 1 litre of soft drink and lets her friend drink $\frac{2}{5}$ of it. How much soft drink is left?

**6** The sum of two fractions is $\frac{11}{9}$. One of the fractions is $\frac{1}{3}$. What is the other fraction?

**7** The difference of two fractions is $\frac{1}{4}$. The larger of the fractions is $\frac{7}{8}$. What is the smaller of the two fractions?

**8** When a particular fraction is added to itself, the result is $\frac{1}{3}$. What is the original fraction?

**9** A unit square is subdivided into 16 squares of equal area. Seven of these squares are painted blue and the remainder are painted red. What fraction of the unit square is red?

**10** Patricia takes 2 slices of a pie that was cut into 7 equal slices, and 3 slices of another identical pie that was cut into 9 equal slices.

**a** What fraction of the first pie did she take?

**b** What fraction of the second pie did she take?

**c** What fraction of a pie would she have if she put the slices together?


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**11** In a game of Australian Rules football, a team scored $\frac{1}{6}$ of its total points in the first quarter, $\frac{2}{9}$ of its points in the second quarter and $\frac{5}{27}$ of its points in the third quarter.

**a** What fraction of the team's points were scored in the first two quarters?

**b** What fraction of the team's points were scored in the first three quarters?

**c** What fraction of the team's points were scored in the final (fourth) quarter?

# 4G Subtraction of mixed numerals

We have already learned how to add mixed numerals. We can subtract mixed numerals in two ways. In the first method, we deal with the whole numbers first. In the second, we convert to improper fractions with the same denominator.

## Example 15

What is $3\frac{1}{2} - 2\frac{2}{5}$?

### Solution

**Method 1**

$$\begin{aligned} 3\tfrac{1}{2} - 2\tfrac{2}{5} &= \left(3 + \frac{1}{2}\right) - \left(2 + \frac{2}{5}\right) \\ &= 3 - 2 + \frac{1}{2} - \frac{2}{5} \\ &= 1 + \frac{1}{2} - \frac{2}{5} \\ &= 1 + \frac{5}{10} - \frac{4}{10} \\ &= 1\tfrac{1}{10} \end{aligned}$$

**Method 2**

$$\begin{aligned} 3\tfrac{1}{2} - 2\tfrac{2}{5} &= \frac{7}{2} - \frac{12}{5} \\ &= \frac{35}{10} - \frac{24}{10} \\ &= \frac{11}{10} \\ &= 1\tfrac{1}{10} \end{aligned}$$

If the first proper fraction is smaller than the second, then an additional step is needed in Method 1, as shown on page 121. What is a fraction?


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### Example 16

Calculate $4\frac{1}{6} - 2\frac{1}{2}$.

### Solution

**Method 1**

Deal with the whole numbers first.

$$\begin{aligned} 4\tfrac{1}{6} - 2\tfrac{1}{2} &= 3 + \frac{6}{6} + \frac{1}{6} - 2 - \frac{1}{2} \\ &= 3 + \frac{7}{6} - 2 - \frac{1}{2} \\ &= 3 - 2 + \frac{7}{6} - \frac{3}{6} \\ &= 1\tfrac{4}{6} \\ &= 1\tfrac{2}{3} \end{aligned}$$

**Method 2**

Convert to improper fractions.

$$\begin{aligned} 4\tfrac{1}{6} - 2\tfrac{1}{2} &= \frac{25}{6} - \frac{5}{2} \\ &= \frac{25}{6} - \frac{15}{6} \\ &= \frac{10}{6} \\ &= \frac{5}{3} \\ &= 1\tfrac{2}{3} \end{aligned}$$

## Exercise 4G

Example 15, 16

**1** Perform these subtractions, giving your answers as mixed numerals in simplest form where possible.

**a** $4\frac{4}{5} - 3\frac{3}{5}$ **b** $3\frac{8}{9} - 2\frac{5}{9}$ **c** $11\frac{1}{4} - 7\frac{3}{4}$

**d** $8\frac{7}{8} - 3\frac{1}{2}$ **e** $5\frac{1}{2} - 3\frac{3}{4}$ **f** $3\frac{5}{8} - 2\frac{1}{2}$

**g** $8\frac{2}{5} - 3\frac{1}{4}$ **h** $6\frac{7}{8} - 6\frac{3}{5}$ **i** $4\frac{1}{20} - 3\frac{7}{8}$

**2** Calculate each difference.

**a** $1\frac{1}{8} - \frac{7}{9}$ **b** $1\frac{1}{5} - \frac{1}{2}$ **c** $1\frac{1}{5} - \frac{3}{4}$

**d** $13\frac{1}{4} - 12\frac{1}{2}$ **e** $37\frac{1}{3} - 35\frac{5}{9}$ **f** $67\frac{7}{8} - 67\frac{2}{3}$

**g** $87\frac{1}{5} - 45\frac{2}{3}$ **h** $101\frac{1}{5} - 98\frac{3}{7}$ **i** $2\frac{1}{13} - 1\frac{11}{12}$

**j** $3\frac{2}{11} - \frac{7}{10}$ **k** $57\frac{7}{11} - 1\frac{9}{12}$ **l** $23\frac{2}{3} - 22\frac{4}{5}$

**3** What number when subtracted from $115\frac{7}{11}$ will leave $37\frac{3}{4}$?

**4** The difference between two numbers is $11\frac{3}{28}$, and the smaller number is $15\frac{5}{21}$. Find the larger number.

**5** The sum of $1\frac{1}{5}$ and a larger number is $5\frac{7}{10}$. Find the larger number.

**6** Evaluate:

**a** $2\frac{7}{8} - 1\frac{1}{4} + 3 - 2\frac{1}{2}$ **b** $6\frac{7}{10} - 3\frac{2}{5} - 1\frac{1}{10}$

**7** From the sum of $1\frac{1}{8}$ and $2\frac{1}{3}$, take the difference between $5\frac{1}{4}$ and $4\frac{7}{8}$.


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# Review exercise

**1** Represent these fractions on a number line and also using disections of squares or rectangles.

**a** $\frac{4}{5}$ **b** $\frac{3}{8}$

**2** Write the fraction that represents each situation.

**a** three weeks in December

**b** five vowels in the alphabet

**c** five bruised fingers on two hands

**d** 343 sheets out of a ream of 500 sheets of paper

**3** Write the highest common factor of each pair of numbers.

**a** 12 and 44 **b** 8 and 24 **c** 16 and 18

**d** 36 and 44 **e** 108 and 72 **f** 144 and 50

**4** Write the lowest common multiple of each pair of numbers.

**a** 2 and 3 **b** 4 and 6 **c** 15 and 8

**5** Express each as a proper fraction or mixed numeral in simplest form.

**a** $\frac{18}{24}$ **b** $\frac{16}{12}$ **c** $\frac{105}{100}$ **d** $\frac{64}{54}$ **e** $\frac{98}{56}$ **f** $\frac{17}{51}$

**6** Evaluate:

**a** $\frac{2}{3}+\frac{7}{8}$ **b** $\frac{7}{12}+\frac{1}{3}$ **c** $\frac{11}{15}+\frac{2}{3}$ **d** $\frac{5}{23}+\frac{1}{10}$

**e** $\frac{3}{11}+\frac{2}{3}$ **f** $\frac{1}{2}+\frac{5}{11}$ **g** $\frac{2}{11}+\frac{5}{3}$ **h** $\frac{9}{10}+\frac{10}{11}$

**7** Evaluate:

**a** $3\frac{1}{2}+2\frac{7}{8}$ **b** $3\frac{3}{4}+11\frac{1}{5}$ **c** $2\frac{7}{8}+5\frac{1}{7}$ **d** $4\frac{1}{3}+2\frac{7}{8}$

**e** $1\frac{1}{3}+2\frac{3}{4}$ **f** $5\frac{5}{7}+1\frac{1}{3}$ **g** $1\frac{1}{11}+2\frac{1}{3}$ **h** $7\frac{7}{10}+22\frac{2}{5}$

**8** Evaluate:

**a** $\frac{7}{8}-\frac{4}{5}$ **b** $\frac{17}{10}-\frac{7}{8}$ **c** $\frac{5}{12}-\frac{1}{3}$ **d** $\frac{11}{15}-\frac{11}{16}$

**e** $\frac{4}{3}-\frac{9}{10}$ **f** $\frac{12}{23}-\frac{1}{10}$ **g** $\frac{7}{3}-\frac{3}{4}$ **h** $\frac{16}{15}-\frac{3}{4}$


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**9** Evaluate:

**a** $3\frac{2}{3} - 1\frac{1}{3}$ **b** $5\frac{3}{4} - 2\frac{1}{4}$ **c** $6\frac{7}{8} - 3\frac{2}{3}$ **d** $6\frac{1}{3} - 2\frac{1}{4}$

**e** $5\frac{1}{3} - 2\frac{3}{4}$ **f** $6\frac{1}{2} - 2\frac{3}{4}$ **g** $5\frac{7}{12} - 4\frac{3}{4}$ **h** $22\frac{7}{12} - 16\frac{7}{10}$

**10** Write in the boxes to show what number the numerator and denominator were multiplied by to arrive at the equivalent fraction.

**a** $\frac{\square}{3} = \frac{54}{\square 81}$ **b** $\frac{\square 5}{7} = \frac{25}{\square 35}$ **c** $\frac{33}{48} = \frac{\square 99}{\square 144}$ **d** $\frac{17}{18} = \frac{\square 119}{\square 126}$

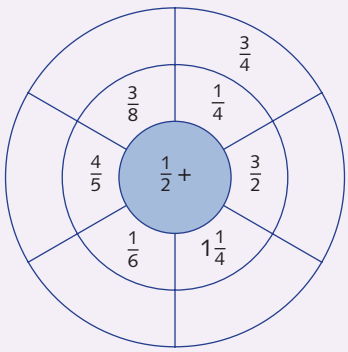
**11** Copy the statements below and fill in each square, using either a + sign or a – sign, to make the statements correct.

**a** $1\frac{1}{4} \square \frac{1}{4} \square \frac{3}{4} = \frac{3}{4}$ **b** $1\frac{7}{8} \square \frac{1}{4} \square \frac{3}{8} = 1\frac{1}{4}$

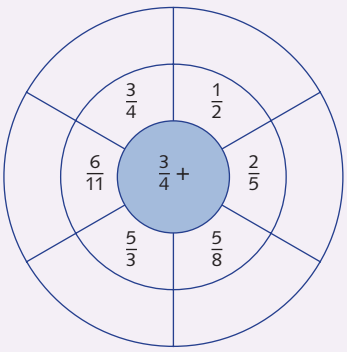
**c** $2 \square \frac{1}{2} \square \frac{1}{4} \square \frac{3}{4} = 2$ **d** $1 \square \frac{1}{2} \square \frac{1}{4} \square \frac{3}{8} = \frac{7}{8}$

**12** Copy each of the target diagrams below into your workbook. Add the fraction in the centre circle to each fraction in the second circle and write your answer in the outermost circle. One part of the first diagram has been done for you.

**a**




**b**




**13** In an apartment block, $\frac{7}{8}$ of the people speak English as their native language and $\frac{1}{16}$ speak Chinese as their native language. What fraction speak a language other than English or Chinese as their native language?

**14** The sum of two fractions is $\frac{13}{12}$. If one fraction is $\frac{1}{4}$, what is the other?

**15** In a class of 25 students, 12 are boys.

**a** What fraction of the class is girls?

**b** If 5 more girls join the class, what fraction of the class is girls?


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# Challenge exercise

**1** Which of the two fractions $\frac{3004}{3005}$ and $\frac{3005}{3006}$ is larger? Carefully explain why.

**2** We can write $\frac{1}{6}$ as the sum of two fractions, each with 1 as the numerator, as $\frac{1}{7}+\frac{1}{42}$. This can be done in four other ways. Find these.

**3** Express $\frac{1}{8}$ as the sum of two fractions, each with 1 as the numerator, in four ways.

**4** Find three fractions between $\frac{1}{4}$ and $\frac{1}{2}$.

**5** Find $x$ if $\frac{1}{3}+\frac{1}{4}+\frac{1}{x}=1$.

**6** Two fractions are equally spaced between $\frac{1}{4}$ and $\frac{4}{5}$. Find the two fractions.

**7** Complete the following magic squares, in which each row, column and diagonal must add up to the same sum.

**a**

| | | |
|---|---|---|
| $2\frac{1}{2}$ | | |
| | $1\frac{5}{6}$ | |
| 3 | | $1\frac{1}{6}$ |

**b**

| | | |
|---|---|---|
| 3 | | $2\frac{4}{5}$ |
| | $2\frac{7}{10}$ | |
| $2\frac{3}{5}$ | | |

**8** Evaluate:

**a** $1-\left(\frac{1}{2}+\frac{1}{4}\right)$ **b** $1-\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\right)$ **c** $1-\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}\right)$

**d** $1-\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64}+\frac{1}{128}\right)$ **e** What do you notice?

**9** Of the marbles in a bag, $\frac{1}{5}$ are blue and $\frac{2}{3}$ are red. The remaining marbles are yellow, and there are 6 of these. How many marbles are in the bag?

**10** There are 50 black sheep and 42 white sheep in a paddock. How many black sheep must be removed so that $\frac{1}{3}$ of the remaining sheep are black?


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CHAPTER

5

Number and Algebra

# Fractions – part 2

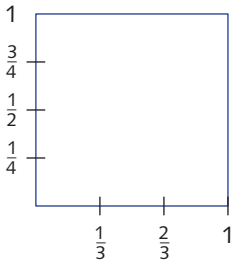
We have already learned about multiplying and dividing whole numbers, and we turn now to multiplying and dividing fractions. This will complete our understanding of the four operations applied to fractions and we can use these to solve problems.

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# 5A Multiplication of fractions

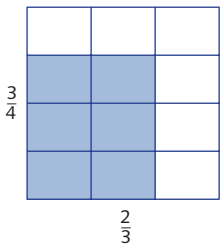
How do we multiply two fractions such as $\frac{2}{3} \times \frac{3}{4}$?

We will use the area model for multiplication that we used in Chapter 1. Begin with a square of side length 1. Its area will also be 1.




Now divide the square into rectangles by drawing vertical lines and horizontal lines, as shown.

Thus, we divide the height into quarters and the base into thirds. Each rectangle has area $\frac{1}{12}$. Next, shade the rectangle with side lengths $\frac{2}{3}$ and $\frac{3}{4}$, as shown in the diagram.




We can see that the shaded region is made up of $2 \times 3 = 6$ rectangles, each of area $\frac{1}{12}$, so the area of the shaded region is the product of the sides, $\frac{2}{3} \times \frac{3}{4}$.

But this is also equal to 6 lots of $\frac{1}{12} = \frac{6}{12}$. We conclude that:

$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$$

From this, we see that the rule for multiplying two fractions is multiply the numerators, multiply the denominators. This can be written as:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

Notice that when we find a fraction 'of' a whole number, we get the same result as if we were multiplying the fraction by that whole number. For example, finding $\frac{1}{2}$ of 6 is the same as evaluating $\frac{1}{2} \times 6$. The same applies when dealing with two fractions. For example, when we say $\frac{2}{3}$ of $\frac{3}{4}$, we can write $\frac{2}{3} \times \frac{3}{4}$.

## Multiplication of fractions

- To multiply two fractions together, multiply the two numerators together and the two denominators together to form the new fraction and simplify if possible.

  In symbols: $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$

  For example: $\frac{2}{3} \times \frac{7}{8} = \frac{14}{24} = \frac{7}{12}$

- The whole number $n$ should be written as $\frac{n}{1}$. For example: $\frac{3}{10} \times 3 = \frac{3}{10} \times \frac{3}{1} = \frac{9}{10}$

- Multiplication of two fractions and the corresponding 'of' statement give the same answer.

  For example: $\frac{2}{3}$ of $\frac{3}{4} = \frac{1}{2}$ is the same as $\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$


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**Example 1**

Evaluate:

**a** $\frac{3}{4} \times \frac{10}{7}$ **b** $\frac{9}{8} \times 4$

**Solution**

**a**
$$\begin{aligned}\frac{3}{4} \times \frac{10}{7} &= \frac{3 \times 10}{4 \times 7} \\ &= \frac{30}{28} \\ &= \frac{15}{14} \\ &= 1\frac{1}{14}\end{aligned}$$

**b**
$$\begin{aligned}\frac{9}{8} \times 4 &= \frac{9}{8} \times \frac{4}{1} \\ &= \frac{9 \times 4}{8 \times 1} \\ &= \frac{36}{8} \\ &= \frac{9}{2} \\ &= 4\frac{1}{2}\end{aligned}$$

## Cancelling

We saw earlier that we can sometimes cancel before multiplying.

Let us look at the multiplication $\frac{2}{3} \times \frac{3}{4}$.

$$\begin{aligned}\frac{2}{3} \times \frac{3}{4} &= \frac{6}{12} \\ &= \frac{1}{2}\end{aligned}$$

There is a shorthand way of writing this, which often simplifies the process of multiplication.

$$\begin{aligned}\frac{2}{3} \times \frac{3}{4} &= \frac{2}{\cancel{3}^{1}} \times \frac{\cancel{3}^{1}}{4} \\ &= \frac{2}{4} \\ &= \frac{\cancel{2}^{1}}{\cancel{4}^{2}} \\ &= \frac{1}{2}\end{aligned}$$

This process is called **cancelling**. We can cancel because we are dividing the numerator and the denominator by the same whole number. We know that this gives an equivalent fraction.

### Cancelling

- **Cancelling** is used to simplify fractions.
- To simplify a product of fractions, find a common factor of a numerator and a denominator and cancel. Repeat if possible.


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### Example 2

**a** Evaluate $\frac{5}{6} \times \frac{2}{3}$.

**b** Evaluate $\frac{16}{27} \times \frac{18}{20}$.

#### Solution

**a**
$$\frac{5}{6} \times \frac{2}{3} = \frac{5}{6^{3}} \times \frac{2^{1}}{3}$$
$$= \frac{5}{3} \times \frac{1}{3}$$
$$= \frac{5 \times 1}{3 \times 3}$$
$$= \frac{5}{9}$$

**b**
$$\frac{16}{27} \times \frac{18}{20} = \frac{16^{4}}{27^{3}} \times \frac{18^{2}}{20^{5}}$$
$$= \frac{4}{3} \times \frac{2}{5}$$
$$= \frac{4 \times 2}{3 \times 5}$$
$$= \frac{8}{15}$$

Cancelling should be done before multiplication. Cancelling is very helpful when three or more fractions are involved. Note that, as for multiplication of whole numbers, order does not matter.

### Example 3

Calculate:

**a** $\frac{2}{3} \times \frac{3}{4} \times \frac{5}{8}$

**b** $\frac{11}{5} \times \frac{10}{14} \times \frac{7}{11}$

#### Solution

**a**
$$\frac{2}{3} \times \frac{3}{4} \times \frac{5}{8} = \frac{2^{1}}{3^{1}} \times \frac{3^{1}}{4^{2}} \times \frac{5}{8}$$
$$= \frac{5}{16}$$

**b**
$$\frac{11}{5} \times \frac{10}{14} \times \frac{7}{11} = \frac{11^{1}}{5^{1}} \times \frac{10^{2}}{14^{2}} \times \frac{7^{1}}{11^{1}}$$
$$= \frac{2}{2}$$
$$= 1$$

## Exercise 5A

Example 1

**1** Evaluate each product, simplifying where possible.

**a** $\frac{1}{4} \times \frac{3}{5}$ **b** $\frac{3}{4} \times \frac{5}{8}$ **c** $\frac{1}{2} \times \frac{1}{8}$

**d** $\frac{1}{3} \times \frac{2}{3}$ **e** $\frac{5}{4} \times \frac{3}{7}$ **f** $\frac{5}{11} \times \frac{3}{8}$

**g** $\frac{7}{2} \times \frac{7}{5}$ **h** $\frac{3}{2} \times \frac{3}{7}$ **i** $\frac{7}{6} \times \frac{5}{7}$


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Example 1

**2** Find:

**a** $4 \times \frac{5}{7}$ **b** $7 \times \frac{3}{11}$ **c** $\frac{5}{12} \times 7$

**d** $11 \times \frac{2}{3}$ **e** $\frac{3}{4} \times 5$ **f** $\frac{4}{11} \times 3$

**3** Find:

**a** $\frac{1}{3}$ of $\frac{1}{2}$ **b** $\frac{1}{6}$ of $\frac{2}{3}$ **c** $\frac{5}{8}$ of $\frac{3}{4}$ **d** $\frac{2}{3}$ of $\frac{3}{2}$

**e** $\frac{5}{4}$ of $\frac{3}{5}$ **f** $\frac{2}{9}$ of $\frac{4}{3}$ **g** $\frac{2}{3}$ of 4 **h** $\frac{4}{5}$ of 13

Example 2

**4** Find the value of each of:

**a** $\frac{4}{3} \times \frac{6}{8}$ **b** $\frac{5}{9} \times \frac{3}{7}$ **c** $\frac{6}{14} \times \frac{7}{8}$ **d** $\frac{8}{21} \times \frac{3}{40}$

**e** $\frac{17}{15} \times \frac{10}{34}$ **f** $\frac{12}{7} \times \frac{49}{60}$ **g** $\frac{44}{28} \times \frac{21}{66}$ **h** $\frac{30}{54} \times \frac{27}{75}$

Example 3

**5** Complete each multiplication.

**a** $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{6}$ **b** $\frac{3}{4} \times \frac{5}{6} \times \frac{3}{8}$ **c** $\frac{1}{2} \times \frac{3}{4} \times \frac{7}{12}$

**d** $\frac{3}{4} \times \frac{5}{6} \times \frac{2}{5}$ **e** $\frac{7}{12} \times \frac{6}{7} \times \frac{5}{9}$ **f** $\frac{2}{3} \times \frac{5}{6} \times \frac{4}{15}$

**g** $\frac{3}{8} \times \frac{5}{9} \times \frac{8}{15}$ **h** $\frac{24}{7} \times \frac{14}{15} \times \frac{5}{16}$ **i** $\frac{8}{9} \times \frac{5}{18} \times \frac{9}{10}$

**j** $\frac{3}{10} \times \frac{5}{9} \times \frac{4}{5}$ **k** $\frac{14}{15} \times \frac{16}{21} \times \frac{5}{8}$ **l** $\frac{8}{11} \times \frac{22}{25} \times \frac{15}{32}$

# 5B Division of fractions

## Dividing by whole numbers

We saw in Section 4C how to divide two whole numbers.

For example, $4 \div 3 = \frac{4}{3} = 1\frac{1}{3}$ and $17 \div 5 = \frac{17}{5} = 3\frac{2}{5}$.

Thus, dividing 4 by 3 gives the same answer as finding $4 \times \frac{1}{3} = \frac{4}{3}$, and dividing 17 by 5 is the same as $17 \times \frac{1}{5} = \frac{17}{5}$.


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In general, for whole numbers $m$ and $n$ dividing $m$ by $n$ is the same as multiplying $m$ by $\frac{1}{n}$.

$$m \div n = m \times \frac{1}{n}$$

The fraction $\frac{1}{n}$ is called the **reciprocal** of $n$.

Thus, the reciprocal of 3 is $\frac{1}{3}$ and the reciprocal of 5 is $\frac{1}{5}$.

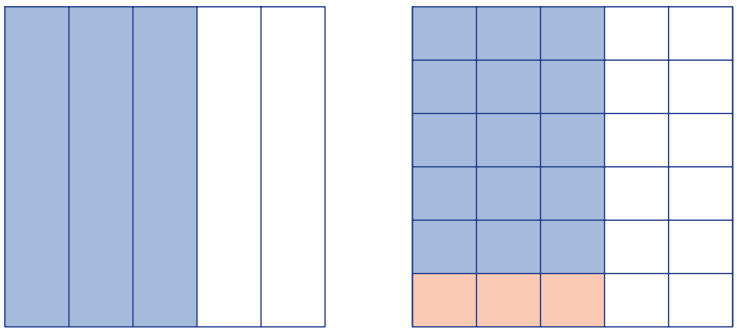
## Division of a fraction by a whole number

The idea of performing a division by multiplying by the reciprocal can also be used to divide a fraction by a whole number. For example:

$$\frac{3}{5} \div 6$$

This means that we take $\frac{3}{5}$ and divide it into 6 equal parts.

This can be illustrated by drawing a unit square divided into fifths.



Shade $\frac{3}{5}$, as shown in the first diagram, and further divide the square into 6 equal horizontal strips, as shown in the second diagram.

The orange shaded area represents both $\frac{3}{5} \div 6$ and $\frac{3}{30}$.

Notice that $\frac{3}{30} = \frac{3}{5} \times \frac{1}{6}$, so the rule of multiplying by the reciprocal also holds.

## Dividing by a fraction

Suppose we wish to divide 8 by $\frac{1}{2}$. We can express this question as 'How many halves in 8 wholes?' Since there are 2 halves in a whole, there will be 16 halves in 8 wholes. Thus, $8 \div \frac{1}{2} = 16$. Once again, the idea of multiplying by the reciprocal gives us the correct answer, since:

$$8 \div \frac{1}{2} = 8 \times \frac{2}{1} = 16$$

Similarly, 6 divided by $\frac{2}{3}$ means that we divide 6 into 18 thirds, and then divide this by 2.


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Thus:

$$6 \div \frac{2}{3} = 6 \times \frac{3}{2} = \frac{18}{2} = 9$$

In both examples, we turn the second fraction upside down and multiply.

The **reciprocal** of a fraction is the fraction obtained by swapping the numerator and denominator.

To divide a number by a fraction, we multiply the number by the reciprocal.

## Dividing a fraction by a fraction

One way of dividing a fraction by a fraction is to express each fraction using a common denominator so that we are dividing two fractions of the same type.

For example, to find $\frac{3}{5} \div \frac{2}{3}$, we write:

$$\frac{3}{5} \div \frac{2}{3} = \frac{9}{15} \div \frac{10}{15}$$

Since the denominators are the same, we divide the numerators to obtain $\frac{9}{10}$.

Notice that $\frac{3}{5} \times \frac{3}{2} = \frac{9}{10}$.

Thus:

$$\frac{3}{5} \div \frac{2}{3} = \frac{3}{5} \times \frac{3}{2} = \frac{9}{10}$$

So dividing $\frac{3}{5}$ by $\frac{2}{3}$ is the same as multiplying it by $\frac{3}{2}$. We say that $\frac{3}{2}$ is the **reciprocal** of $\frac{2}{3}$. It is the fraction obtained when the numerator and denominator are swapped.

Thus, the rule '**To divide by a fraction, multiply by the reciprocal**' holds in all situations.

### Division of fractions

- The **reciprocal** of a fraction is the fraction we get by swapping the numerator and denominator. The product of a fraction and its reciprocal is 1.
- Division and multiplication of fractions are **inverse** operations.
  For example: $\frac{2}{9} \div \frac{1}{8}$ means the fraction that, when multiplied by $\frac{1}{8}$, gives $\frac{2}{9}$.
- Dividing a fraction by a fraction is the same as multiplying the first fraction by the reciprocal of the second. So:

$$\frac{2}{9} \div \frac{1}{8} = \frac{2}{9} \times 8 = \frac{16}{9} \quad \text{and} \quad \frac{3}{7} \div \frac{4}{5} = \frac{3}{7} \times \frac{5}{4} = \frac{15}{28}$$

In general: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$


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## Example 4

Work out $\frac{2}{3} \div \frac{3}{5}$ by multiplying by the reciprocal. Check that the answer you get is the number that, when multiplied by the divisor $\frac{3}{5}$, gives $\frac{2}{3}$.

### Solution

$$\frac{2}{3} \div \frac{3}{5} = \frac{2}{3} \times \frac{5}{3} \quad \left(\frac{5}{3} \text{ is the reciprocal of } \frac{3}{5}\right)$$
$$= \frac{2 \times 5}{3 \times 3}$$
$$= \frac{10}{9}$$

Checking:
$$\frac{3}{5} \times \frac{10}{9} = \frac{30}{45}$$
$$= \frac{2}{3} \text{ (common factor 15)}$$

## Example 5

Find:

**a** $6 \div \frac{5}{7}$

**b** $\frac{3}{5} \div \frac{4}{7}$

### Solution

**a**
$$6 \div \frac{5}{7} = 6 \times \frac{7}{5}$$
$$= \frac{6}{1} \times \frac{7}{5}$$
$$= \frac{42}{5}$$
$$= 8\frac{2}{5}$$

**b**
$$\frac{3}{5} \div \frac{4}{7} = \frac{3}{5} \times \frac{7}{4}$$
$$= \frac{3 \times 7}{5 \times 4}$$
$$= \frac{21}{20}$$
$$= 1\frac{1}{20}$$

## Example 6

Evaluate:

**a** $\frac{3}{4} \div \frac{1}{4}$

**b** $\frac{7}{10} \div \frac{3}{5}$

**c** $\frac{3}{7} \div \frac{9}{14}$

### Solution

**a**
$$\frac{3}{4} \div \frac{1}{4} = \frac{3}{\cancel{4}^{1}} \times \frac{\cancel{4}^{1}}{1}$$
$$= \frac{3}{1}$$
$$= 3$$

**b**
$$\frac{7}{10} \div \frac{3}{5} = \frac{7}{\cancel{10}^{2}} \times \frac{\cancel{5}^{1}}{3}$$
$$= \frac{7}{6}$$
$$= 1\frac{1}{6}$$

**c**
$$\frac{3}{7} \div \frac{9}{14} = \frac{\cancel{3}^{1}}{\cancel{7}^{1}} \times \frac{\cancel{14}^{2}}{\cancel{9}^{3}}$$
$$= \frac{1}{1} \times \frac{2}{3}$$
$$= \frac{2}{3}$$


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### Example 7

Evaluate (work from left to right):

**a** $\frac{3}{4} \times \frac{11}{3} \div \frac{11}{2}$

**b** $\frac{7}{8} \div \frac{1}{2} \div \frac{2}{7}$

### Solution

**a** $\frac{3}{4} \times \frac{11}{3} \div \frac{11}{2} = \frac{\cancel{3}^{1}}{\cancel{4}^{2}} \times \frac{\cancel{11}^{1}}{\cancel{3}^{1}} \times \frac{2^{1}}{\cancel{11}^{1}}$

$= \frac{1}{2}$

**b** $\frac{7}{8} \div \frac{1}{2} \div \frac{2}{7} = \frac{7}{8} \times \frac{2}{1} \times \frac{7}{2}$

$= \frac{7}{8} \times \frac{\cancel{2}^{1}}{1} \times \frac{7}{\cancel{2}^{1}}$

$= \frac{49}{8}$

$= 6\frac{1}{8}$

## Exercise 5B

Example 5a

**1** Calculate:

**a** $4 \div \frac{3}{5}$ **b** $2 \div \frac{3}{7}$ **c** $2 \div \frac{5}{6}$ **d** $5 \div \frac{4}{7}$

**e** $3 \div \frac{8}{9}$ **f** $5 \div \frac{6}{11}$ **g** $7 \div \frac{2}{3}$ **h** $11 \div \frac{5}{6}$

Example 5b

**2** Find:

**a** $\frac{3}{4} \div \frac{5}{7}$ **b** $\frac{2}{3} \div \frac{5}{11}$ **c** $\frac{3}{11} \div \frac{5}{8}$ **d** $\frac{3}{11} \div \frac{2}{3}$

**e** $\frac{5}{8} \div \frac{2}{3}$ **f** $\frac{1}{2} \div \frac{3}{7}$ **g** $\frac{5}{12} \div \frac{7}{11}$ **h** $\frac{5}{7} \div \frac{11}{13}$

Example 6

**3** Evaluate:

**a** $\frac{3}{4} \div \frac{3}{8}$ **b** $\frac{3}{8} \div \frac{3}{4}$ **c** $\frac{5}{6} \div \frac{5}{9}$ **d** $\frac{2}{3} \div \frac{5}{6}$

**e** $\frac{3}{4} \div \frac{5}{8}$ **f** $\frac{2}{3} \div \frac{4}{9}$ **g** $\frac{5}{12} \div \frac{7}{16}$ **h** $\frac{9}{16} \div \frac{3}{20}$

**i** $\frac{8}{9} \div \frac{16}{27}$ **j** $\frac{7}{18} \div \frac{14}{27}$ **k** $\frac{5}{24} \div \frac{15}{16}$ **l** $\frac{72}{100} \div \frac{16}{25}$


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Example 7

**4** Evaluate:

**a** $\frac{4}{9} \times \frac{3}{4} \div \frac{2}{3}$ **b** $\frac{5}{6} \times \frac{2}{3} \div \frac{5}{9}$ **c** $\frac{2}{3}$ of $\frac{3}{4} \div \frac{8}{9}$ **d** $\frac{7}{9} \div \frac{3}{4} \times \frac{14}{27}$

**e** $\frac{2}{9} \div \frac{3}{4} \times \frac{2}{3}$ **f** $\frac{5}{12} \div \frac{2}{3} \times \frac{5}{8}$ **g** $\frac{3}{4} \div \frac{9}{16} \times \frac{1}{6}$ **h** $\frac{5}{9} \div \frac{2}{3} \times \frac{3}{5}$

**i** $\frac{3}{12} \times \frac{4}{5} \div \frac{3}{25}$ **j** $4 \div \frac{2}{3} \times \frac{3}{5}$ **k** $\frac{2}{3} \div \frac{5}{12} \div 4$ **l** $\frac{4}{9} \times \frac{3}{5} \div \frac{7}{15}$

**m** $\frac{1}{2} \div \frac{1}{2} \div \frac{1}{2} \div \frac{1}{2}$ **n** $\frac{1}{2} \times \frac{1}{2} \div \frac{1}{2} \times \frac{1}{2} \div \frac{1}{2}$

**o** $\frac{4}{7} \times \frac{14}{100} \div \frac{34}{110} \div \frac{11}{17}$ **p** $\frac{24}{81} \div \frac{96}{18} \times \frac{132}{72} \div \frac{55}{24}$

**5** To how many people can I give $\frac{2}{3}$ of a chocolate bar if I have 4 chocolate bars?

**6** I have 12 oranges. To how many people can I give $\frac{3}{4}$ of an orange?

**7** A ribbon that is 20 m long is to be divided into lengths of $\frac{5}{8}$ of a metre. How many such lengths are there?

# 5C Multiplication and division of mixed numerals

When multiplying and dividing mixed numerals, convert to improper fractions and then proceed as you have already learned.

## Example 8

**a** Evaluate $1\frac{2}{3} \times 4\frac{1}{2}$.

**b** Evaluate $7\frac{3}{4} \div 2\frac{1}{3}$.

## Solution

**a** $1\frac{2}{3} \times 4\frac{1}{2} = \frac{5}{\cancel{3}^{1}} \times \frac{\cancel{9}^{3}}{2}$ (Change to improper fractions.)

$= \frac{15}{2}$

$= 7\frac{1}{2}$ (Convert to a mixed numeral.)

**b** $7\frac{3}{4} \div 2\frac{1}{3} = \frac{31}{4} \div \frac{7}{3}$ (Change to improper fractions.)

$= \frac{31}{4} \times \frac{3}{7}$ (Multiply by the reciprocal of $\frac{7}{3}$.)

$= \frac{93}{28}$

$= 3\frac{9}{28}$ (Convert to a mixed numeral.)


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### Operations with mixed numbers

When dealing with multiplication and division and mixed numerals, the following strategy is normally used.
Convert each mixed numeral to an improper fraction and proceed as usual.

## Exercise 5C

Example 8a

**1** Find:

**a** $5\frac{1}{2} \times 6\frac{1}{4}$ **b** $4\frac{4}{5} \times 3\frac{3}{4}$ **c** $2\frac{2}{9} \times 1\frac{1}{3}$ **d** $\frac{1}{3} \times 3\frac{2}{3}$

**e** $\frac{1}{7} \times 4\frac{1}{2}$ **f** $\frac{7}{8} \times 3\frac{3}{5}$ **g** $5\frac{1}{9} \times \frac{2}{3}$ **h** $2\frac{1}{4} \times \frac{1}{5}$

**i** $3\frac{1}{3} \times 5$ **j** $8\frac{1}{2} \times 4$ **k** $6 \times 2\frac{3}{7}$ **l** $5 \times 3\frac{3}{5}$

Example 8b

**2** Find:

**a** $1\frac{1}{3} \div \frac{1}{3}$ **b** $2\frac{2}{3} \div \frac{5}{6}$ **c** $1\frac{5}{12} \div \frac{3}{4}$ **d** $6 \div 1\frac{1}{3}$

**e** $7\frac{1}{4} \div 1\frac{1}{5}$ **f** $2\frac{1}{5} \div 1\frac{1}{3}$ **g** $5\frac{1}{3} \div 4\frac{1}{4}$ **h** $3\frac{2}{5} \div \frac{5}{8}$

**3** Find:

**a** $2\frac{1}{2} \div 1\frac{1}{4}$ **b** $4\frac{1}{3} \div 1\frac{1}{2}$ **c** $6\frac{3}{5} \div 2\frac{1}{3}$ **d** $10\frac{1}{2} \div 5\frac{7}{8}$

**e** $5\frac{1}{3} \div 6\frac{1}{4}$ **f** $4\frac{1}{5} \div 1\frac{1}{4}$ **g** $2\frac{1}{8} \div 7\frac{1}{5}$ **h** $5\frac{1}{2} \div 7\frac{1}{3}$

**i** $2\frac{1}{9} \div 2\frac{1}{4}$ **j** $1\frac{1}{8} \div 7\frac{1}{2}$ **k** $5\frac{1}{25} \div 4\frac{1}{5}$ **l** $21\frac{1}{2} \div 14\frac{1}{4}$

**4** Five boys each have $1\frac{1}{4}$ litres of lemonade. How many litres of lemonade do the boys have in total?

**5** Find:

**a** $\frac{2}{3}$ of $3\frac{1}{4}$ **b** $\frac{3}{4}$ of $5\frac{1}{12}$ **c** $\frac{7}{8}$ of $6\frac{1}{5}$

**6** Jake shares his liquorice strap equally between himself and five friends. If the strap is $1\frac{2}{3}$ metres long, how much will each person get?

**7** Alexa cuts a yellow ribbon, of length $2\frac{5}{6}$ metres, into four equal pieces. How long is each piece?

**8** Sarah's pension is $105 per fortnight. How much is this per day?

**9** Fran's family want to make their 689 Easter eggs last for one year.

**a** How many Easter eggs can they eat per week?

**b** If there are 7 people in Fran's family, how many Easter eggs will one person eat per week if the eggs are shared equally?


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# 5D Word problems involving fractions

There are a number of things to consider when tackling word problems involving fractions. We need to think about which operations are required, and to think carefully about what is 'the whole'.

## Example 9

Two-thirds of the chocolates in a box contain nuts. There are 54 chocolates in the box.

**a** How many chocolates in the box contain nuts?

**b** What fraction of the chocolates in the box do not contain nuts?

### Solution

**a** $\frac{2}{3}$ of $54 = \frac{2}{\cancel{3}^{1}} \times \frac{\cancel{54}^{18}}{1}$

$= 2 \times 18$

$= 36$

36 of the chocolates in the box contain nuts.

**b** $1 - \frac{2}{3} = \frac{1}{3}$

$\frac{1}{3}$ of the chocolates in the box do not contain nuts.

## Example 10

At Toocastle College, $\frac{3}{5}$ of the students are boys.

**a** What fraction of students are girls?

**b** If there are 360 boys, how many girls are there?

### Solution

**a** $\frac{3}{5}$ of the students are boys. Therefore, $1 - \frac{3}{5} = \frac{2}{5}$ of the students are girls.

**b** $\frac{3}{5}$ is 360. This means that $\frac{1}{5}$ of the number of students is $360 \div 3 = 120$. Therefore, the number of girls is $2 \times 120 = 240$.


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## Exercise 5D

Example 9

**1** Two-thirds of the jelly beans in a jar are black. There are 96 jelly beans in the jar. How many of them are black?

**2** Five-twelfths of the students in a particular school are girls. There are 1344 students in the school.

**a** How many girls are there in the school?

**b** What fraction of the students in the school are boys?

**3** Seven-eighths of the trees in a forest are eucalypts. It is known that there are 5000 trees in the forest.

**a** How many eucalypts are there in the forest?

**b** What fraction of the trees are not eucalypts?

Example 10

**4** In a box of chocolates, $\frac{3}{5}$ of the chocolates have soft centres.

**a** What fraction of the chocolates do not have soft centres?

**b** If there are 15 chocolates with soft centres, how many chocolates are there altogether?

**5** A piece of land is divided among Jan, David and Greg. Jan is allocated $\frac{7}{12}$ of the land and David receives $\frac{1}{6}$ of the land.

**a** What fraction of the land does Greg receive?

**b** The land is worth \$96 000. What is the value of each person's share?

**6** In a theatre, $\frac{5}{12}$ of the seats are in the back stalls, $\frac{1}{4}$ are in the front stalls and the remainder are in the balcony.

**a** What fraction of the seating capacity of the theatre is in the balcony?

**b** If the theatre can hold 1080 people, how many people can sit in each of the sections?

**7** A container of cooking oil is $\frac{3}{5}$ full. A further $4\frac{1}{2}$ L of cooking oil is required to fill it. How much oil does the container hold?

**8** Complete each of the following magic squares. Each row, column and diagonal sums to the same mixed number.

**a**

| | | |
|---|---|---|
| | | |
| | | $\frac{1}{12}$ |
| $\frac{1}{3}$ | $\frac{1}{4}$ | $\frac{2}{3}$ |

**b**

| | | |
|---|---|---|
| 2 | | |
| $\frac{1}{4}$ | $1\frac{1}{4}$ | $2\frac{1}{4}$ |
| | | |

**9** Of a group of 50 students, $\frac{3}{5}$ are girls.

**a** What fraction of the group is boys?

**b** How many of the students are boys?


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**10** In a school, $\frac{2}{5}$ of the students are girls. There are 600 boys.

**a** How many girls are there?

**b** How many students are there?

**11** Each month, a family spends $\frac{5}{12}$ of its income on food, $\frac{1}{3}$ on housing and $\frac{1}{24}$ on transport. If there is \$960 left over, calculate the family's income.

**12** A farmer has $\frac{7}{12}$ of his land cultivated. He also has $\frac{1}{6}$ of his land used for buildings. Of the remaining land, $\frac{1}{3}$ is marsh and is unable to be farmed. The rest of the land is forest. What fraction of his land is forest?

# 5E Order of operations with fractions

The conventions of order of operations that we discussed for whole numbers in Chapter 1 apply also to the arithmetic of fractions.

## Order of operations

- Evaluate expressions inside brackets first.
- In the absence of brackets, carry out operations in the following order:
  - powers
  - multiplication and division from left to right, then
  - addition and subtraction from left to right.

### Example 11

Evaluate:

**a** $1 - \frac{1}{4} - \frac{1}{3}$

**b** $\frac{7}{8} - \left(\frac{1}{4} + \frac{1}{3}\right)$

### Solution

**a** $1 - \frac{1}{4} - \frac{1}{3} = 1 - \frac{3}{12} - \frac{4}{12}$

$= \frac{12}{12} - \frac{3}{12} - \frac{4}{12}$

$= \frac{5}{12}$

**b** $\frac{7}{8} - \left(\frac{1}{4} + \frac{1}{3}\right) = \frac{7}{8} - \left(\frac{3+4}{12}\right)$

$= \frac{21}{24} - \frac{14}{24}$

$= \frac{7}{24}$


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### Example 12

Evaluate:

**a** $\frac{1}{6}+\left(\frac{2}{3}\right)^2$

**b** $\frac{4}{3}-\frac{3}{5}\div\frac{6}{11}$

### Solution

**a** $\frac{1}{6}+\left(\frac{2}{3}\right)^2=\frac{1}{6}+\frac{2}{3}\times\frac{2}{3}$

$=\frac{1}{6}+\frac{4}{9}$

$=\frac{3}{18}+\frac{8}{18}$

$=\frac{11}{18}$

**b** $\frac{4}{3}-\frac{3}{5}\div\frac{6}{11}=\frac{4}{3}-\frac{\cancel{3}^{1}}{5}\times\frac{11}{\cancel{6}^{2}}$

$=\frac{4}{3}-\frac{11}{10}$

$=\frac{40}{30}-\frac{33}{30}$

$=\frac{7}{30}$

## Exercise 5E

Example 11

**1** Evaluate:

**a** $\frac{11}{8}-\left(\frac{1}{4}+\frac{1}{3}\right)$

**b** $1-\left(\frac{1}{5}+\frac{1}{3}\right)$

**c** $1-\left(\frac{1}{3}-\frac{1}{5}\right)$

Example 12

**2** Evaluate:

**a** $\frac{3}{4}+\frac{3}{4}\times\frac{3}{4}$

**b** $\frac{1}{2}\div\frac{4}{3}+\frac{3}{4}$

**c** $\frac{3}{4}-\frac{1}{2}\times\frac{1}{4}$

**d** $\frac{11}{18}\div\frac{44}{3}+\frac{7}{9}$

**e** $\frac{5}{6}+\left(\frac{2}{3}\right)^2$

**f** $\left(\frac{3}{4}\right)-\left(\frac{1}{2}\right)^2$

**g** $\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}-\frac{1}{2}$

**h** $\frac{3}{4}+\frac{5}{4}\times\frac{1}{4}\div\frac{3}{4}$

**i** $\frac{16}{15}\div\frac{8}{25}+\frac{5}{4}$

**j** $\frac{7}{8}+\frac{5}{4}\times\frac{4}{15}-\frac{5}{12}$

**k** $\frac{2}{3}\times\frac{1}{4}\div\frac{5}{9}$

**l** $\frac{3}{5}\div\frac{8}{20}\times\frac{1}{4}$

**m** $\left(\frac{2}{3}+\frac{5}{6}\right)\times\frac{5}{9}$

**n** $\frac{5}{4}\div\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\right)$

**o** $\left(\frac{4}{5}-\frac{2}{3}\right)\times\left(\frac{2}{6}+\frac{5}{8}\right)\div\left(\frac{1}{2}+\frac{2}{3}\right)$

**p** $\frac{2}{3}-\frac{4}{5}\times\frac{2}{6}+\frac{5}{8}\div\frac{1}{4}+\frac{2}{3}$



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# Review exercise

**1** Evaluate:

**a** $\frac{2}{3} \times \frac{3}{4}$ **b** $5\frac{1}{6} \times 2$ **c** $3\frac{1}{4} \div 13$ **d** $\frac{4}{11} \times \frac{22}{25}$

**e** $6 \div \frac{1}{11}$ **f** $5\frac{1}{2} \times 3\frac{1}{4}$ **g** $3\frac{1}{4} \div 1\frac{2}{3}$ **h** $5\frac{2}{3} \div \frac{1}{5}$

**2** Evaluate:

**a** $\frac{2}{3} \times \frac{3}{5} \div \frac{1}{4}$ **b** $\frac{5}{11} \times \frac{3}{5} + 2$ **c** $6 \div \frac{1}{3} + \frac{1}{2}$ **d** $1\frac{1}{3} \times 1\frac{1}{4} + 2$

**3** Find:

**a** $6\frac{2}{3} \times 1\frac{1}{4}$ **b** $5\frac{2}{3} \div 1\frac{1}{4}$ **c** $6\frac{7}{8} \times 5\frac{1}{3}$ **d** $6\frac{1}{3} \div 5\frac{2}{3}$

**4** Find $\frac{2}{3}$ of 63.

**5** $\frac{3}{4}$ of a number is 27. Find the number.

**6** A rope 6 m long is divided into 4 equal pieces. How long is each piece?

**7** Evaluate $\frac{2}{3} \times \left(\frac{3}{4} - \frac{1}{2}\right)$.

**8** **a** Find $\frac{3}{5}$ of 20 m.

**b** Find $\frac{3}{4}$ of \$20.

**c** Find $\frac{7}{8}$ of 1600 kg.

**9** What is the sum of $\frac{1}{5}$ and half of $\frac{1}{5}$?

**10** Omar reads $\frac{2}{11}$ of the pages of a book one day, $\frac{3}{5}$ the next day and the remaining 84 pages the following day. How many pages are in the book?

**11** $\frac{3}{7}$ of a sum of money is \$21. How much is $\frac{4}{5}$ of the sum of money?

**12** Five-sixths of a farm covers 325 hectares (ha). What is the area of the whole farm?


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# Challenge exercise

**1** A half is a third of the number $x$. What is the number $x$?

**2** A quarter is a third of the number $x$. What is the number $x$?

**3** What is the value of $\left(1-\frac{1}{2}\right)\times\left(1-\frac{1}{3}\right)\times\left(1-\frac{1}{4}\right)\times\left(1-\frac{1}{5}\right)$?

**4** What is the value of $\left(1+\frac{1}{5}\right)\times\left(1+\frac{1}{6}\right)\times\left(1+\frac{1}{7}\right)\times\left(1+\frac{1}{8}\right)\times\left(1+\frac{1}{9}\right)$?

**5** Evaluate:

**a** $1+\dfrac{1}{1+\frac{1}{2}}$

**b** $1+\dfrac{1}{1+\dfrac{1}{1+\frac{1}{2}}}$

**6** Using the numbers 1, 2 and 3, fill in the boxes to make these statements true.

**a** $\square\frac{\square}{\square}\times\frac{2}{3}=\square\frac{\square}{\square}$

**b** $\square\frac{\square}{\square}\times 1\frac{2}{5}=\square\frac{\square}{\square}$

**7** How many times do you need to write the fraction $\frac{1}{3}$ to make this statement true?

$\frac{1}{3}\div\frac{1}{3}\div\frac{1}{3}\div\cdots\div\frac{1}{3}=81$

**8** The fractions $\frac{1}{7}$ and $\frac{1}{8}$ are special because when they are multiplied the answer is the same as the difference between them. That is, $\frac{1}{7}\times\frac{1}{8}=\frac{1}{7}-\frac{1}{8}=\frac{1}{56}$. Find five other such pairs of numbers.

**9** Johan was hungry late one night. He looked in the freezer and found a 4-litre tub of ice cream. Johan started eating the ice cream, but so he would not get into trouble for eating it all, he stopped when he had eaten half of it. The next night he was hungry again, so he went back to the ice cream and again stopped when he had eaten half of what was there. This went on for five nights in a row before his mother finally caught him. How much ice cream was left at the end of each of the five nights? If his mother had not caught him, would he ever have finished all the ice cream?


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**10** The school lap-a-thon circuit is $2\frac{3}{4}$ kilometres in length. Benjamin walked $4\frac{1}{3}$ laps, Nathan walked $5\frac{1}{2}$ laps and Alicia walked $3\frac{1}{4}$ laps.

**a** Who covered the greatest distance?

**b** What was the total distance walked by the three students?

**c** How much farther than Benjamin did Nathan walk?

**11** In a cricket test match lasting 5 days, $\frac{1}{6}$ of the runs were scored in the first day, $\frac{1}{5}$ in the second day, $\frac{2}{5}$ in the third day, $\frac{7}{32}$ in the fourth day and remaining 21 runs on the last day. How many runs were scored on each day?

**12** Calculate $\frac{1}{96} + \frac{97 \times 95}{96} - 95$.

**13** Find the whole numbers $a$, $b$ and $c$ if $\frac{37}{13} = 2 + \cfrac{1}{a + \cfrac{1}{b + \cfrac{1}{c}}}$.

**14** A train from Alston to Brampton stops at two intermediate stations. At the first of these, $\frac{1}{2}$ of the train's passengers leave and 135 new passengers board. At the second station, $\frac{1}{3}$ of the passengers who arrived on the train at the first station leave, and 110 new passengers board. The train arrives at Brampton with 350 passengers. How many passengers were on the train when it left Alston?


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CHAPTER

# 6

Measurement and Geometry

# An introduction to geometry

Geometry starts with some of the things that we see around us:

- the sharp line of the horizon out to sea
- the surfaces and edges of a city building
- the circular disc of the moon
- the path of a stone thrown through the air
- the wavy surface of the ocean
- the patterns on a cat's fur coat
- the complicated patterns that clouds often make against the blue sky.

As in all of mathematics, we start with the very simplest patterns and gradually examine more complicated ones.

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# 6A Points, lines and planes

The simplest objects of geometry are points, lines and planes. Because they are so simple, it is hard to give precise definitions of them, just as it is hard to say precisely what a number is. The following descriptions do not really say what points, lines and planes are, but they will help us to talk about them with some agreement about what they mean in our imagination.

## Points

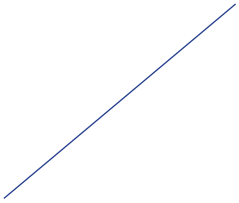
A **point** marks a position, but has no size. The mark shown below has a definite width, so it is not really a point, but it represents a point in our imagination. We usually use capital letters such as $P$, $Q$, $A$ and $B$ to name points.

• $P$

## Lines

A **line** has no width, but extends infinitely in both directions. The drawing below has width and has ends, so it is not really a line, but it represents a line in our imagination.

The word 'line' always means **straight line**, and does not include curves such as circles or squiggles.



## Planes

A **plane** has no thickness, but extends infinitely. The drawing below is intended to represent a plane, but the plane does not stop at the four edges that have been drawn.

The word 'plane' always means **flat plane**, and does not include curved surfaces such as cylinders, cones, spheres or the wavy surface of the ocean.



### Points, lines and planes

- Points, lines and planes are the fundamental objects of geometry.
- Points, lines and planes are idealisations of everyday objects – they exist in our imaginations, even though they do not exist in the physical world.

We draw the diagrams for geometry in a plane on a piece of paper, a whiteboard or a screen. We will sometimes have to imagine that the paper or whiteboard or screen extends infinitely.


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Geometry requires neat and accurate drawings. In this chapter you will need:

- a sharp pencil and an eraser
- a ruler with a straight edge
- a protractor.

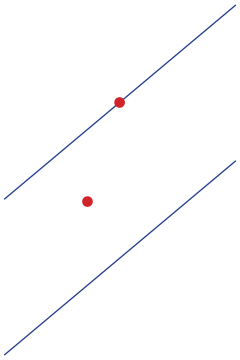
In later geometry chapters, you will also need compasses.

Here are some simple observations we can make about points and lines in a plane. Some of them may seem very obvious, but that is as it should be – good mathematics should always start as simply as possible.

## A point and a line

Given a point and a line, there are two possibilities:

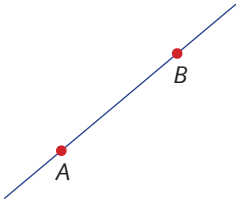
- The point lies on the line.
- The point does not lie on the line.



## Two distinct points

Given two distinct points $A$ and $B$, there is one line, and only one line, that passes through both points $A$ and $B$. This line through $A$ and $B$ can be named either $AB$ or $BA$.

A straight edge, such as a ruler, is used to construct the line. Bring the ruler up against the two points, then draw a line through the two points.

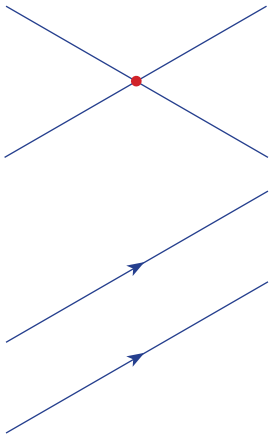



## Two distinct lines – lines meet at a point or are parallel lines

Given two distinct lines in a plane, there are two possibilities:

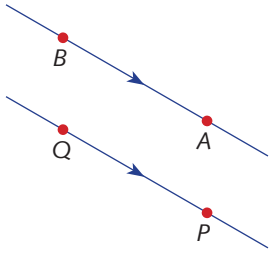
- The two lines meet at a single point. This point is called the **intersection** of the two lines.
- The two lines have no point in common, no matter how far the lines are **produced** in either direction (the word 'produced' means 'extended'). The two lines are then called **parallel lines**.

The arrows on the two lines in the second diagram are used to indicate that the lines are parallel.



## Notation for parallel lines

In the diagram to the right, the lines $AB$ and $PQ$ are parallel. This is written as $AB \parallel PQ$.



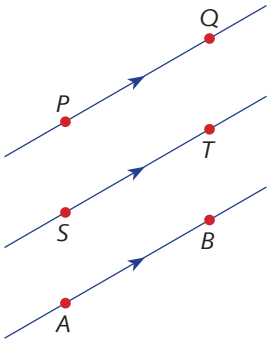


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## Three parallel lines

If two distinct lines $AB$ and $PQ$ are each parallel to a third line $ST$, then the lines $AB$ and $PQ$ are parallel to each other.

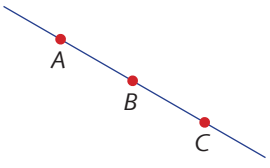
This is written as $AB \parallel PQ \parallel ST$.

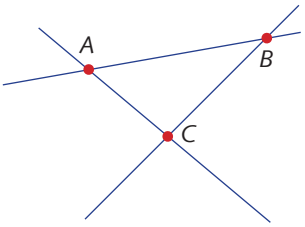



## Three distinct points – collinear points or non-collinear points

Given three distinct points $A$, $B$ and $C$ in a plane, there are two possibilities:

- The three points all lie on one line. The three points are then called **collinear points**.
- The three lines $AB$, $BC$ and $CA$ are all distinct and enclose a triangular region. The three points $A$, $B$ and $C$ are then called **non-collinear points**.

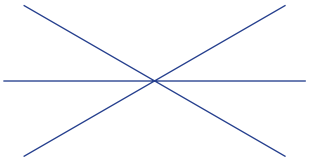


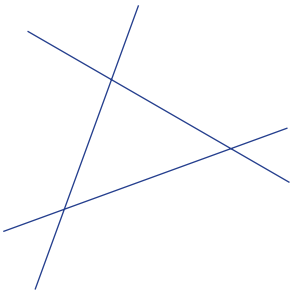




## Three distinct lines – concurrent lines or non-concurrent lines

Given three distinct lines in a plane, with no two lines parallel, there are two possibilities:

- The three lines all pass through one point. The three lines are then called **concurrent lines**.
- The three lines enclose a triangular region. The three lines are then called **non-concurrent lines**.





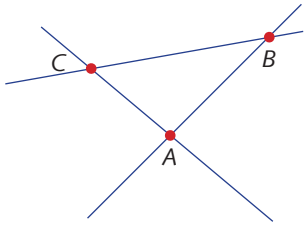
### Points and lines in a plane

- A point either lies on a line or does not lie on a line.
- There is exactly one line passing through two distinct points.
- Two distinct lines in the plane either meet at a point or are parallel.
- Two lines in a plane are called **parallel** if they never meet, no matter how far they are **produced** (meaning *extended*). This is written as $AB \parallel PQ$.
- Two lines parallel to a third line are parallel to each other.
- Three points that all lie on one line are called **collinear**.
- Three lines that all pass through one point are called **concurrent**.


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## Exercise 6A

**1** The three lines in the first diagram to the right enclose a triangular region. Name the three lines in the figure.




**2** The second diagram to the right shows two pairs of parallel lines.

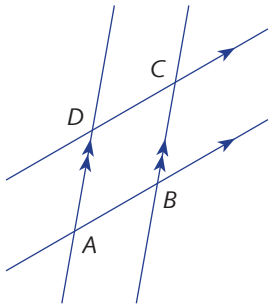
**a** In symbols, write '$AB$ is parallel to $DC$'.

**b** In symbols, write '$BC$ is parallel to $AD$'.

**c** What is the intersection of the lines $AB$ and $BC$?

**d** What point lies on both $BC$ and $CD$?

**e** Do any points lie on both lines $AB$ and $CD$?




**3** Look at the third diagram to the right.

**a** Name the four concurrent lines passing through the point $X$.

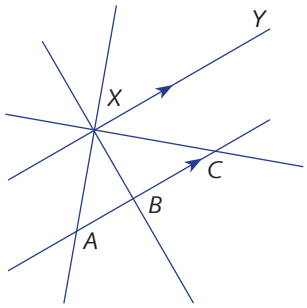
**b** Name the two parallel lines, using the correct symbol for 'is parallel to'.

**c** Name the intersection of the lines $BX$ and $AC$.

**d** Name the line joining $C$ and the intersection of $AX$ and $BX$.

**e** What points belong to both lines $AC$ and $CB$?

**f** What points belong to both lines $AC$ and $XY$?




**4** Trace the six-point diagram to the right carefully into your exercise book.

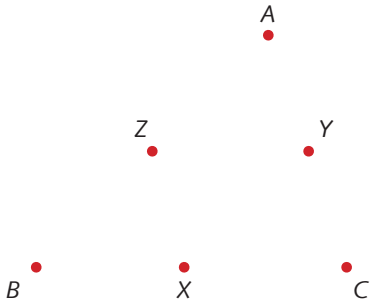
**a** Use your ruler to construct lines showing that:

**i** the points $B$, $X$ and $C$ are collinear

**ii** the points $C$, $Y$ and $A$ are collinear

**iii** the points $A$, $Z$ and $B$ are collinear

**b** Draw the lines $AX$, $BY$ and $CZ$. What can you say about these three lines?



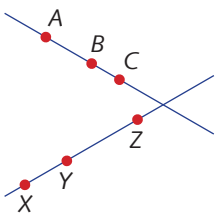

**5** Draw a half-page version of the diagram to the right. Then use your ruler to carry out each of the following constructions.

**a** Join $AY$ and $XB$ and let them intersect at $R$.

**b** Join $BZ$ and $YC$ and let them intersect at $P$.

**c** Join $CX$ and $ZA$ and let them intersect at $Q$.

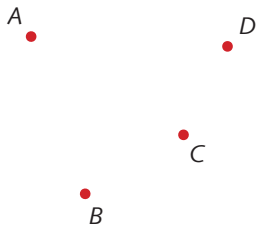
If your diagram has been drawn accurately, $P$, $Q$ and $R$ should be collinear. (This is a famous result called *Pappus' theorem*, after the ancient Greek mathematician Pappus.)





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**6** **a** Trace the four-point diagram to the right carefully into your exercise book.




**b** Use your ruler to construct the lines $AB$ and $DC$, and produce them until they intersect at $X$.

**c** Use your ruler to construct the lines $AD$ and $BC$, and produce them until they intersect at $Y$.

**d** Use your ruler to construct the lines $AC$ and $BD$, and let them intersect at $Z$.

**7** **a** There are infinitely many points on every line. Explain the use of the word *infinitely* in this statement.

**b** There are infinitely many lines passing through every point. Explain the use of the word *infinitely* in this statement.

**8** There are four possible ways to place three distinct lines in a plane. Two of these configurations involve parallel lines, the other two do not. Draw all four of them, and find the one that encloses a region.

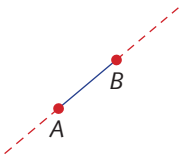
**9** There are eight possible ways to place four distinct lines in a plane. Five of these configurations involve parallel lines; the other three do not. Draw all eight of them.

# 6B Intervals, rays and angles

This section introduces intervals, rays and angles, and explains how to name them.

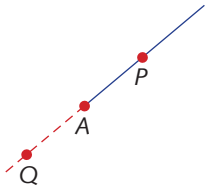
## Intervals

Let $A$ and $B$ be two points on a line. The **interval** $AB$ is the part of the line between $A$ and $B$, including the two endpoints.




## Rays

The point $A$ in the diagram to the right divides the line $QAP$ into two pieces, called **rays**. The ray $AP$ is the piece that contains the point $P$; it is drawn unbroken. The **opposite ray**, $AQ$, is the other piece of the line and contains the point $Q$.




Each ray includes the endpoint $A$, which is called the **vertex** of the ray. The two opposite rays $AP$ and $AQ$ have only the vertex $A$ in common.

Thus, the ray $AP$ starts at $A$ and goes on forever in a fixed direction, like a thin ray of light coming from a torch held at the vertex $A$.


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## Angles

In the diagram on the left below, the two rays $OA$ and $OB$ have a common vertex, $O$.

We say that we have formed two **angles**. The first one is 'between' the two rays; the second is 'outside' the two rays.

We can describe the situations above using the idea of the amount of turning.

Imagine that the ray $OA$ is rotated about the point $O$ until it lies along $OB$. There are two ways of doing this:

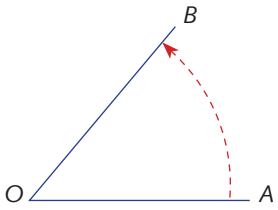



**Figure 1**

and

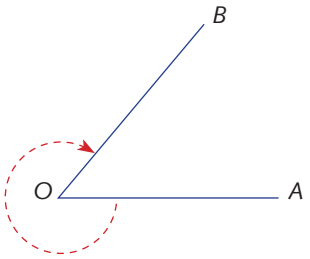


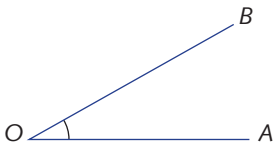

**Figure 2**

The amount of turning in Figure 1 is smaller, and is called the size of the angle between $OA$ and $OB$. The amount of turning in Figure 2 is called the size of the **reflex angle** formed by $OA$ and $OB$.

In this chapter, we will usually work with the smaller angle between two rays.

## Naming an angle

The angle shown to the right can be named either $\angle AOB$ or $\angle BOA$. Notice that the vertex $O$ must go in the middle, but the points $A$ and $B$ can be written either way around. If we are to refer to the larger angle, we will call it reflex $\angle AOB$.

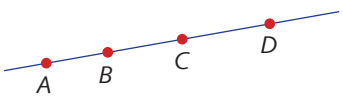



### Intervals, rays and angles

- An **interval** $AB$ is the section of line $AB$ between $A$ and $B$, and includes the points $A$ and $B$.
- A point on a line divides the line into two **opposite rays**. The point is the **vertex** of both rays.
- The name of an angle has the vertex in the middle, for example, $\angle AOB$. This refers to the smaller of the two angles unless otherwise specified.

## Exercise 6B

**1** **a** In the diagram to the right, name all the labelled points in:




**i** the line $BC$ **ii** the ray $BC$

**iii** the interval $BC$ **iv** the ray $DB$

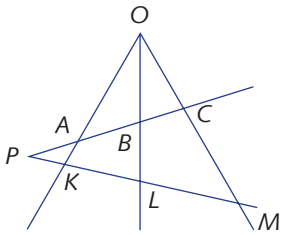
**v** the interval $CA$ **vi** the interval $AD$

**b** Copy and complete: The rays $BD$ and $BA$ are _____ rays.

**c** What is the intersection of the ray $BC$ and the ray $CB$? (That is, what do they have in common?)


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**2** There are five lines in the diagram to the right.




**a** Copy and complete: The lines $AK$, $BL$ and $CM$ are _____.

**b** Does $L$ lie on the ray $OB$?

**c** Does $L$ lie on the interval $PK$?

**d** Name the two rays with vertex $P$.

**e** Name the four rays with vertex $A$.

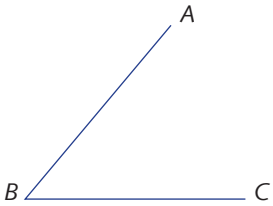
**3** **a** Draw two distinct points, $L$ and $M$, and construct the line $LM$. Choose a point, $N$, between $L$ and $M$. Then choose another point, $P$, so that $M$ is between $L$ and $P$.

**b** Does $N$ lie in the interval $LM$?

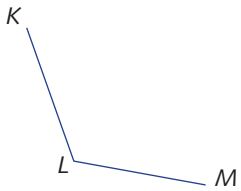
**c** Does $P$ lie in the interval $LM$?

**4** Name each angle below, using three letters (for example, $\angle AOB$).

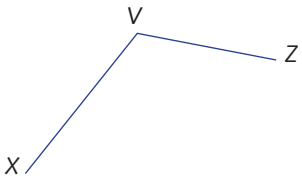
**a**




**b**

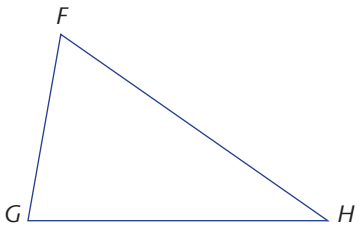



**c**

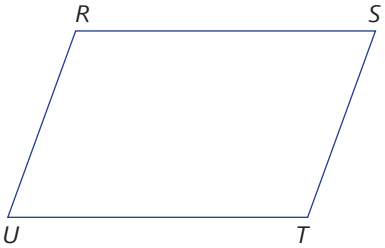



**5** Name all the angles of each figure below.

**a**

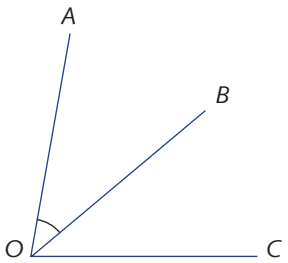



**b**

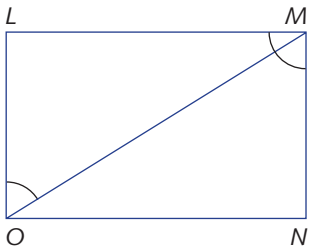



**6** Name each angle indicated in the diagrams below.

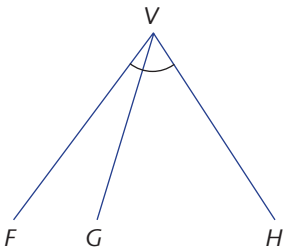
**a**



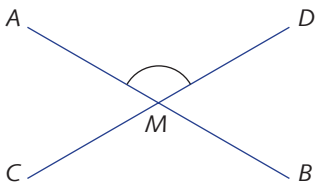

**b**




**c**



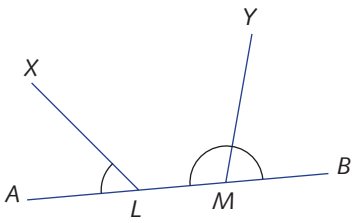

**d**



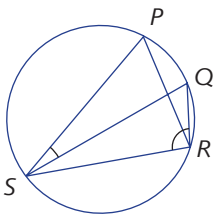


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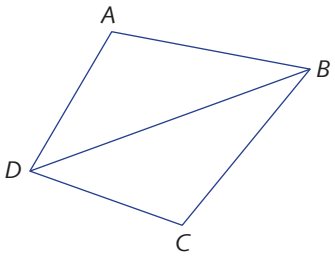
**e**



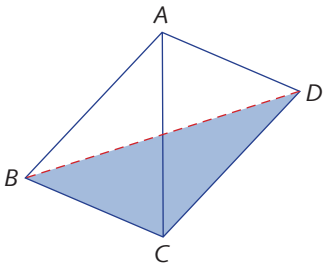

**f**




**7** **a** Name all eight angles associated with the figure below.




**b** Name all 12 angles formed at the vertices of the tetrahedron below.




# 6C Measuring angles

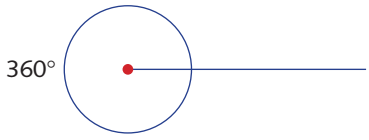
In this section, we discuss how to measure angles. We also describe some important types and properties of angles.

## Revolution

A **revolution** is the amount of turning required to rotate a ray about its endpoint until it falls back onto itself.

There are several systems of units for measuring the size of an angle. The best-known system divides the full revolution into 360 equal parts, called **degrees**, and was developed by Babylonian mathematicians in the ancient world. This system has an astronomical basis – there are about $365\frac{1}{4}$ days in the year, so the Sun moves approximately 1 degree (written as 1°) against the fixed stars every day.

The size of 1 revolution is 360 degrees, which is written as 360°.

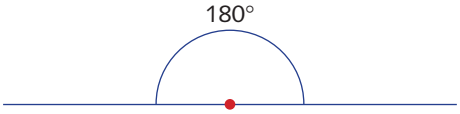




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## Straight angle

A **straight angle** is the angle formed by taking a ray and its opposite ray. A straight angle is half of a revolution, and so has size equal to 180°.

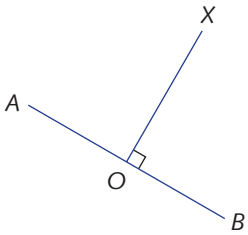



## Right angle

Let $AOB$ be a line, and let $OX$ be a ray making equal angles with the ray $OA$ and the ray $OB$. Then the equal angles $\angle AOX$ and $\angle BOX$ are called **right angles**.

A right angle is half of a straight angle, and so is equal to 90°.

The conventional way to indicate a right angle is to use a small square at the vertex, as shown in the diagram to the right for $\angle BOX$.

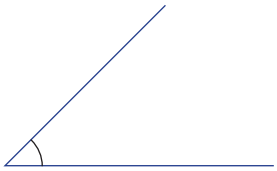



## Acute angles, obtuse angles and reflex angles

These types of angles are defined by comparing them with right angles, straight angles and revolutions.

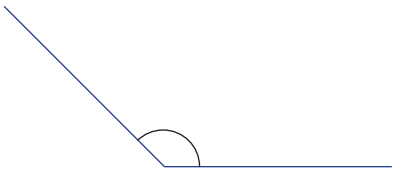
### Acute angle

An **acute angle** is an angle that is larger than 0° and less than a right angle.



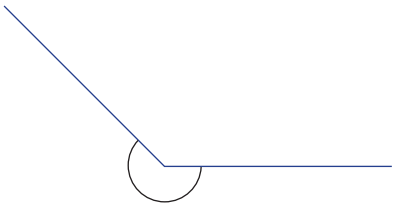
### Obtuse angle

An **obtuse angle** is an angle that is larger than a right angle and less than a straight angle.



### Reflex angle

A **reflex angle** is greater than a straight angle but smaller than a revolution.



## A right angle is not an acute angle

A right angle is neither acute nor obtuse – it is the boundary between the two. Similarly, a straight angle is neither obtuse nor reflex, a revolution is not a reflex angle, and an angle of size 0° is not an acute angle.

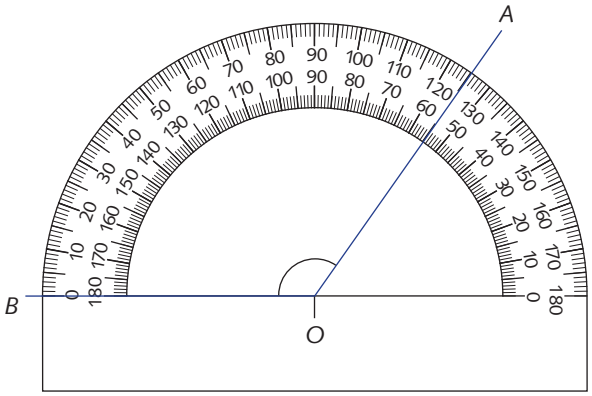
## Using a protractor to measure and construct angles

A **protractor** allows angles to be measured and constructed correct to about the nearest degree. The most common version consists of a semicircular piece of clear plastic with 181 equally spaced markers placed around its circumference, allowing the measurement of angles from 0° to 180°. Another type of protractor is a full circle with 360 equally spaced markers, allowing the measurement of angles from 0° to 360°.


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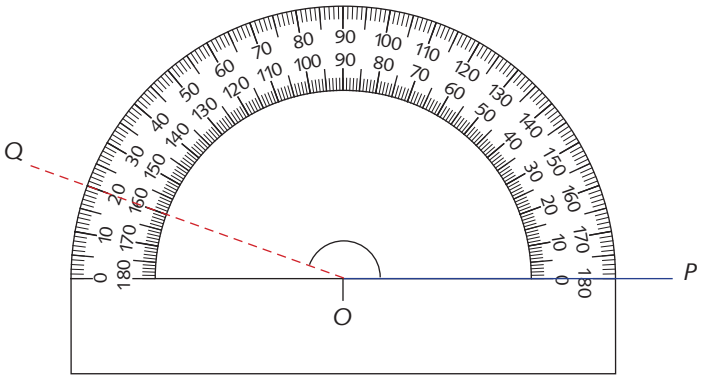
The steps for measuring or constructing an angle are almost the same.

- Identify the centre of the semicircle or circle on the protractor.
- Place the centre precisely over the vertex of the angle.
- Rotate the protractor until its baseline lies along one arm of the angle.
- The protractor has two scales, going in opposite directions. They are there for your convenience – make sure that they do not become your downfall! The two angles at each marker add to 180° (for example, 70° and 110°). Such angles are said to be **supplementary** and will be discussed in the next section.




In the diagram above, the protractor is being used to measure $\angle AOB$. The angle is obtuse, with size of about 125°.

If you read the wrong scale on the previous diagram, you would read the size as the supplement 55°, which is an acute angle. It is a good idea to establish first whether the angle is acute, obtuse or reflex.




In the diagram above, a protractor is being used to construct an angle of size 160°, given that the ray $OP$ is one arm. First, the base line of the protractor is lined up with $OP$. Then a point, $Q$, is placed at the 160° marker, using a sharp pencil. Finally, the ray from $O$ to $Q$ is drawn.

This angle is also obtuse – a wrong move here will result in an acute angle of 20°, the supplement of 160°.

## Complementary angles and supplementary angles

Pairs of angles that add to 90° or 180° occur so often that it is helpful to have special words to describe them.

Two angles that add to 90° are called **complementary angles**. For example, 30° and 60° are complementary angles, since 30° + 60° = 90°.


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Two angles that add to 180° are called **supplementary angles**. For example, 30° and 150° are supplementary angles, since 30° + 150° = 180°.

Notice how the two scales on the protractor are related to each other. At each marker, the two scales (the inner and the outer one) give supplementary angles, such as 70° and 110°.

### Example 1

Write down the complement and the supplement of 25°.

### Solution

The **complement** of 25° is 90° − 25° = 65°.

The **supplement** of 25° is 180° − 25° = 155°.

## Angles

- A **revolution** is 360°, a **straight angle** is 180° and a **right angle** is 90°.
- An **acute angle** has a size between 0° and 90°.
- An **obtuse angle** has a size between 90° and 180°.
- A **reflex angle** has a size between 180° and 360°.
- The size of an angle is normally taken to be between 0° and 180°.
- When the reflex angle is required, it is written as 'reflex $\angle AOB$'. The reflex size of an angle is between 180° and 360°. Notation such as $\angle AOB$ normally refers to the angle between 0° and 180°.
- Two angles are called **complementary** if they add to 90°.
- Two angles are called **supplementary** if they add to 180°.
- Angles can be approximately measured and constructed with a **protractor**.

## Exercise 6C

Example 1

**1** Write down the complement of:

**a** 15° **b** 35° **c** 72° **d** 88° **e** 56°

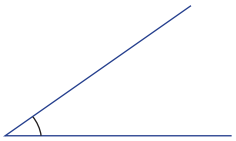
Example 1

**2** Write down the supplement of:

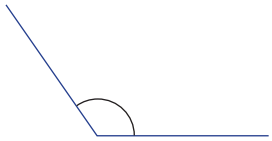
**a** 160° **b** 100° **c** 75° **d** 88° **e** 146°

**3** Classify each angle as acute, obtuse, reflex or right.

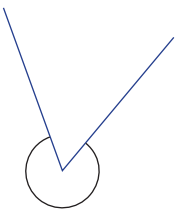
**a**



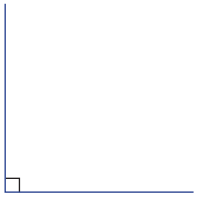
**b**



**c**



**d**




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**4** What is the size of the angle between the hour hand and the minute hand at:

**a** 6 a.m.? **b** 3 a.m.? **c** 1 p.m.? **d** 2 p.m.?

**e** 4 a.m.? **f** 8 p.m.? **g** 10 a.m.? **h** 7 a.m.?

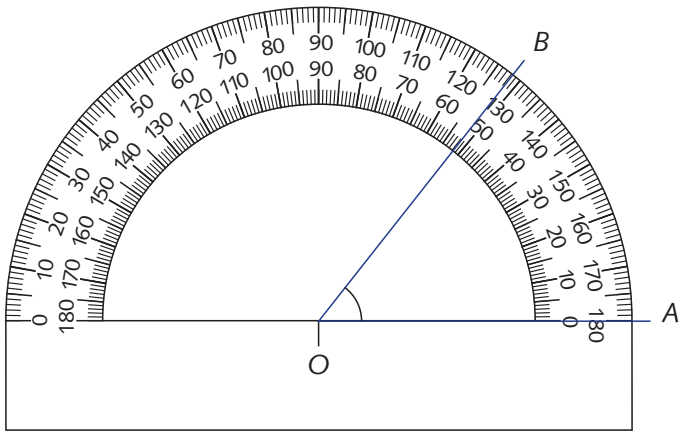
**5** Through how many degrees does the minute hand of a clock move in:

**a** 1 hour? **b** 30 minutes? **c** 45 minutes? **d** 5 minutes?

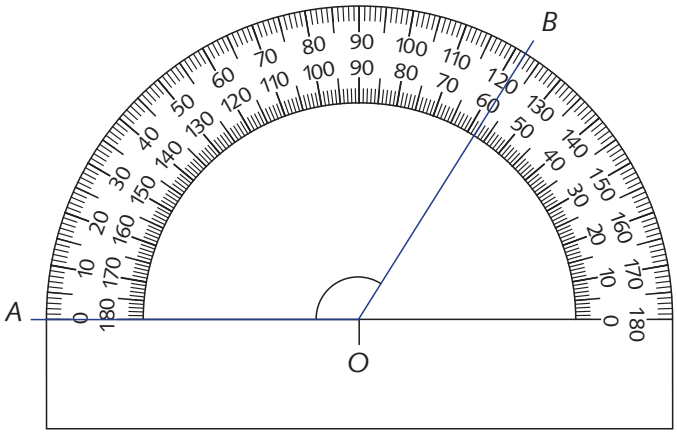
**e** 50 minutes? **f** 25 minutes? **g** 2 hours? **h** $1\frac{1}{2}$ hours?

**6** Read the angle $AOB$ that is being measured by the protractor in each of these diagrams.

**a**

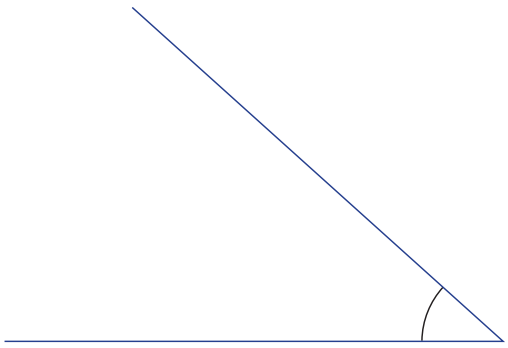



**b**

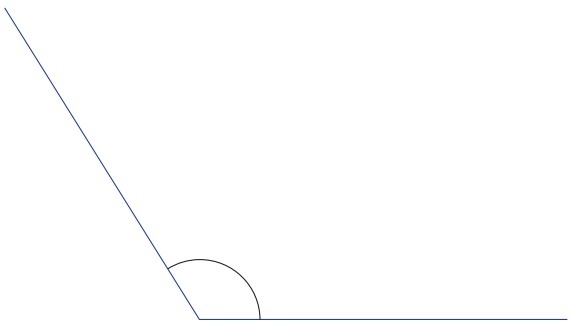



**7** **a** Use your protractor to find the approximate size of each of these angles. Be careful to place the centre of the semicircle precisely over the vertex of the angle.

**i**

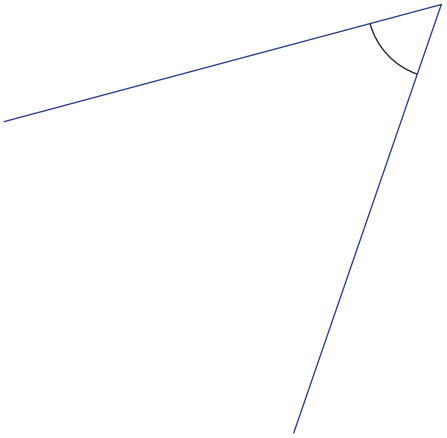


**ii**

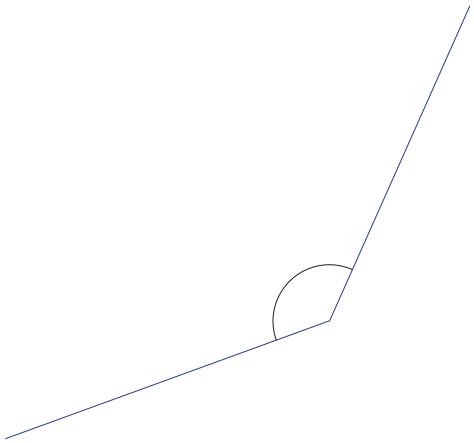



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**iii**



**iv**



**b** Write down the size of the reflex angle in each of the subparts **i–iv** of part **a**.

**8** **a** Copy the diagram below, leaving about 6 cm of space above the diagram for angles to be drawn.




**b** Use your protractor to construct rays, pointing upwards, that make angles with the ray $AP$ of:

**i** 24° **ii** 160° **iii** 70° **iv** 170° **v** 82° **vi** 116° **vii** 5° **viii** 133°

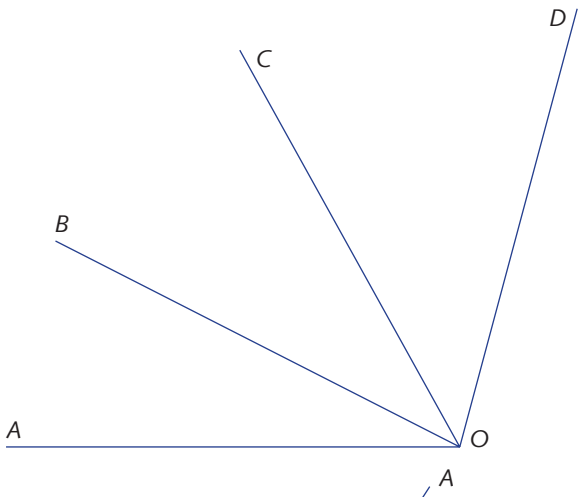
**9** **a** Measure the sizes of angles $\angle AOB$, $\angle BOC$ and $\angle COD$.

**b** Measure the size of $\angle AOD$.

**c** Complete this sentence: The size of angle $\angle AOD$ is the sum of _____.

**d** What is the size of reflex $\angle AOD$?

**e** What is the size of reflex $\angle AOB$?



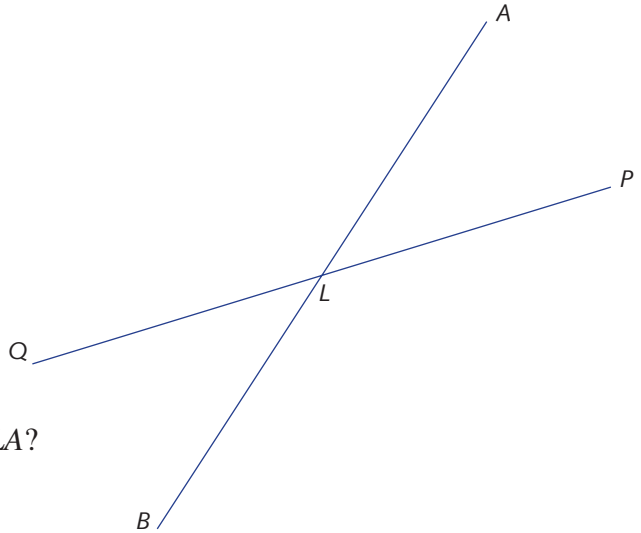

**10** **a** Measure the sizes of $\angle ALP$ and $\angle ALQ$ in the diagram.

**b** Complete this sentence: The angles $\angle ALP$ and $\angle ALQ$ are _____.

**c** Measure the sizes of $\angle ALP$ and $\angle BLQ$ in the diagram.

**d** Complete this sentence: The angles $\angle ALP$ and $\angle BLQ$ are _____.

**e** What is $\angle ALP + \angle PLB + \angle BLQ + \angle QLA$?





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**11** **a** Measure $\angle PQA$ and $\angle PRL$ in the diagram.

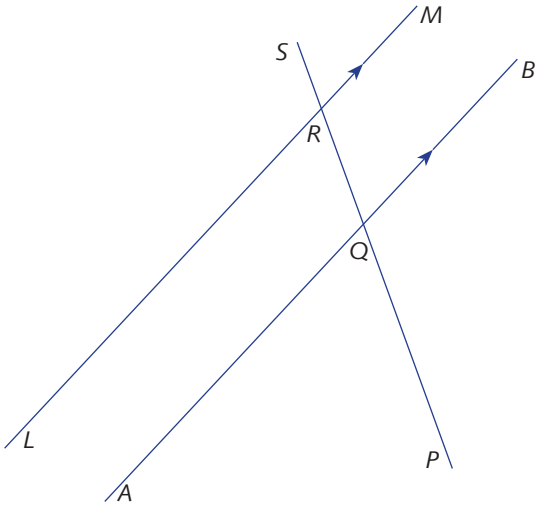
**b** Complete this sentence: The angles $\angle PQA$ and $\angle PRL$ are _____.

**c** Measure $\angle AQR$ and $\angle LRQ$ in the diagram.

**d** Complete this sentence: The angles $\angle AQR$ and $\angle LRQ$ are _____.

**e** Measure $\angle BQR$ and $\angle QRL$ in the diagram.

**f** Complete this sentence: The angles $\angle BQR$ and $\angle QRL$ are _____.




**12** **a** Measure $\angle URS$ and $\angle UTS$ in the diagram.

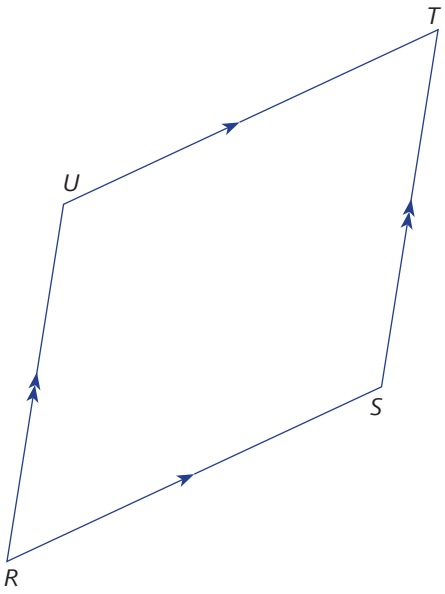
**b** Complete this sentence: The angles $\angle URS$ and $\angle UTS$ are _____.

**c** Measure $\angle RUT$ and $\angle STU$ in the diagram.

**d** Complete this sentence: The angles $\angle RUT$ and $\angle STU$ are _____.

**e** Measure all four interior angles and add them up. What do they add to?

**f** Classify the four interior angles as acute or obtuse.




# 6D Angles at a point – geometric arguments

Clear, logical reasoning has always been an important part of geometry. Diagrams make it easier to understand and construct convincing arguments in geometry and in many other branches of mathematics.

When setting out geometric arguments, we state the appropriate reason in brackets after any geometric statement. Reasons should always be as specific as possible and should name any figure that they refer to.

This section develops four reasons that can be given when dealing with problems about angles at a point.

## Adjacent angles

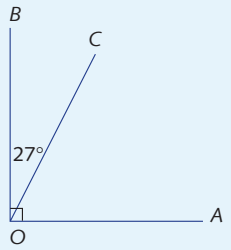
Two angles at a point are said to be **adjacent** if they share a common ray.

Adjacent angles can be added and subtracted in the obvious ways.


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**Example 2**

Find $\angle AOC$ in the diagram shown.



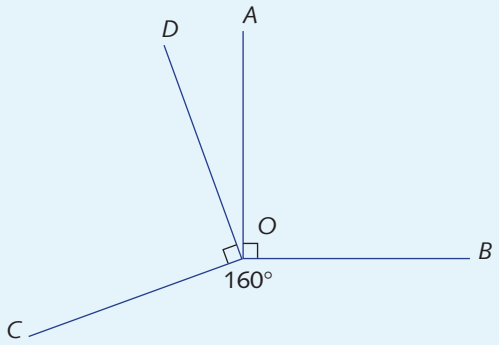

**Solution**

$\angle AOB = 90°$
$\angle AOC + 27° = 90°$ (adjacent angles at $O$)
so $\angle AOC = 63°$

## Angles in a revolution add to 360°

**Example 3**

Find $\angle AOD$ in the diagram shown.



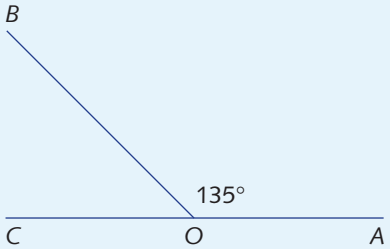

**Solution**

$\angle AOD + 90° + 160° + 90° = 360°$ (revolution at $O$)
so $\angle AOD = 20°$

## Angles in a straight angle add to 180°

**Example 4**

Find $\angle BOC$ in the diagram shown.





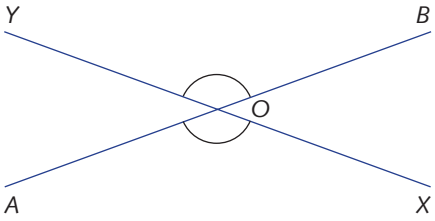
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**Solution**

$\angle BOC + 135° = 180°$ (straight angle at $O$)

so $\angle BOC = 45°$

## Vertically opposite angles are equal



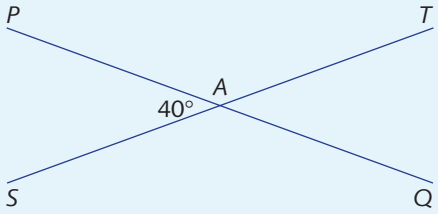

When two lines intersect, four angles are formed at the point of intersection. The two marked angles $\angle AOX$ and $\angle BOY$ are called **vertically opposite angles**, because they are *opposite* each other across the vertex $O$.

These two angles are always equal. To see this, note that:

$\angle AOX + \angle BOX = 180°$
$\angle BOY + \angle BOX = 180°$
therefore: $\angle AOX = \angle BOY$

**Example 5**

Find $\angle TAQ$ and $\angle SAQ$ in the diagram shown.




**Solution**

$\angle TAQ = 40°$ (vertically opposite angles at $A$) and $\angle SAQ + 40° = 180°$ (straight angle at $A$)
so $\angle SAQ = 140°$

## Using Greek letters for angle size

Letters are very useful in geometry, as in all mathematics. In this book, angle sizes will usually be represented by lower-case Greek letters. Four Greek letters will be sufficient at this stage:

- $\alpha$, called *alpha* (the Greek letter a)
- $\beta$, called *beta* (the Greek letter b)
- $\gamma$, called *gamma* (the Greek letter g)
- $\theta$, called *theta* (the Greek letter for th).

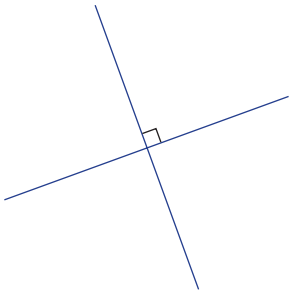
The English word *alphabet* comes from the first two letters of the Greek alphabet – alpha and beta.


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## Perpendicular lines

Two lines are called **perpendicular** if they intersect so that the four angles formed are right angles.

Because adjacent angles on a straight line are supplementary, and vertically opposite angles are equal, it is only ever necessary to prove that one of the four angles is a right angle.



### Angles at a point – geometric arguments

- Four reasons to be used in arguments (preferably with the names of vertices):
  1. Adjacent angles can be added and subtracted.
  2. Angles in a revolution add to 360°.
  3. Angles in a straight angle add to 180°.
  4. Vertically opposite angles are equal.
- Two lines are called **perpendicular** if they meet at right angles.
- Four Greek letters – $\alpha$ (alpha), $\beta$ (beta), $\gamma$ (gamma), $\theta$ (theta) – are often used to represent angle sizes in geometry.

## Exercise 6D

**1** **a** In your exercise book, place $O$ to the left of your page. Allow at least 10 cm of space above, below and to the right of point $O$.

**b** Draw three rays, $OA$, $OB$ and $OC$, going off to the right in different directions.

**c** Using your protractor, measure the sizes of $\angle AOB$, $\angle BOC$ and $\angle AOC$.

**d** Check that they obey the statement, 'Adjacent angles can be added'.

**2** **a** Draw a horizontal line, $AOB$. Allow at least 10 cm of space around your line.

**b** Draw another ray, $OP$, going diagonally upwards from $O$.

**c** Measure $\angle AOP$ and $\angle BOP$.

**d** Check that they obey the statement, 'Angles in a straight angle add to 180°'.

**3** **a** Place $O$ in the centre of your page. Allow at least 10 cm of space around $O$.

**b** Draw two rays, $OA$ and $OB$, going off to the right; one upwards, one downwards.

**c** Draw a third ray, $OC$, going to the left.

**d** Measure the sizes of $\angle AOB$, $\angle BOC$ and $\angle COA$.

**e** Check that they obey the statement, 'Angles in a revolution add to 360°'.

**4** **a** Draw two lines, $AB$ and $PQ$, intersecting at $O$.

**b** Measure the sizes of $\angle AOP$, $\angle POB$, $\angle BOQ$ and $\angle QOA$.

**c** Check that they obey the statement, 'Vertically opposite angles are equal'.

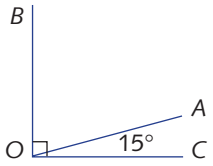

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*Note*: In the following question, you will begin to learn how to set out a mathematical argument. Reasons must be given in geometrical exercises, as in the examples on the previous pages. The correct reasons are as important as the correct answers.

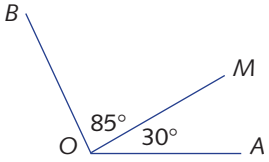
Example 2, 3, 4

**5** Find $\angle AOB$ in each diagram below, giving reasons for your answer.

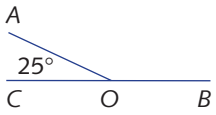
**a**



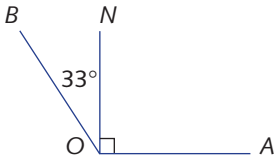

**b**



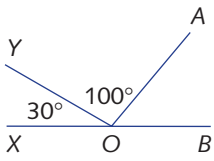

**c**




**d**



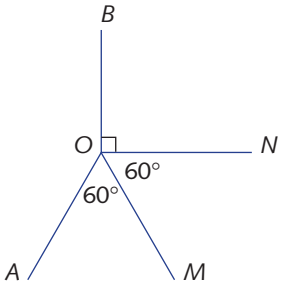

**e**



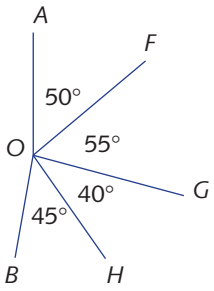

**f**



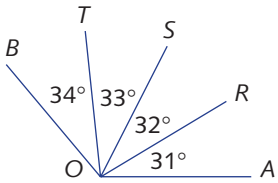

**g**



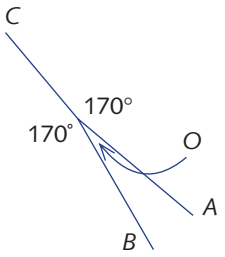

**h**



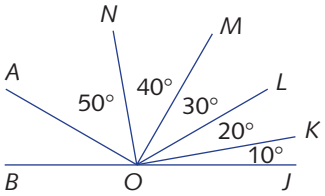

**i**



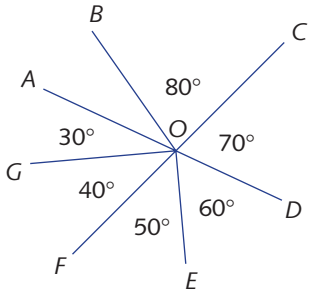

**j**




**k**

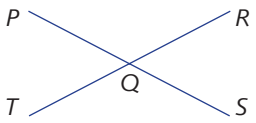



**l**

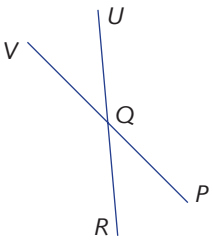



**6** Name the angle that is vertically opposite to $\angle PQR$ in each diagram below.

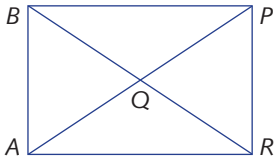
**a**



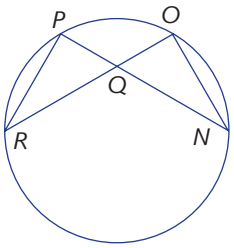

**b**




**c**




**d**



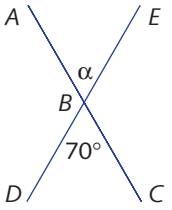


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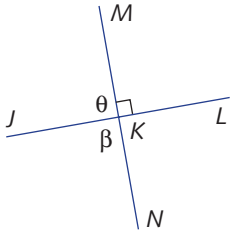
Example 5

**7** Find the values of $\alpha$, $\beta$, $\gamma$ and $\theta$ in each diagram below, carefully giving reasons for all of your statements.

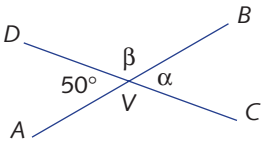
**a**



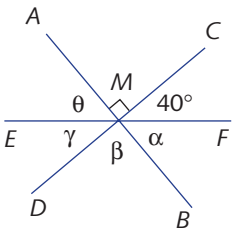

**b**



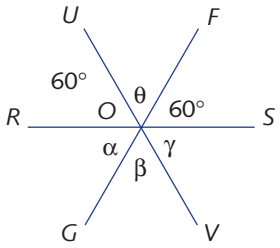

**c**



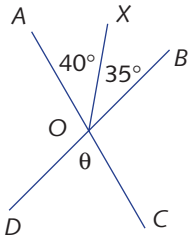

**d**



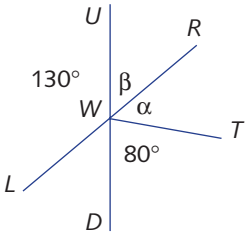

**e**



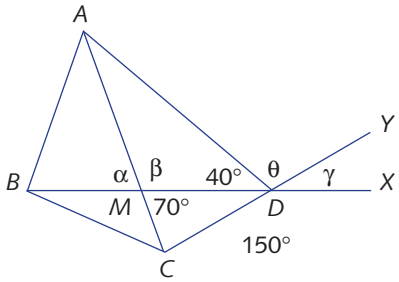

**f**




**g**

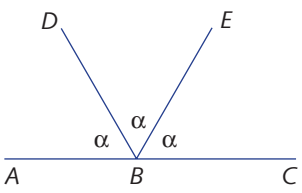



**h**

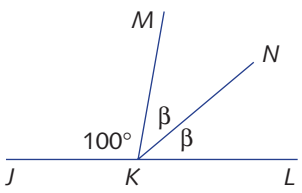



**8** In each diagram below, angles marked with the same Greek letter are equal in size. Find the value of each, giving reasons for your answers.

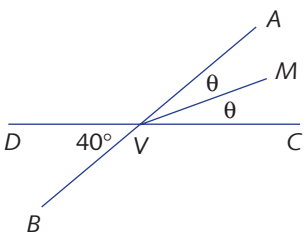
**a**



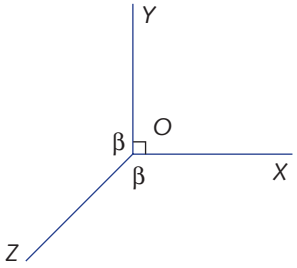

**b**



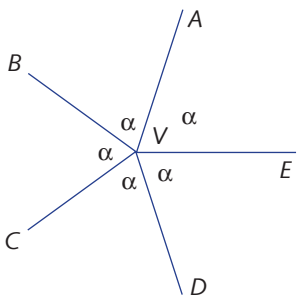

**c**



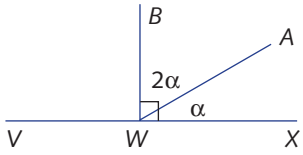

**d**




**e**




**f**

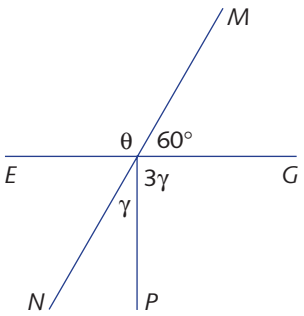




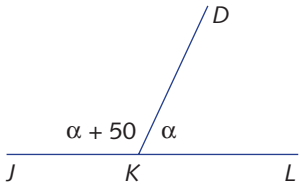



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**g**




**h**



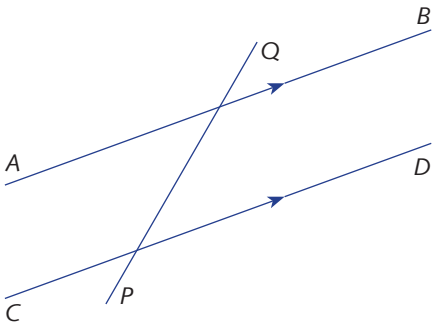

# 6E Angles associated with transversals

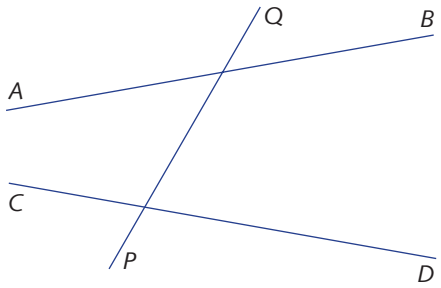
This section and the next involve the relationships between angles and parallel lines.

Reasons are important in this section – make sure that all reasons are as specific as possible. In particular, *always name any parallel lines that you are using in your argument.*

## Transversal

A **transversal** is a line that crosses two other lines. In both diagrams below, the line $PQ$ is a transversal to the lines $AB$ and $CD$.

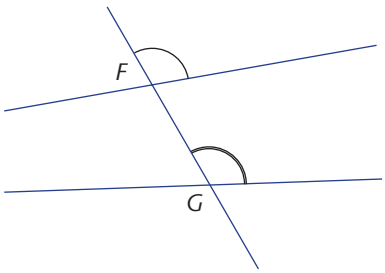


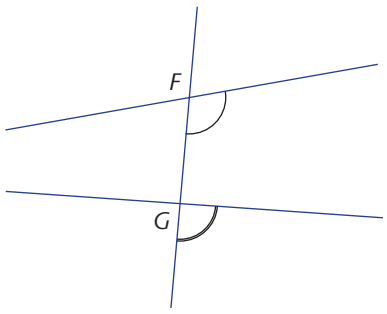




Notice that $PQ$ is a transversal whether or not the other two lines are parallel.

## Corresponding angles

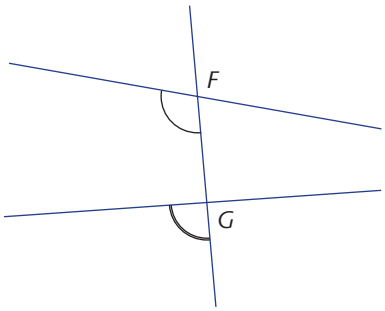
In each diagram below, the two marked angles are called **corresponding angles**, because they are in *corresponding* positions around the two vertices $F$ and $G$.

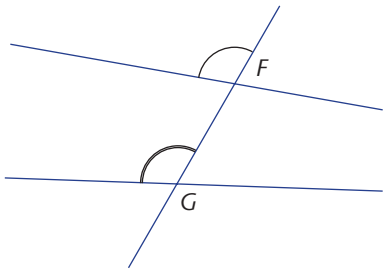




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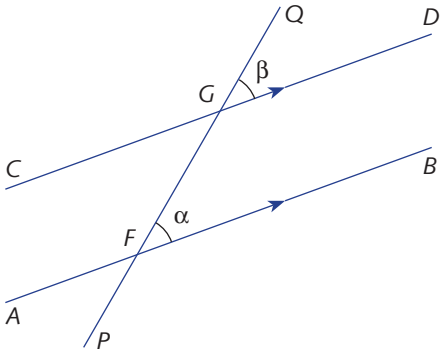



## Corresponding angles and parallel lines

The situation becomes interesting when the two lines are parallel, as in the diagram below.

If lines $AB$ and $CD$ are parallel, then the corresponding angles $\angle BFQ$ and $\angle DGQ$, marked $\alpha$ and $\beta$, are equal.

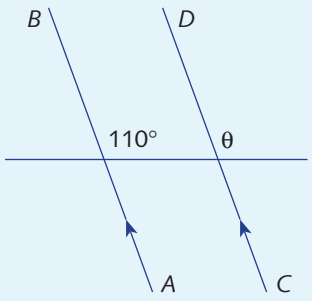



This result actually needs to be taken as an assumption of our geometry – it cannot be proven from what we have developed so far.

The following example shows how to set out your reasoning when solving a problem involving parallel lines and corresponding angles. Your solution must mention corresponding angles, and it must mention that the lines are parallel.

### Example 6

Find $\theta$ in the diagram shown.




### Solution

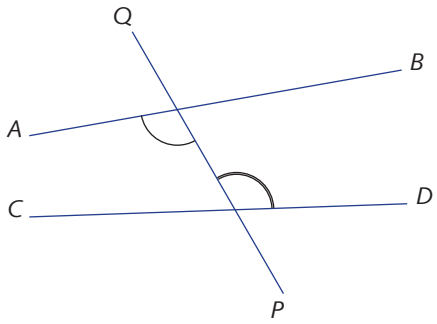
$\theta = 110°$ (corresponding angles, $AB \parallel CD$)

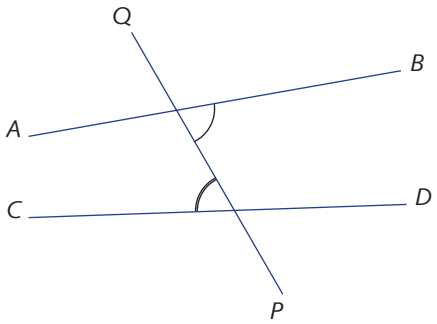


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## Alternate angles

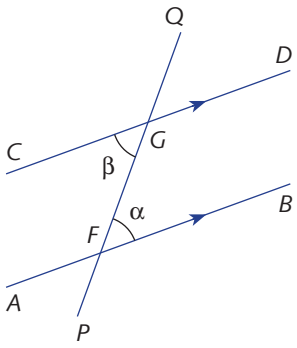
In each diagram below, the two marked angles are called **alternate angles**, because they are on *alternate* sides of the transversal $PQ$. The two angles must also be between the two lines.




## Alternate angles and parallel lines

When the lines $AB$ and $CD$ are parallel, as in the diagram to the right, the alternate angles $\angle BFG$ and $\angle FGC$, marked $\alpha$ and $\beta$, are equal.




We can prove the result using the previous result that showed that the corresponding angles are equal.

$\angle DGQ = \alpha$ (corresponding angles, $AB \parallel CD$)

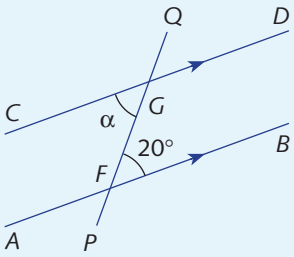
and $\angle DGQ = \beta$ (vertically opposite angles at $G$).

Hence, $\alpha = \beta$.

When this result is given as a reason, you must mention alternate angles and name the parallel lines, as in the next example.

### Example 7

Find $\alpha$ in the diagram shown.



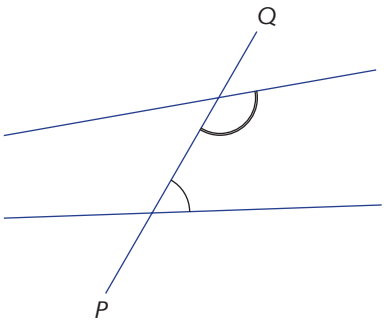

### Solution

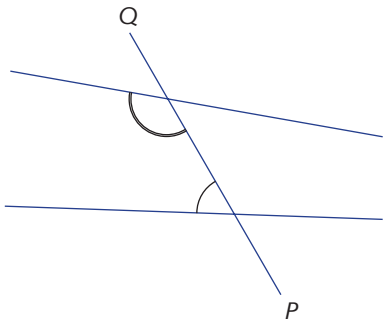
$\alpha = 20°$ (alternate angles, $AB \parallel CD$)


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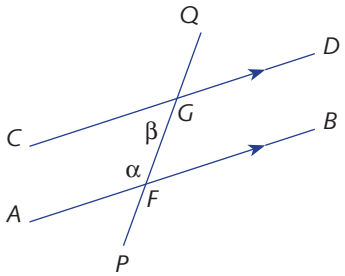
## Co-interior angles

In each diagram below, the two marked angles are called **co-interior angles**, because they are between the two lines and on the same side of the transversal $PQ$.




## Co-interior angles and parallel lines




Suppose now that the two lines are parallel, as in the diagram shown. In this situation, the co-interior angles $\angle AFG$ and $\angle CGF$, marked $\alpha$ and $\beta$, cannot be equal, because one is acute and the other obtuse (unless they are both right angles).

The co-interior angles are **supplementary**; that is, their sum is 180°.

To prove this, we note that:

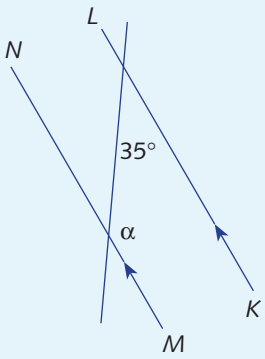
$\angle BFG = \beta$ (alternate angles, $AB \parallel CD$).

Hence, $\alpha + \beta = 180°$ (straight angle at $F$).

When you use this result, you will need to mention co-interior angles and name the parallel lines, as in the next example.

### Example 8

Find $\alpha$ in the diagram shown.




### Solution

$\alpha + 35° = 180°$ (co-interior angles, $KL \parallel MN$)

so $\alpha = 145°$



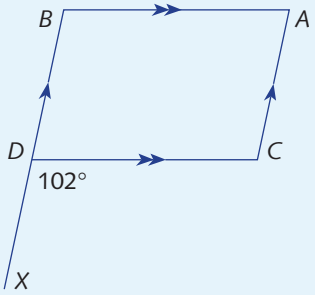
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## Problems involving two steps

The solution to the problem below needs two steps, with a reason for each step. Notice that a different pair of parallel lines is used in each step. As with many geometrical problems, different proofs are available.

### Example 9

Find $\angle BAC$ in the diagram shown.




### Solution

First, $\angle DCA = 102°$ (alternate angles, $AC \parallel BD$)

so $\angle BAC = 78°$ (co-interior angles, $AB \parallel CD$)

### Angles associated with transversals

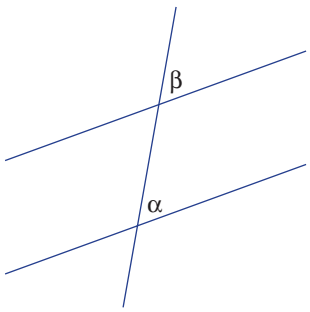
A **transversal** is a line that crosses two other lines. If the lines crossed by the transversal are parallel, then:

- the **corresponding** angles are equal
- the **alternate** angles are equal
- the **co-interior** angles are supplementary.

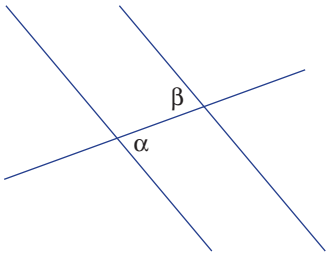
## Exercise 6E

**1** In each diagram, identify each pair of angles marked with $\alpha$ and $\beta$ as corresponding angles, alternating angles or co-interior angles.

**a**



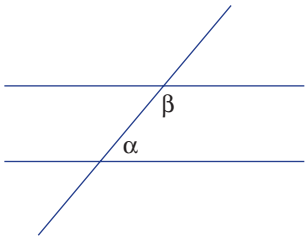

**b**



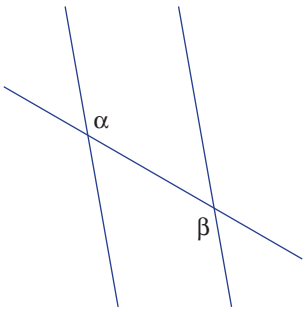


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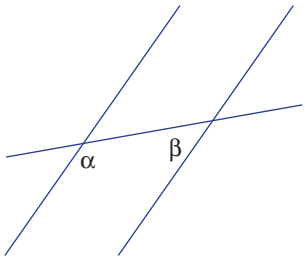
**c**



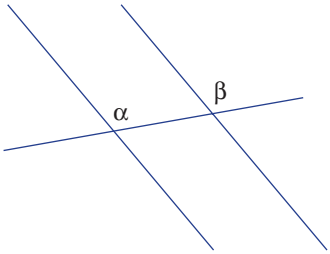

**d**




**e**



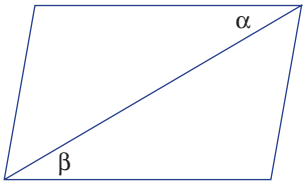

**f**



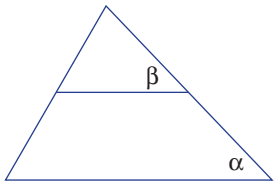

**g**



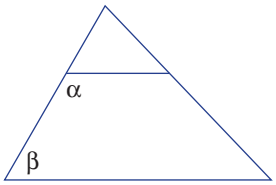

**h**



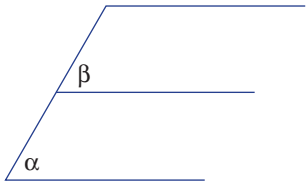

**i**



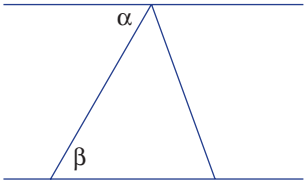

**j**




**k**




**l**




**2** **a** Draw a large capital **Z** and mark two alternate angles with $\alpha$ and $\beta$.

**b** Draw a large capital **N** and mark two alternate angles with $\alpha$ and $\beta$.

**c** Draw a large capital **H** and mark two co-interior angles with $\alpha$ and $\beta$.

**d** Draw a large capital **H** and mark two alternate angles with $\alpha$ and $\beta$.

**e** Draw a large capital **F** and mark two corresponding angles with $\alpha$ and $\beta$.

**f** Draw a large capital **E** and mark two co-interior angles with $\alpha$ and $\beta$.

**g** Draw a large capital **E** and mark two corresponding angles with $\alpha$ and $\beta$.

**h** Draw a large capital **W** and mark two alternate angles with $\alpha$ and $\beta$.


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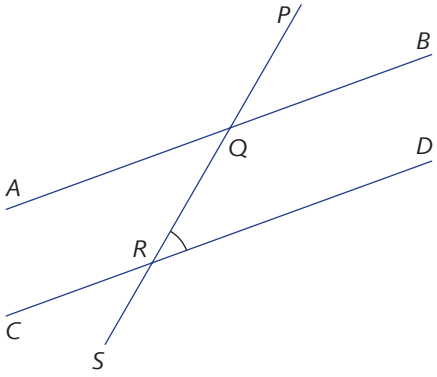
**3** For each diagram below, name:

**a** the angle corresponding to the marked angle

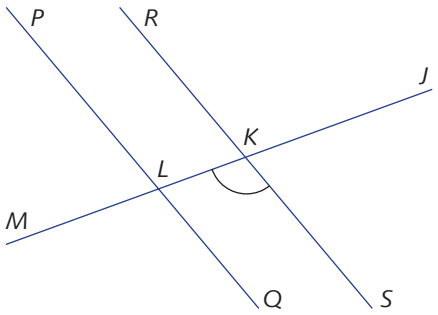
**b** the angle alternate to the marked angle

**c** the angle co-interior to the marked angle.

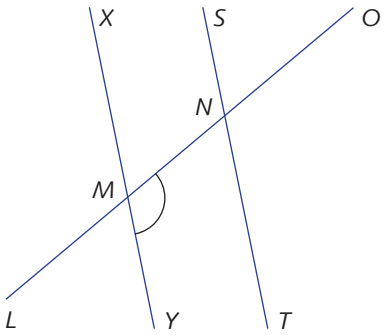
**i**



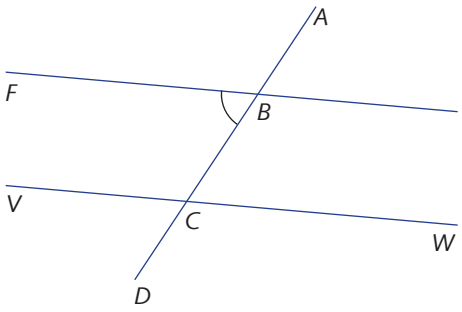

**ii**




**iii**




**iv**



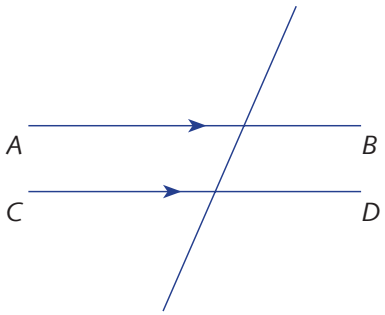

**4** **a** Draw two horizontal parallel lines labelled $AB$ and $CD$, using opposite edges of your ruler.

**b** Draw a transversal crossing these two parallel lines at an angle.

**c** Let the transversal meet $AB$ at $X$ and $CD$ at $Y$.

**d** Measure $\angle AXY$ and $\angle XYD$.

**e** Write down the result that these measurements confirm.



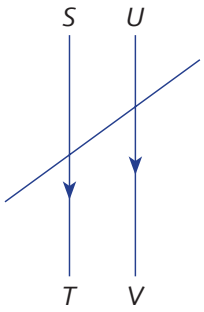

**5** **a** Draw two vertical parallel lines, $ST$ and $UV$, using opposite edges of your ruler.

**b** Draw a transversal, $JK$, crossing these two parallel lines at an angle.

**c** Let the transversal meet $ST$ at $X$ and $UV$ at $Y$.

**d** Measure $\angle JXS$ and $\angle JYU$.

**e** Write down the result that these measurements confirm.





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**6** **a** Draw two oblique parallel lines, $AB$ and $CD$, using opposite edges of your ruler.

**b** Draw a transversal crossing these two parallel lines at an angle.

**c** Let the transversal meet $AB$ at $X$ and $CD$ at $Y$.

**d** Measure $\angle AXY$ and $\angle CYX$.

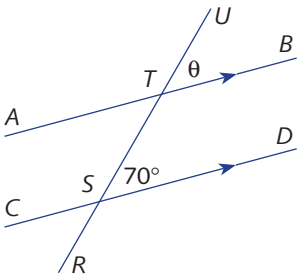
**e** Write down the result that these measurements confirm.

*Note*: This is the second exercise in which you are setting out mathematical arguments. Write your reasons carefully, as in the four examples on the previous pages. Some questions require more than one step, each with its own reason.

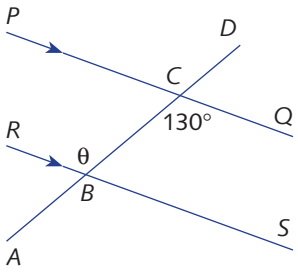
Example 6, 7, 8

**7** Find the values of $\alpha$, $\beta$, $\gamma$ and $\theta$ in the diagrams below. Give careful reasons for all your statements, mentioning the relevant parallel lines.

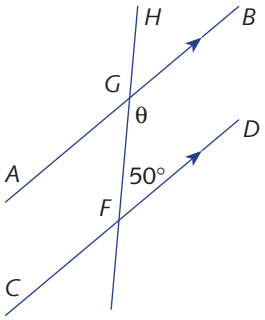
**a**



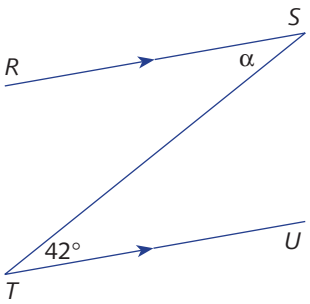

**b**



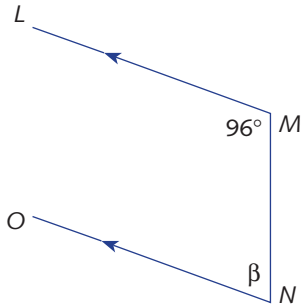

**c**



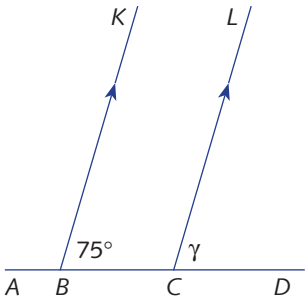

**d**



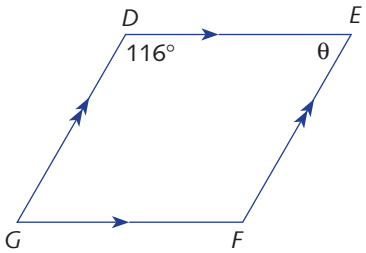

**e**



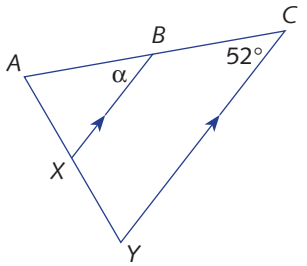

**f**



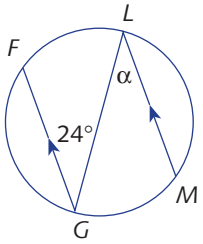

**g**



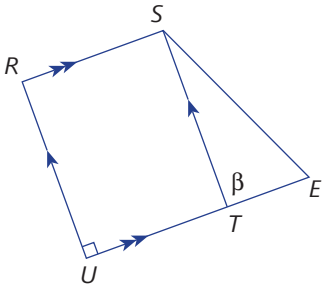

**h**



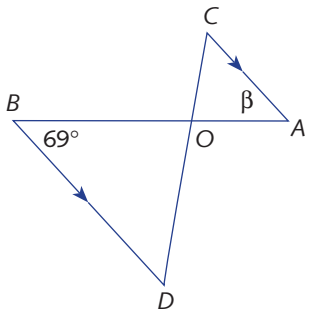

**i**



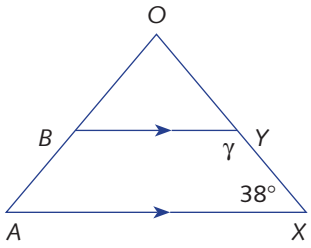

**j**




**k**




**l**



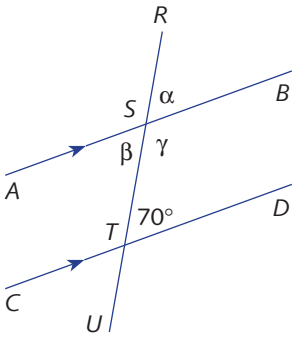


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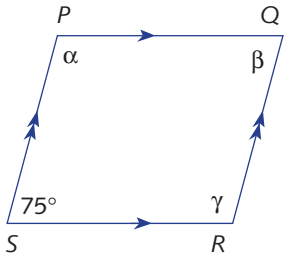
Example 6, 7, 8, 9

**8** Find the values of the letters $\alpha$, $\beta$, $\gamma$ and $\theta$ in the diagrams below. Give a reason for each step in your argument, and name the relevant parallel lines.

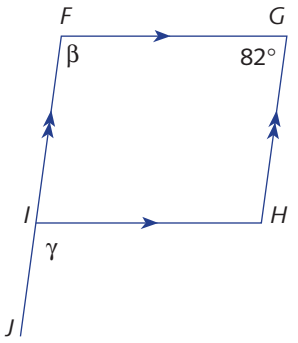
**a**



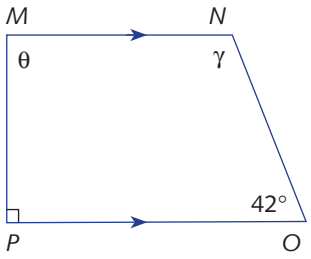

**b**



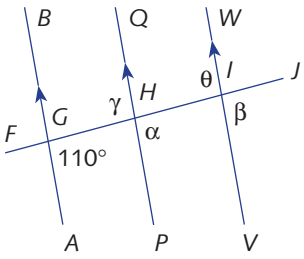

**c**



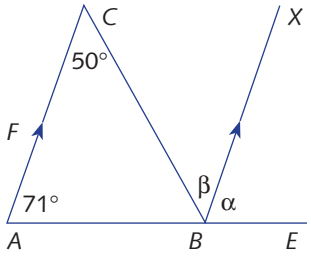

**d**



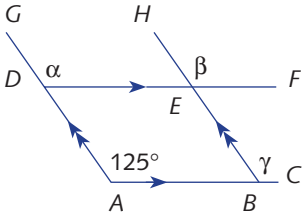

**e**



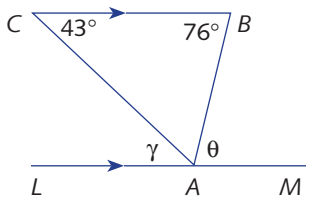

**f**




**g**




**h**

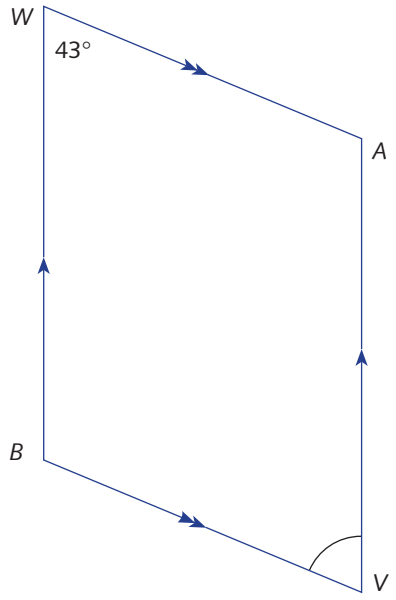




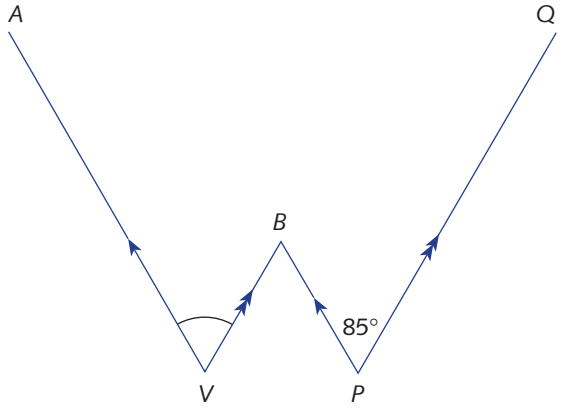
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**9** Find the size of the marked angle $\angle AVB$ in each diagram below. The solution to each part will require at least two steps, each with its own reason.

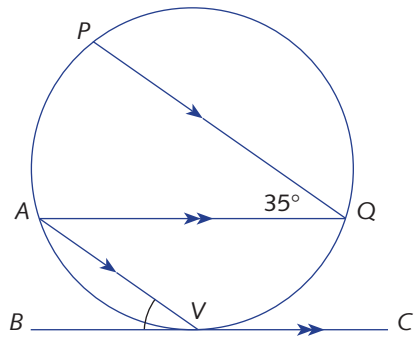
**a**



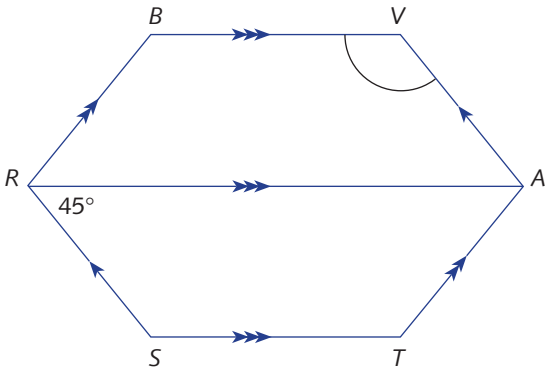

**b**




**c**




**d**




# 6F Further problems involving parallel lines

This section deals with more complicated problems involving parallel lines:

- problems where construction lines need to be added
- problems involving algebra.

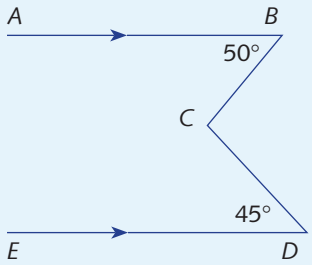
## Adding construction lines to solve a problem

Some problems cannot be solved until one or more extra lines, called **construction lines**, have been added to the diagram, as in the example on the next page.


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## Example 10

Find $\angle BCD$ in the diagram shown.



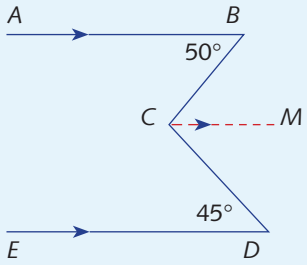

### Solution

Construct the line $CM$ through $C$ parallel to $AB$ and $ED$, as shown in the diagram.

Then $\angle BCM = 50°$ (alternate angles, $AB \parallel CM$)

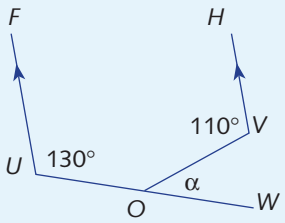
and, $\angle DCM = 45°$ (alternate angles, $ED \parallel CM$)

hence, $\angle BCD = 95°$ (adjacent angles at $C$)




## Example 11

Find the value of $\alpha$ in the diagram shown.




### Solution

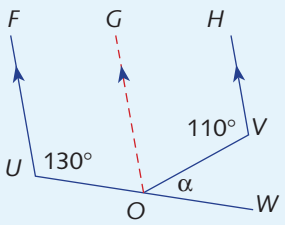
Construct $OG \parallel UF$, as shown.

Then $\angle VOG = 70°$ (co-interior angles, $HV \parallel GO$)

and $\angle UOG = 50°$ (co-interior angles, $FU \parallel GO$)

$\alpha + 50° + 70° = 180°$ (straight angle at $O$)

so $\alpha = 60°$





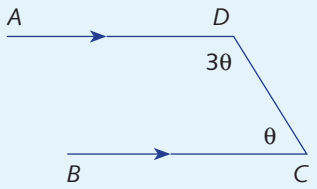
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## Problems involving algebra

In the example below, the value of $\theta$ is found by using geometric arguments and algebra.

### Example 12

Find $\theta$ in the diagram shown.




### Solution

$\theta + 3\theta = 180°$ (co-interior angles, $AD \parallel BC$)
$4\theta = 180°$
$\theta = 45°$

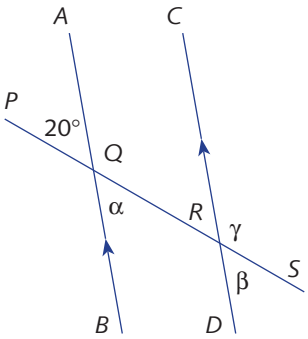
## Exercise 6F

*Note*: Each problem in this exercise requires you to set out a mathematical argument, with carefully written reasons. Some questions require two or more steps, each with their own reasons.

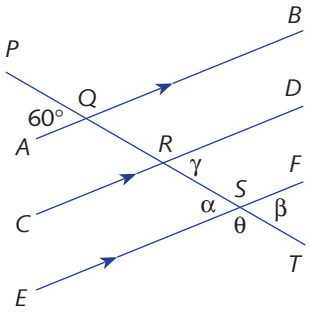
Example 10, 11

**1** Find the values of $\alpha$, $\beta$, $\gamma$ and $\theta$ in each of the diagrams below and on the next page. Give careful reasons for all your statements.

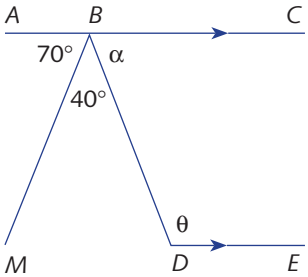
**a**



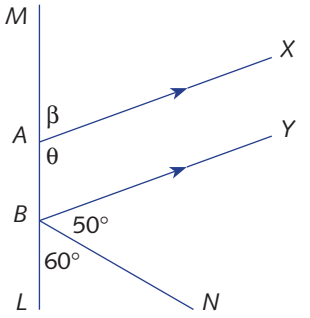

**b**




**c**



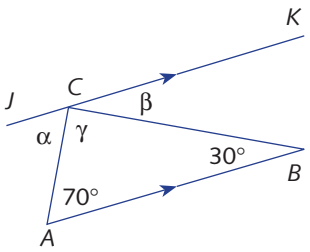

**d**



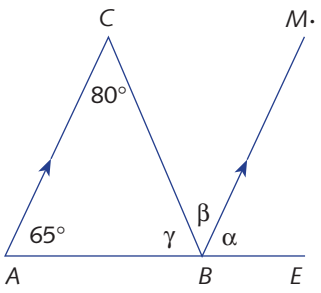


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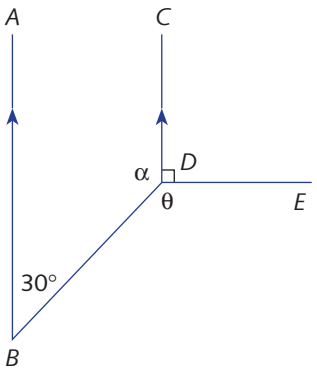
**e**



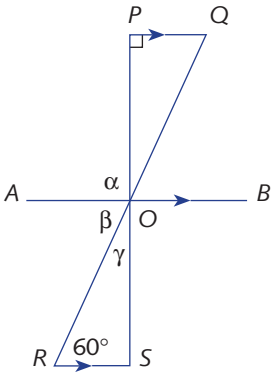

**f**




**g**

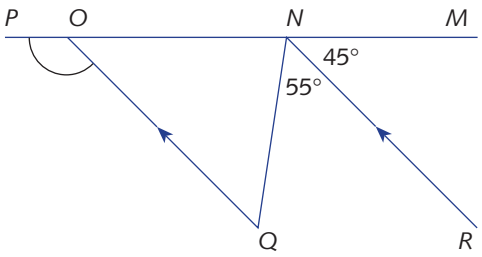



**h**

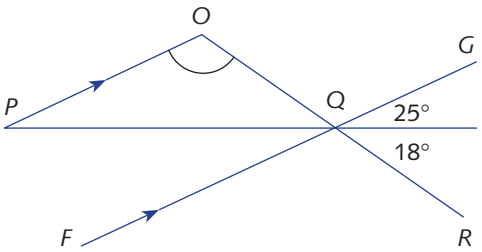



**2** Find the size of the marked angle $\angle POQ$ in each diagram below. Each solution will require two or more steps, each with its own reason.

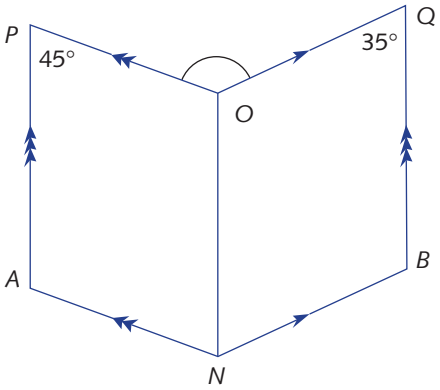
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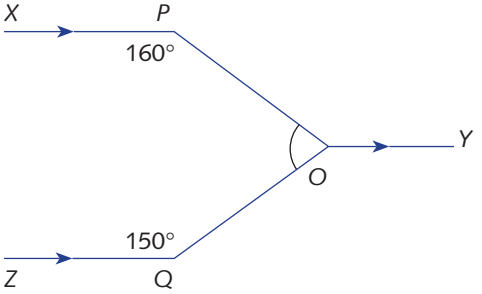

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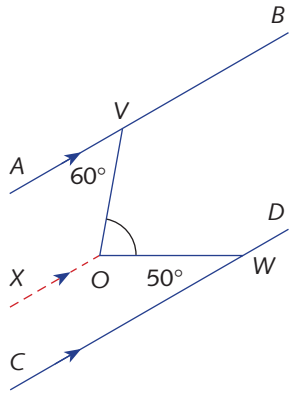




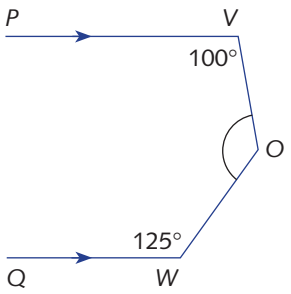
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**3** Copy each diagram below, then add a suitable construction line in order to find the size of the marked angle $\angle VOW$. The construction line has been drawn for you in the first one. Give careful reasons for all your statements.

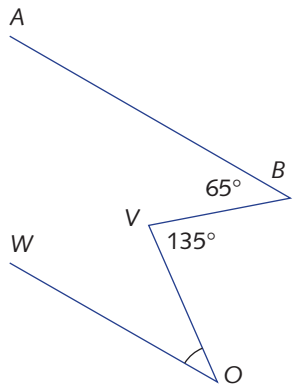
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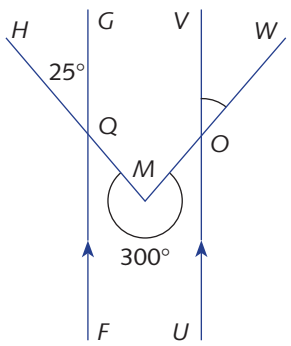

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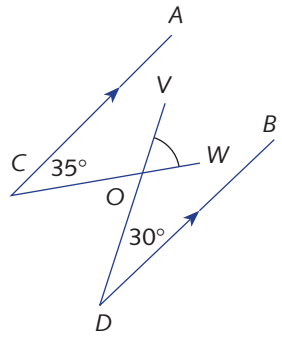

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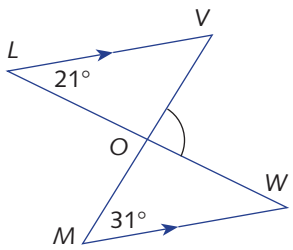

**d**



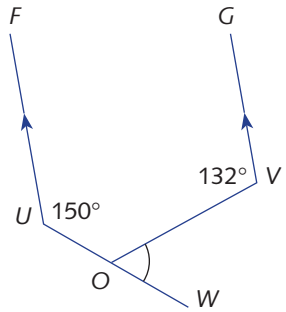

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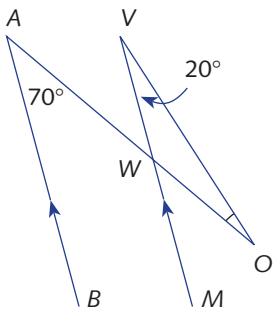

**f**




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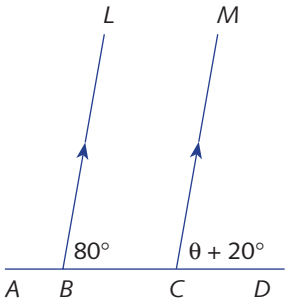


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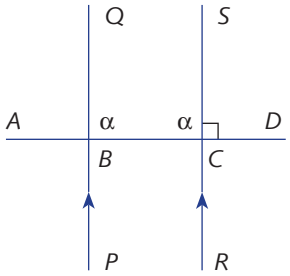
Example 12

**4** Find the values of $\alpha$, $\beta$, $\gamma$ and $\theta$, giving reasons for your answers.

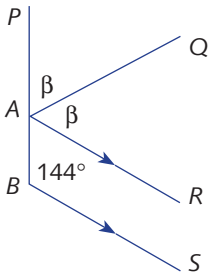
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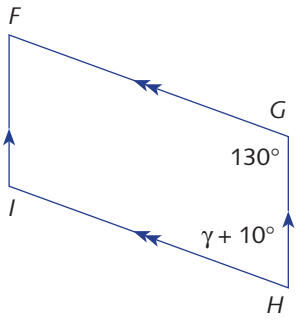

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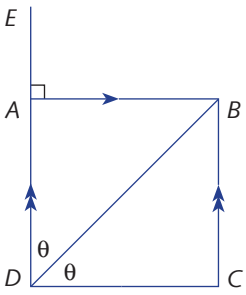

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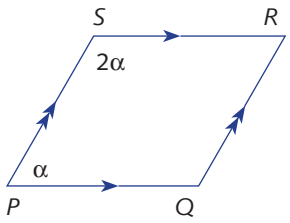

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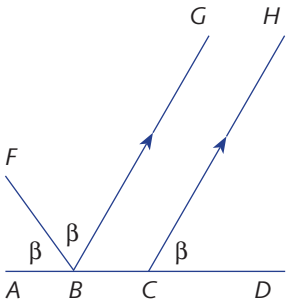

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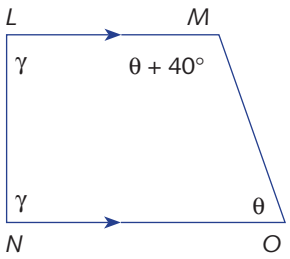

**f**




**g**




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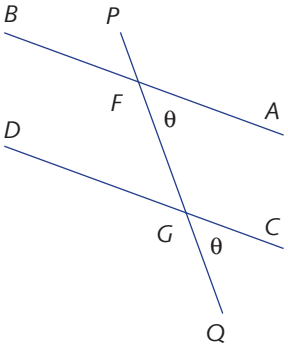
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# 6G Proving that two lines are parallel

Corresponding, alternate and co-interior angles can be used to prove that two lines are parallel.

## Equal corresponding angles mean the lines are parallel

In the diagram at the right, we are told that the corresponding angles $\angle AFQ$ and $\angle CGQ$, both marked $\theta$, are equal. In this situation, we can conclude that the two lines are parallel.



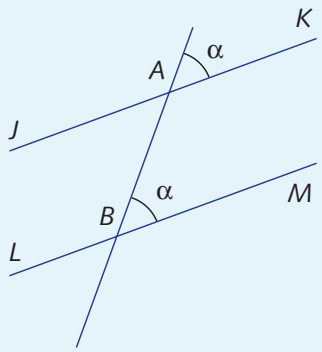

We know that there is only one line through $F$ that is parallel to $DC$. For that line, the angle corresponding to $\angle QGC$ is also $\theta$, so that line must coincide with the line $BFA$.

When using equal corresponding angles to prove that two lines are parallel, the reason must be stated as 'corresponding angles are equal'.

The example below is a typical problem.

**Example 13**

Find any parallel lines in the diagram shown.

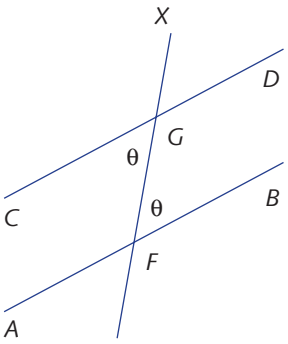



**Solution**

$JK \parallel LM$ (corresponding angles are equal)

## Equal alternate angles mean the lines are parallel

In the diagram to the right, the two alternate angles $\angle BFG$ and $\angle FGC$, both marked $\theta$, are equal.




Hence, $\angle DGX = \theta$ (vertically opposite at $G$).

Then $\angle BFG = \angle DGX = \theta$.

That is, corresponding angles are equal.

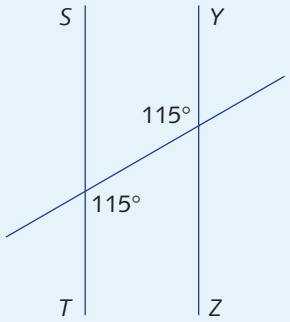
Therefore, the lines $CD$ and $AB$ are parallel.

When using equal alternate angles to prove that two lines are parallel, the reason must be stated as 'alternate angles are equal', as in the next example.


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**Example 14**

Find any parallel lines in the diagram shown.

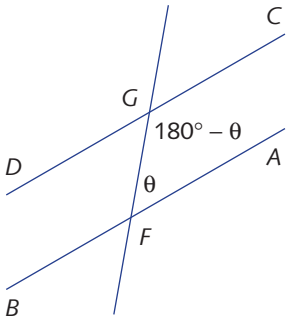



**Solution**

$ST \parallel YZ$ (alternate angles are equal)

## Supplementary co-interior angles mean the lines are parallel

In the diagram to the right, we are told that the two co-interior angles $\angle AFG$ and $\angle CGF$, marked $\theta$ and $180° - \theta$, are supplementary. This again means that the two lines are parallel.




This can be demonstrated as follows.

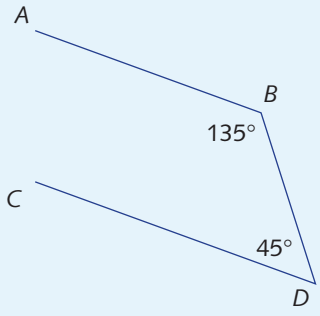
$\angle FGD = \theta$ (straight angle at $G$)

so $AB \parallel CD$ (alternate angles are equal)

This reason should be stated as 'co-interior angles are supplementary' when used in arguments.

**Example 15**

Find any parallel lines in the diagram shown.




**Solution**

$AB \parallel CD$ (co-interior angles are supplementary)

All other lines intersect and so are not parallel.


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## Proving that two lines are parallel

Suppose that a transversal crosses two other lines.

- If the corresponding angles are equal, then the lines are parallel.
- If the alternate angles are equal, then the lines are parallel.
- If the co-interior angles are supplementary, then the lines are parallel.

### A statement and its converse

The statements in this section are the **converses** of the statements in Section 6E, meaning that they are formed from the previous statements by reversing the logic. For example:

*Statement:* If the lines are parallel, then the corresponding angles are equal.

*Converse:* If the corresponding angles are equal, then the lines are parallel.

Pairs such as these, consisting of a statement and its converse, occur routinely throughout mathematics, and are particularly prominent in geometry. In this case, both the statement and its converse are true.

It is most important to realise that, in general, a statement and its converse are quite different. *Never assume that because a statement is true, then the converse must be true*. For example, consider the following statement and its converse.

*Statement:* If a number is a multiple of 4, then it is even.

*Converse:* If a number is even, then it is a multiple of 4.

The first statement is clearly true, but its converse is false because, for example, 10 is even but is not a multiple of 4.

Here is an example from surfing.

*Statement:* If you catch a wave, then you will be happy.

*Converse:* If you are happy, then you will catch a wave.

Many people would agree with the first statement, but everyone knows that its converse is plain silly – you need skill to catch waves, not happiness.

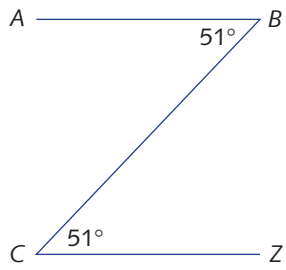
Thus, the truth of a statement has little to do with the truth of its converse. Just as much care has been taken in justifying the converse statements in this section as was taken in justifying the original statements of the previous section.

## Exercise 6G

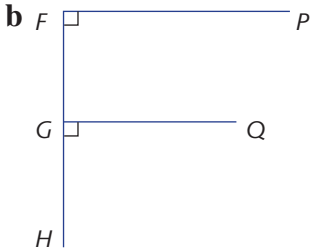
Example 13, 14, 15

**1** For each diagram below, name all pairs of parallel lines, giving reasons.

**a**



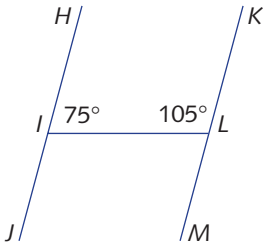

**b**



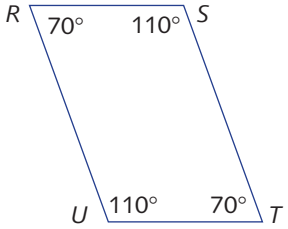


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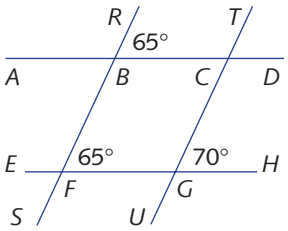
**c**



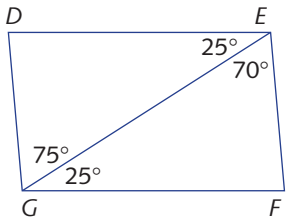

**d**



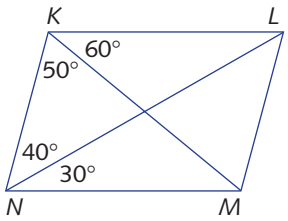

**e**



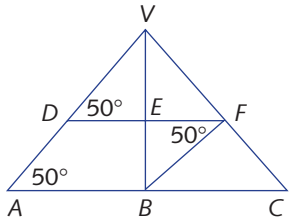

**f**




**g**

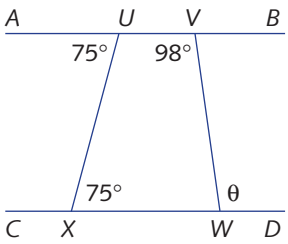



**h**

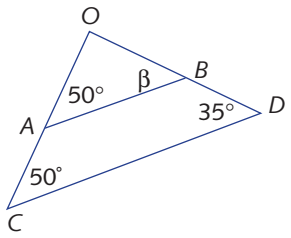



**2** In each diagram below, give a reason why $AB \parallel CD$. Hence, find the values of $\alpha$, $\beta$, $\gamma$ and $\theta$. Give all reasons.

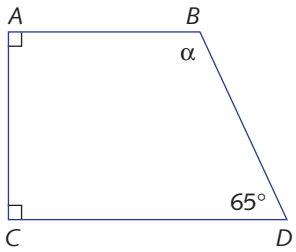
**a**



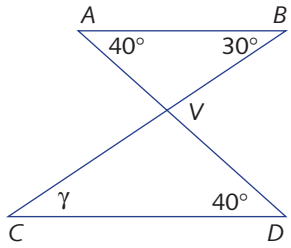

**b**




**c**

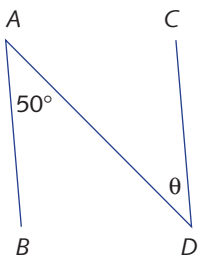



**d**

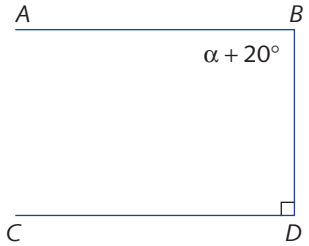



**3** In each diagram, find the values of $\alpha$, $\beta$, $\gamma$ and $\theta$ that will make $AB$ parallel to $CD$. Give all reasons.

**a**



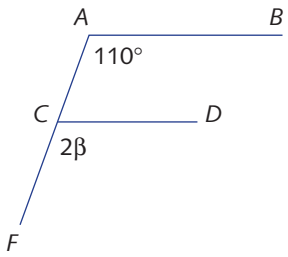

**b**



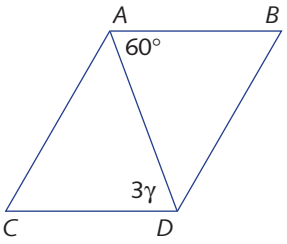


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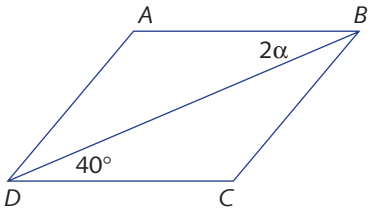
**c**



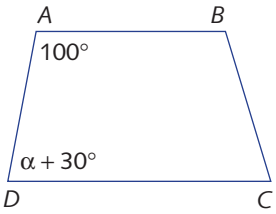

**d**



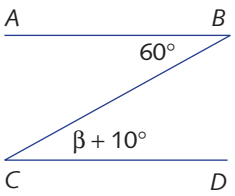

**e**



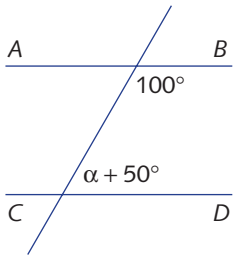

**f**




**g**




**h**




**4** State whether each statement below is true or false, then write down its converse. Then state whether the converse is true or false.

**a** If you can run, then you can walk.

**b** If a number is greater than 10, then it is greater than 1000.

**c** If a man lives in Australia, then he lives in Melbourne.

**d** If a number is divisible by 5, then its last digit is 5.

**e** If a woman has a daughter, then she is a mother.

**f** If a whole number has fewer than four digits, then it is less than 1000.

**g** If a dog has black and white hair, then it can stand on its hind legs.

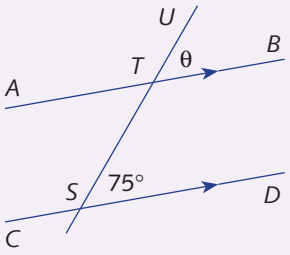
**h** If one side of a rectangle has length 8 cm, then its area is 40 $cm^2$.


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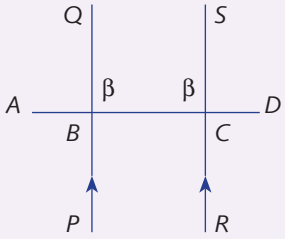
# Review exercise

**1** Find the values of $\alpha$, $\beta$, $\gamma$ and $\theta$ in the diagrams below. Give reasons in each case.

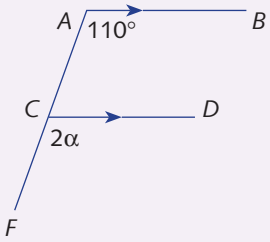
**a**



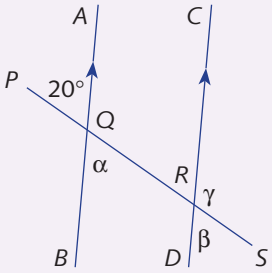

**b**



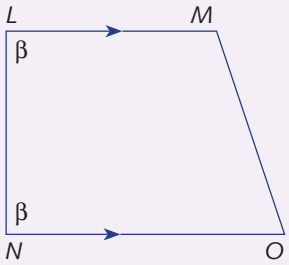

**c**




**d**




**e**




**2** State the complement of each angle.

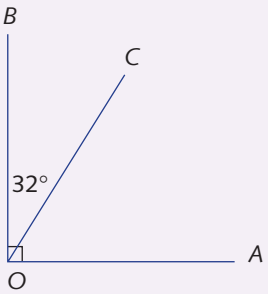
**a** 30° **b** 63° **c** 74° **d** 84°

**3** State the supplement of each angle.

**a** 127° **b** 76° **c** 134° **d** 15°

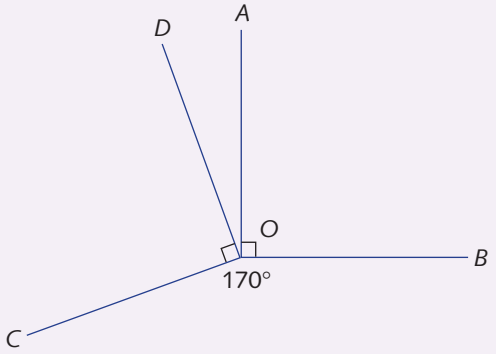
**4** Find $\angle AOC$ in the diagram to the right.

Also give the size of reflex angle $\angle AOC$.




**5** Find $\angle AOD$ in the diagram to the right.

Also give the size of reflex angle $\angle AOD$.

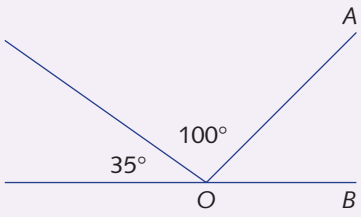




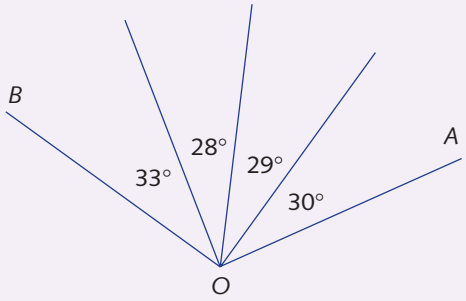

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**6** Find the size of $\angle AOB$ in each diagram below.

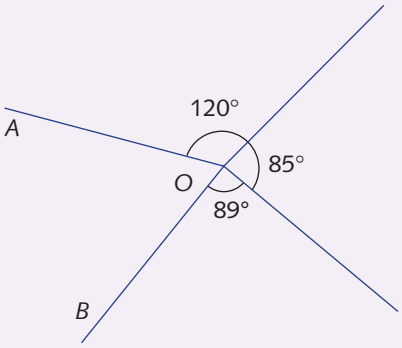
**a**



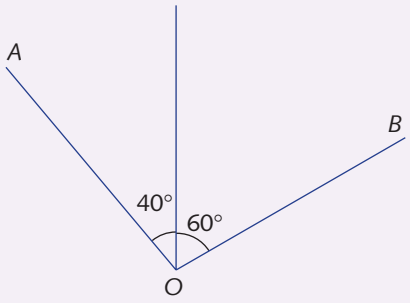

**b**




**c**

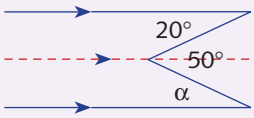



**d**

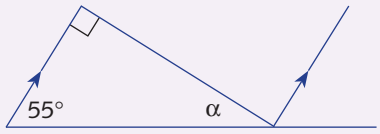



**7** Find the value of $\alpha$ in each diagram below.

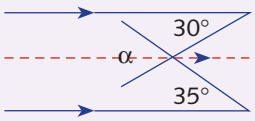
**a**



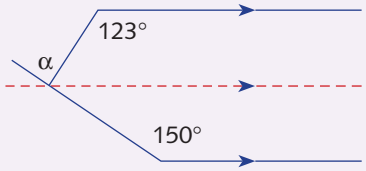

**b**




**c**




**d**



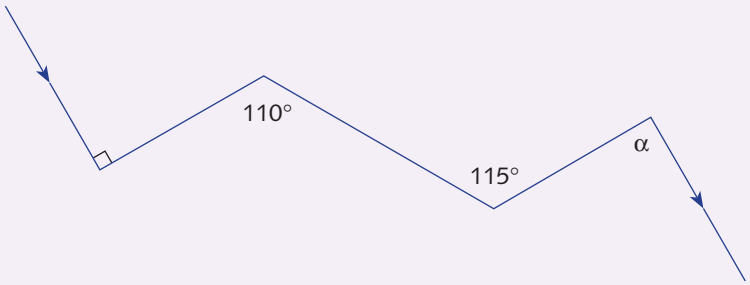




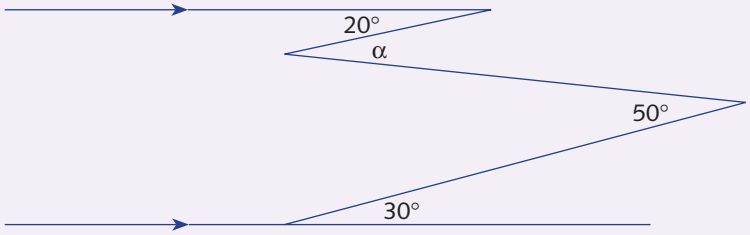
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# Challenge exercise

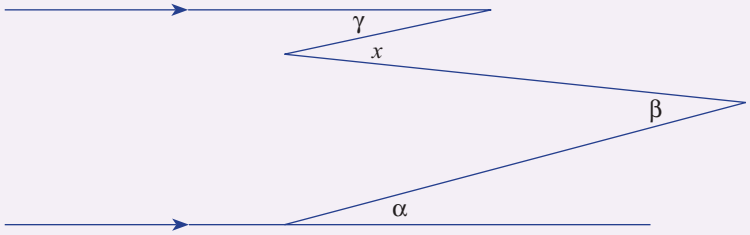
**1** Calculate the unknown angle.



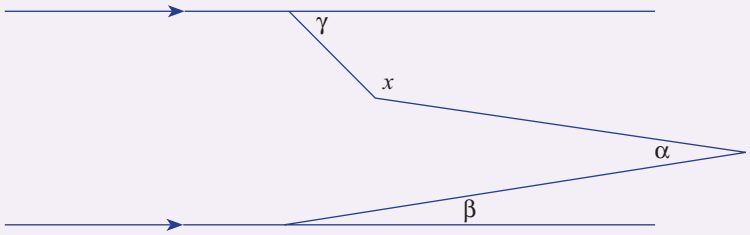

**2** **a** Find the value of $\alpha$.




**b** Find $x$ in terms of $\alpha$, $\beta$ and $\gamma$.

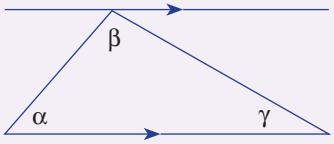



**3** Find $x$ in terms of $\alpha$, $\beta$ and $\gamma$.




**4** There are five possible configurations of three distinct planes in space. Two involve parallel planes, and the other three do not. Draw a picture of each one.

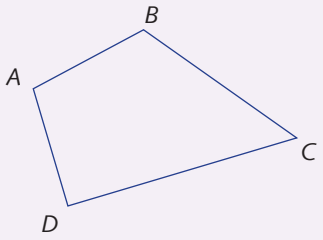
**5** **a** Prove that $\alpha + \beta + \gamma = 180°$; that is, that the sum of the angles in any triangle is 180°.



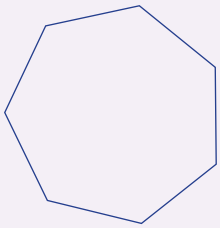


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**b** By dividing the quadrilateral at the right into two triangles, show that the sum of the angles in a quadrilateral is 360°.




**c** A heptagon is a seven-sided figure, as shown below. What is the angle sum of a heptagon?



**6** What is the angle between the minute and hour hand of an analogue clock as 12:15 p.m.? Hint: Why is it **less** than 90°?

**7** How many minutes pass after 12 noon before the angle between the minute and hour hand is exactly 90°?


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CHAPTER

7

Number and Algebra

# Algebra with fractions

Algebra is a very important part of mathematics. It allows us to use letters to represent numbers. In Chapter 3, we used letters to represent whole numbers and we developed some of the rules of algebra. We will now extend our work to include fractions, and use letters in problems involving division.

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# 7A Division in algebra

We begin with some examples to show how division can be used with algebra.

## Example 1

If $x$ oranges are divided equally among 5 people, how many oranges does each person receive if:

**a** $x = 50$?

**b** $x = 37$?

### Solution

**a** $\text{Oranges per person} = \frac{x}{5}$

$= \frac{50}{5}$

$= 10$

**b** $\text{Oranges per person} = \frac{x}{5}$

$= \frac{37}{5}$

$= 7\frac{2}{5}$

## Example 2

Write each expression using algebraic notation.

**a** A number is divided by 5, and 6 is added to the result.

**b** Five is added to a number, and the result is divided by 3.

### Solution

**a** Let $x$ be the number.
Dividing by 5 gives $\frac{x}{5}$.
Adding 6 to this result gives $\frac{x}{5} + 6$.

**b** Let $x$ be the number.
Adding 5 gives $x + 5$.
Dividing this by 3 gives $\frac{x+5}{3}$.

Notice that the fraction line (the **vinculum**) acts like a bracket.

That is, $\frac{x+5}{3} = \frac{(x+5)}{3}$.

## Example 3

If $x = 10$, find the value of:

**a** $\frac{x}{5} + 3$

**b** $\frac{x+4}{3}$


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**Solution**

**a** $\frac{x}{5} + 3 = \frac{10}{5} + 3$
$= 2 + 3$
$= 5$

**b** $\frac{x+4}{3} = \frac{10+4}{3}$
$= \frac{14}{3}$
$= 4\frac{2}{3}$

**Example 4**

A vat initially contains $n$ litres of oil. A further 40 litres of oil are added to the vat.

**a** How many litres of oil are there now in the vat?

**b** The oil is then divided into 50 containers. How much oil is there in each container?

**Solution**

**a** The vat contains $n + 40$ litres of oil.

**b** Each container holds $\frac{n+40}{50}$ litres.

The following table gives the meanings of some commonly occurring types of algebraic expressions.

| Algebraic expression | Meaning |
|---|---|
| $\frac{x+3}{5}$ | 3 is added to $x$, and the result is divided by 5. |
| $\frac{x-5}{7}$ | 5 is subtracted from $x$, and the result is divided by 7. |

## Exercise 7A

Example 2

**1** Write each division as a fraction.

**a** A number, $x$, is divided by 5.

**b** A number, $x$, is divided by 5, and 3 is added to the result.

**c** 7 is added to a number, $m$, and the result is divided by 3.

**d** 6 is subtracted from a number, $p$, and the result is divided by 3.

**e** A number, $m$, is divided by 11, 3 is subtracted from the result, and the result of this is multiplied by 6.

**2** Write each division as a fraction.

**a** $(b + 2) \div 2$ **b** $(c - 5) \div 4$ **c** $(y + 11) \div 4$ **d** $(23 - x) \div 7$


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Example 3

**3** Evaluate each expression for $x = 20$.

**a** $\frac{x}{2}$ **b** $\frac{x}{5} + 7$ **c** $\frac{x + 10}{6}$ **d** $\frac{40 - x}{10}$

**4** Evaluate each expression for $m = 5$ and $n = 10$.

**a** $\frac{m}{n}$ **b** $\frac{n - m}{5}$ **c** $\frac{20}{m} + 3$ **d** $\frac{3m - n}{m}$

Example 4

**5** A pile of $n$ bananas is divided into 5 heaps.

**a** How many bananas are there in each heap?

**b** Three bananas are added to each heap. How many bananas are there in each heap now?

**6** $n$ tonnes of coal are stored in a shed. An extra 1000 tonnes are then added.

**a** How many tonnes of coal are there in the shed now?

**b** It is decided to ship the coal in 10 equal loads. How many tonnes of coal are there in each load?

**7** On a maths test paper, there are $x$ questions. Before the test starts, it is announced that the last two questions are wrong and should be ignored.

**a** How many questions must the students complete now?

**b** The students are given an hour to do the test. How many minutes should they spend on each question, on average?

**8** Peter joined three of his friends for dinner. They decided to share the bill of $\$y$ equally.

**a** How much did Peter have to pay?

**b** Peter had a 50-dollar note in his wallet. How much did he have left after the meal?

# 7B Multiplication and division in algebra

The following examples introduce algebraic expressions involving multiplication and division.

## Example 5

Write each of these expressions using algebraic notation.

**a** A number, $x$, is multiplied by 3 and divided by 2.

**b** $\frac{1}{4}$ of $x$.

**c** A number, $x$, is multiplied by $\frac{2}{3}$.

**d** A number, $x$, is divided by 3, 5 is added to the result, and the result of this is multiplied by 7.


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**Solution**

**a** If $x$ is the number, multiplying by 3 gives the result $3x$.

Dividing the result by 2 gives $\frac{3x}{2}$.

**b** $\frac{1}{4}$ of $x = \frac{1}{4} \times \frac{x}{1}$

$= \frac{x}{4}$

**c** Multiply $x$ by $\frac{2}{3}$.

$\frac{2}{3}$ of $x = \frac{2}{3} \times \frac{x}{1}$

$= \frac{2x}{3}$

**d** Dividing by 3 gives the result $\frac{x}{3}$.

Adding 5 gives $\frac{x}{3} + 5$.

Finally, multiplying this by 7 gives $7\left(\frac{x}{3} + 5\right)$.

Note that brackets must be used here because the whole expression $\frac{x}{3} + 5$ is multiplied by 7.

The following table gives the meanings of some commonly occurring types of algebraic expressions.

| Algebraic expression | Meaning |
|---|---|
| $\frac{2x}{3}$ | $x$ is multiplied by 2 and divided by 3.<br>$x$ is multiplied by $\frac{2}{3}$.<br>$\frac{2}{3}$ of $x$<br>$\frac{1}{3}$ of $2x$ |
| $\frac{2x+3}{5}$ | $x$ is multiplied by 2, and 3 is added. The result is divided by 5. |
| $\frac{3x-5}{7}$ | $x$ is multiplied by 3, and 5 is subtracted. The result is divided by 7. |
| $\frac{2x}{3} + 6$ | 6 is added to $\frac{2}{3}$ of $x$.<br>6 is added to $\frac{2}{3} \times x$.<br>6 is added to $\frac{1}{3}$ of $2x$. |
| $\frac{x^2}{3}$ | $x$ is squared and the result is divided by 3. |
| $6\left(\frac{x}{4} + 5\right)$ | $x$ is divided by 4, 5 is added to the result, and the result of this is multiplied by 6. |


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## Substitution

We recall that the process in which we replace a letter by a particular value is called substitution.

### Example 6

Evaluate each of these expressions for $x = 24$.

**a** $\frac{3x}{2}$ **b** $\frac{2(x+6)}{5}$

### Solution

**a** $\frac{3x}{2} = \frac{3 \times 24^{12}}{2^{1}}$
$= 36$

**b** $\frac{2(x+6)}{5} = \frac{2(24+6)}{5}$
$= \frac{2 \times 30^{6}}{5^{1}}$
$= 2 \times 6$
$= 12$

## Exercise 7B

Example 5

**1** Write each of these expressions using algebraic notation. In each part, use the pronumeral $x$.

**a** A number is multiplied by 7 and then divided by 3.

**b** A number is divided by 4 and then multiplied by 3.

**c** A number is multiplied by 3, then divided by 2, and 5 is added to the result.

**d** A number is multiplied by 5, then 2 is subtracted from the result, and the result of this is then divided by 5.

**e** $\frac{3}{4}$ of a number.

**f** 6 is added to $\frac{7}{8}$ of a number.

**g** 7 is added to a number multiplied by $\frac{3}{8}$.

**h** A number is divided by 3, then 7 is added to the result, and the result of this is then multiplied by 7.

**i** A number is divided by 8, then 5 is subtracted from the result, and the result of this is then multiplied by 9.

**j** 3 is taken away from $\frac{2}{5}$ of a number, and the result of this is then multiplied by 4.

**k** $\frac{4}{9}$ of a number is subtracted from 23, and the result of this is then multiplied by $\frac{3}{7}$.

**l** $\frac{4}{9}$ of a number is subtracted from $\frac{2}{3}$, and the result of this is then multiplied by $\frac{2}{13}$.

**m** A number is multiplied by itself, and the result is then multiplied by $\frac{2}{3}$.


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Example 6

**2** Evaluate each expression for $x = 12$.

**a** $\frac{3x}{4}$ **b** $\frac{3x}{4} - 9$ **c** $\frac{3(x+2)}{7}$

**3** Evaluate these expressions for $x = 2$.

**a** $\frac{2(x+5)}{7}$ **b** $\frac{2x}{10} + 10$ **c** $7\left(\frac{x-1}{5}\right)$ **d** $4 - \frac{3x}{2}$

**4** Evaluate these expressions for $x = 60$.

**a** $\frac{2(x+50)}{5}$ **b** $\frac{2x}{15} + 10$ **c** $3\left(\frac{x-20}{5}\right)$ **d** $40 - \frac{3x}{5}$

**e** $5 - \frac{360}{2x}$ **f** $\frac{2x+54}{11} + 10$ **g** $\frac{3(200-3x)}{5}$ **h** $3\left(\frac{260-2x}{7} - 8\right)$

**5** Evaluate these expressions for $m = 5$ and $n = \frac{1}{2}$.

**a** $m + n$ **b** $2m + n$ **c** $2m - 3n$ **d** $\frac{m}{4}$

**e** $n \div 5$ **f** $mn$ **g** $n \div m$ **h** $\frac{m+n}{11}$

**6** The perimeter of a triangle, with all sides of equal length, is $x$ cm. What is the length of each side, in terms of $x$? If $x = 22$, what is the length of each side?

**7** A piece of string is $x$ metres in length. It is divided into 5 equal parts.

**a** Find the length of each part, in terms of $x$.

**b** Find the length of each part for:

**i** $x = 20$

**ii** $x = 42$

**iii** $x = 96$

**8** A number, $x$, is doubled and the result is divided by 8. Write this using algebraic notation. If $x = 7$, what is the final result?

**9** A number, $x$, is multiplied by $\frac{4}{7}$, and 5 is subtracted from the result. Write this using algebraic notation. If $x = 10$, find the final result.

**10** For $m = 6$ and $n = \frac{1}{2}$, evaluate these expressions.

**a** $3\left(\frac{m}{n} + 10\right)$ **b** $n\left(\frac{35-m}{3}\right)$ **c** $m + \frac{3m - 34n}{4}$ **d** $\frac{m+23}{n} - 4m$

**11** A supermarket charges $\frac{b}{3}$ dollars for $b$ kg of potatoes, and $\frac{c}{2}$ dollars for $c$ kg of oranges. If 12 kg of potatoes and 10 kg of oranges are bought, what is the cost?

**12** A ribbon of length 12 m is divided into $x$ parts of equal length.

**a** What is the length of each part, in terms of $x$?

**b** If $x = 10$, what is the length of each part?


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**13** $x$ kg of a swimming pool chemical cost \$90. How much, in terms of $x$, did 1 kg cost?

**14** The area of a rectangle is $6x^2$ cm$^2$. The width of the rectangle is 2 cm.

**a** What is the length of the rectangle, in terms of $x$?

**b** What is the length of the rectangle if $x = 4$?

**15** The area of a rectangle is $6x^2$ cm$^2$. The width of the rectangle is 5 cm.

**a** What is the length of the rectangle, in terms of $x$?

**b** What is the length of the rectangle if $x = 4$?

**16** Evaluate these expressions for $a = \frac{2}{3}$ and $b = \frac{3}{5}$.

**a** $a - b$ **b** $3a$ **c** $2a + 3b$ **d** $4a - b$

**e** $ab$ **f** $a \div b$ **g** $b \div a$ **h** $2b \div a$

# 7C Dividing and cancelling

In Chapters 2 and 3, indices were used as a shorthand notation. For example:

$x \times x \times x = x^3$ and $y \times y = y^2$

In Chapter 4, we learned how to cancel fractions. The methods of arithmetic, introduced in earlier chapters, can all be used in the same way with algebra.

It is assumed in the following examples that pronumerals in the denominator cannot take the value 0.

## Example 7

Simplify:

**a** $\frac{8x}{2}$ **b** $\frac{7xy}{x}$ **c** $\frac{42xyz}{xz}$ **d** $\frac{56abc}{35c}$

## Solution

**a** $\frac{8x}{2} = \frac{\cancel{8}^{4}x}{\cancel{2}^{1}}$

$= 4x$

**b** $\frac{7xy}{x} = \frac{7\cancel{x}^{1}y}{\cancel{x}^{1}}$

$= 7y$

**c** $\frac{42xyz}{xz} = \frac{42\cancel{x}^{1}y\cancel{z}^{1}}{\cancel{x}^{1}\cancel{z}^{1}}$

$= 42y$

**d** $\frac{56abc}{35c} = \frac{\cancel{56}^{8}ab\cancel{c}^{1}}{\cancel{35}^{5}\cancel{c}^{1}}$

$= \frac{8ab}{5}$


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## Example 8

Simplify:

**a** $\frac{x}{x}$ **b** $\frac{x^2}{x}$ **c** $\frac{x^3}{x^2}$

### Solution

**a** $\frac{x}{x} = 1$

**b** $\frac{x^2}{x} = \frac{x \times x^1}{x^1}$
$= x$

**c** $\frac{x^3}{x^2} = \frac{x \times x^1 \times x^1}{x^1 \times x^1}$
$= x$

## Example 9

Simplify:

**a** $\frac{60p^2q}{12p}$ **b** $\frac{50x^2y^2}{20xa}$

### Solution

**a** $\frac{60p^2q}{12p} = \frac{60^5 \times p^1 \times p \times q}{12^1 \times p^1}$
$= 5pq$

**b** $\frac{50x^2y^2}{20xa} = \frac{50^5 \times x^1 \times x \times y \times y}{20^2 \times x^1 \times a}$
$= \frac{5xy^2}{2a}$

## Example 10

Simplify:

**a** $\frac{3}{4} \times \frac{8a}{3}$ **b** $\frac{2a}{3} \times \frac{3b}{4}$ **c** $\frac{a}{5} \times \frac{3a}{7}$

### Solution

**a** $\frac{3}{4} \times \frac{8a}{3} = \frac{3^1 \times 8^2 a}{4^1 \times 3^1}$
$= 2a$

**b** $\frac{2a}{3} \times \frac{3b}{4} = \frac{2^1 a \times 3^1 b}{3^1 \times 4^2}$
$= \frac{ab}{2}$

**c** $\frac{a}{5} \times \frac{3a}{7} = \frac{a \times 3a}{5 \times 7}$
$= \frac{3a^2}{35}$


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## Exercise 7C

Example 7

**1** Simplify:

**a** $\frac{4x}{8}$ **b** $\frac{20x}{5}$ **c** $\frac{4x}{20}$ **d** $\frac{2de}{d}$

**e** $\frac{4xy}{y}$ **f** $\frac{7a}{21}$ **g** $\frac{5xy}{x}$ **h** $\frac{3a}{12a}$

**2** Simplify:

**a** $\frac{6x}{x}$ **b** $\frac{8xy}{y}$ **c** $\frac{24xyz}{xz}$ **d** $\frac{mnp}{5m}$

**e** $\frac{9zx}{3z}$ **f** $\frac{18xy}{y}$ **g** $\frac{18xyz}{yz}$ **h** $\frac{72abc}{16c}$

**i** $\frac{36zyx}{4zya}$ **j** $\frac{42xaby}{7ydc}$ **k** $\frac{72def}{54dfyz}$ **l** $\frac{34abc}{6cd}$

Example 8

**3** Simplify:

**a** $\frac{a^2}{a}$ **b** $\frac{a^3}{a^2}$ **c** $\frac{a^3}{a}$ **d** $\frac{4ab}{ab}$

**e** $\frac{3a^2b}{a}$ **f** $\frac{5ab}{a^2}$ **g** $\frac{abcd}{defg}$

Example 9

**4** Simplify:

**a** $\frac{48x^2y}{4x}$ **b** $\frac{25x^2y^2}{20xa}$ **c** $\frac{45x^3y^2}{20xy}$

Example 10

**5** Simplify:

**a** $\frac{5}{4} \times \frac{8a}{10}$ **b** $\frac{2a}{3} \times \frac{3b}{8}$ **c** $\frac{2a}{5} \times \frac{a}{3}$ **d** $a \times \frac{a}{3}$

**e** $\frac{2a}{5} \times \frac{5b}{12}$ **f** $\frac{2a}{3} \times \frac{3a}{5}$ **g** $\frac{5}{4}$ of $20a$ **h** $\frac{3}{4}$ of $\frac{4a}{15}$

**6** Evaluate each expression for $x = 2$ and $y = 3$.

**a** $\frac{x^2}{2}$ **b** $\frac{x^2y}{2}$ **c** $\frac{5x^2y^2}{6}$ **d** $\frac{3x + y}{3x}$ **e** $\frac{6}{xy}$ **f** $\frac{3x^2}{2y^2}$

**7** A rectangle has length $x$ cm and width $\frac{x}{3}$ cm. What is the area of the rectangle? Find the area if $x = 9$.

**8** A rectangle has length $\frac{2x}{3}$ m and width $\frac{3x}{4}$ m. What is the area of the rectangle? Find the area if $x = 12$.

**9** The cost of $x$ kg of mince meat is $\frac{2x}{5}$ dollars.

**a** How much would 1 kg cost?

**b** How much would $z$ kg cost?


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**10** David spends $\frac{2}{3}$ of his income on entertainment. If he earns $\$x$ a month, how much does he spend on entertainment each month?

**11** A certain type of material for curtains costs $\$x$ a metre. How much would $1\frac{4}{5}$ m cost?

**12** Twenty people go to the theatre. Each ticket costs $x$ dollars. The cost of the tickets is to be shared equally among 15 people. How much does each of these people pay?

**13** A newspaper agent employs a team of $y$ newspaper delivery boys and girls. The whole team has to distribute $6x$ daily papers to the local residential area every morning.

**a** How many papers, on average, does each person have to distribute?

**b** What is the average number of papers each person distributes each morning if $x = 100$ and $y = 12$?

# Review exercise

**1** Write each of these expressions using algebraic notation.

**a** A number, $x$, is divided by 10.

**b** A number, $m$, is divided by 3, and 4 is added to the result.

**c** 6 is added to a number, $x$, and the result is divided by 5.

**d** A number, $z$, is divided by 7, and 4 is subtracted from the result.

**e** A number, $m$, is multiplied by 7, and divided by 3.

**f** A number, $a$, is divided by 2 and multiplied by 5.

**g** A number, $b$, is multiplied by 8 and divided by 7, and 5 is added to the result.

**h** A number, $c$, is divided by 7, then 5 is subtracted from the result, then the result of this is multiplied by 10.

**i** 6 is taken away from $\frac{2}{5}$ of a number, $p$, then the result of this is multiplied by $\frac{2}{5}$.

**j** $\frac{4}{11}$ of a number, $q$, is subtracted from 23, then the result of this is multiplied by $\frac{3}{8}$.

**k** 3 is taken away from $\frac{3}{5}$ of a number, $r$, then the result of this is multiplied by $\frac{3}{5}$.

**l** A number, $r$, is multiplied by itself, and the result is multiplied by $\frac{3}{4}$.

**2** For $x = 30$, evaluate:

**a** $\frac{x}{3}$ **b** $\frac{x}{5} + 8$ **c** $\frac{x+2}{7}$


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**3** Evaluate these expressions for $x = 8$.

**a** $\frac{5x}{4}$ **b** $\frac{4x}{5}$ **c** $\frac{3(x+5)}{10}$ **d** $\frac{2x}{5} + 12$

**e** $6\left(\frac{x-2}{5}\right)$ **f** $25 - \frac{5x}{2}$ **g** $\frac{5x-8}{5}$ **h** $\frac{3x-4}{3}$

**4** Evaluate these expressions for $m = 6$ and $n = 7$.

**a** $\frac{m}{n}$ **b** $\frac{m+n}{5}$ **c** $\frac{m}{3} + \frac{n}{2}$ **d** $\frac{m-3}{7}$

**5** An artist had $x$ litres of red paint and $y$ litres of blue paint. In order to spray her sculpture in purple, she mixed $\frac{1}{18}$ of the red paint with $\frac{6}{35}$ of the blue paint. How much purple paint did she produce?

**6** Simplify:

**a** $\frac{8x}{4}$ **b** $\frac{6mn}{m}$ **c** $\frac{24mnp}{mn}$ **d** $\frac{xyz}{5x}$

**e** $\frac{16mn}{4m}$ **f** $\frac{11ab}{b}$ **g** $\frac{18abc}{bc}$ **h** $\frac{48abc}{16c}$

**7** Evaluate these expressions for $a = \frac{4}{5}$ and $b = \frac{3}{4}$.

**a** $a - b$ **b** $3a$ **c** $2a + 3b$ **d** $5 - a$

**e** $6a - b$ **f** $ab$ **g** $a \div b$ **h** $4b \div a$

**8** Evaluate these expressions for $a = \frac{3}{7}$ and $b = \frac{7}{8}$.

**a** $b - a$ **b** $2a$ **c** $2b - 2a$ **d** $3(a + b)$

**e** $\frac{1}{2}(a + b)$ **f** $b \div a$ **g** $a^2 + b^2$ **h** $5 - b$

**9** Complete these statements.

**a** $5x \times \square = 5x^2$ **b** $\frac{2x^2}{3} \times \square = \frac{4}{9}x^2$ **c** $\frac{2a}{3} \times \square = 6a^2$

**d** $\frac{45a^2}{7} \div \square = 5a$ **e** $\frac{6a}{11} \div \square = 24a^2$ **f** $\frac{5z}{12} \div \square = z$

**10** Simplify:

**a** $\frac{z^3}{z^2}$ **b** $\frac{3x^2y}{y}$ **c** $\frac{5m^3n^3}{m^2}$ **d** $\frac{xyzw}{yzwx}$

**11** A rectangle has length $x$ cm and width $\frac{2x}{5}$ cm. What is the area of the rectangle? Find the area if $x = 10$.

**12** A rectangle has length $\frac{5x}{4}$ m and width $\frac{3x}{4}$ m. What is the area of the rectangle? Find the area if $x = 12$.


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# Challenge exercise

**1** **a** A rectangle has area 56 $cm^2$ and width 7 cm. What is the length of the rectangle?

**b** A rectangle has area $x$ $cm^2$ and width 3 cm. What is the length of the rectangle?

**c** A rectangle has area $(x + 4)$ $cm^2$ and width 3 cm. What is the length of the rectangle?

**d** A rectangle has area 3 $cm^2$ and width $x$ cm. What is the length of the rectangle?

**2** **a** A rectangle has area $x$ $cm^2$ and width 2 cm. What is the perimeter of the rectangle?

**b** A rectangle has area $(x + 3)$ $cm^2$ and width 2 cm. What is the perimeter of the rectangle?

**3** **a** A car travels at 60 km/h for 3 hours. What is the distance travelled in the 3 hours?

**b** A car travels at 60 km/h for $n$ hours. What is the distance travelled in the $n$ hours?

**c** A car travels at $x$ km/h for 3 hours. What is the distance travelled in the 3 hours?

**d** A car travels at $x$ km/h for $n$ hours. What is the distance travelled in the $n$ hours?

**4** **a** A boat is 50 km due east of Brisbane at 3 p.m. It then travels in an easterly direction at 30 km/h.

**i** How far from Brisbane is the boat at 6 p.m.?

**ii** How far from Brisbane is the boat after travelling for $n$ hours after 3 p.m.?

**b** A boat is 500 km due east of Sydney at 11 a.m. It then travels in a westerly direction at 25 km/h.

**i** How far from Sydney is the boat at 4 p.m.?

**ii** How far from Sydney is the boat after travelling for $n$ hours after 11 a.m. (up to the time it reaches Sydney)?

**5** **a** A car travels at a constant speed for 2 hours. It travels 120 km in this time. What is the speed of the car?

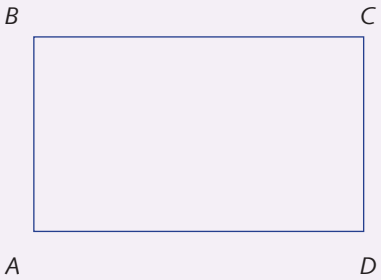
**b** A car travels at a constant speed for 3 hours. It travels $n$ km in this time. What is the speed of the car?

**c** A car travels at a constant speed for $m$ hours. It travels $n$ km in this time. What is the speed of the car?

**6** If a car is travelling at 50 km/h, how long does it take for the car to travel $n$ km?


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**7** The area of the rectangle $ABCD$ is 16 cm$^2$.




The length of $BA$ is $x$ cm.

**a** Find the length of $BC$, in terms of $x$.

**b** Write down an expression for the perimeter of the rectangle, in terms of $x$.

**c** If $x = 4$, find the perimeter of the rectangle.

**d** If $x = \frac{1}{4}$, find the perimeter of the rectangle.

**8** A car travels 20 km from Cranung to Doville. It travels $d$ km at 60 km/h and the remainder at 80 km/h.

**a** How far does the car travel at 80 km/h?

**b** For how long, it terms of $d$, does the care travel at:

**i** 60 km/h?

**ii** 80 km/h?

**c** What is the total time for the journey, in terms of $d$?

**d** Find the total time taken, in minutes, if:

**i** $d = 6$ km

**ii** $d = 12$ km

**iii** $d = 20$ km


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CHAPTER

8

Number and Algebra

# Decimals

You have become accustomed to using fractions to represent quantities that are not whole numbers. We are now going to see how to use decimals to represent certain kinds of fractions. Decimals are very useful for prices and when making measurements, such as the heights of people in your class or the amount of gas used for heating your home. Decimals and percentages, rather than fractions, are used in commerce.

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# 8A Place value and comparison of decimals

## Decimal place value

The word **decimal** comes from the Latin word *decem*, meaning 'ten'. Decimals are an extension of the base-ten place-value number system for whole numbers, which was discussed in Section 1G of Chapter 1. In the decimal system, we use tenths, hundreds, thousandths and so on, as well as units, tens, hundreds, thousands and so on. Here are some basic facts connecting decimals and fractions:

$\frac{1}{10}$ is written as 0.1 $\quad$ $\frac{2}{10}$ is written as 0.2 $\quad$ $\frac{9}{10}$ is written as 0.9

$\frac{1}{100}$ is written as 0.01 $\quad$ $\frac{3}{100}$ is written as 0.03 $\quad$ $\frac{9}{100}$ is written as 0.09

$\frac{1}{1000}$ is written as 0.001 $\quad$ $\frac{4}{1000}$ is written as 0.004 $\quad$ $\frac{9}{1000}$ is written as 0.009

and so on.

The decimal 0.359 means:

$$\frac{3}{10} + \frac{5}{100} + \frac{9}{1000}$$

Similarly, the decimal 8.463 means:

$$8 + \frac{4}{10} + \frac{6}{100} + \frac{3}{1000}$$

The decimal 256.584 is a shorthand for:

$$2 \times 100 + 5 \times 10 + 6 + \frac{5}{10} + \frac{8}{100} + \frac{4}{1000}$$

The **decimal point** separates the units column from the tenths column. To the right of the decimal point, we read the names of the digits individually. For example, 125.2408 is read as 'one hundred and twenty-five point two, four, zero, eight' or 'one, two, five point two, four, zero, eight'.

Here is a chart indicating the place values in the decimal system.

| ↑ Keep going | Millions | Hundreds of thousands | Tens of thousands | Thousands | Hundreds | Tens | Units | Tenths | Hundredths | Thousandths | Ten-thousandths | Hundred-thousandths | Millionths | ↓ Keep going |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | 1 | 2 | 5 • | 2 | 4 | 0 | 8 | | | |
| | | | | | | | | $\frac{1}{10}$ | $\frac{1}{100}$ | $\frac{1}{1000}$ | $\frac{1}{10000}$ | $\frac{1}{100000}$ | $\frac{1}{1000000}$ | |
| | 1000000 | 100000 | 10000 | 1000 | 100 | 10 | 1 | 0.1 | 0.01 | 0.001 | 0.0001 | 0.00001 | 0.000001 | |


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**Example 1**

Using the place values of the digits, write the decimal number 125.2408 as a sum of hundreds, tens, units, tenths, hundredths, thousandths and ten-thousandths.

**Solution**

$$125.2408 = 100 + 2 \times 10 + 5 \times 1 + \frac{2}{10} + \frac{4}{100} + \frac{0}{1000} + \frac{8}{10\,000}$$

The term $\frac{0}{1000}$ may be omitted.

Decimal notation is a convenient and powerful way of writing fractions with denominators as powers of 10.

**Example 2**

Write 3.142 as a fraction with a denominator as a power of 10.

**Solution**

$$\begin{aligned} 3.142 &= 3 + \frac{1}{10} + \frac{4}{100} + \frac{2}{1000} \\ &= \frac{3000}{1000} + \frac{100}{1000} + \frac{40}{1000} + \frac{2}{1000} \\ &= \frac{3142}{1000} \end{aligned}$$

The process can be reversed. A fraction with a denominator that is a power of 10 can be written as a decimal.

**Example 3**

Write $\frac{34}{1000}$ as a decimal.

**Solution**

$$\begin{aligned} \frac{34}{1000} &= \frac{30}{1000} + \frac{4}{1000} \\ &= \frac{3}{100} + \frac{4}{1000} \\ &= \frac{0}{10} + \frac{3}{100} + \frac{4}{1000} \\ &= 0.034 \end{aligned}$$


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## Decimal places

The number of places occupied by the digits after (that is, to the right of) the decimal point is called the number of **decimal places**. Thus, 345.607 has three decimal places, and 0.02 has two decimal places.

## Class discussion

Which is larger:

- 2.12 or 2.4?
- 0.8 or 0.836?
- 4.04 or 4.048?
- 9.52 or 9.029 816 43?
- 0.37 or 0.370 82?
- 5.9 or 5.897?
- 0.35 or 0.6?

How can you tell which is larger? What is a useful strategy for comparing decimal numbers?

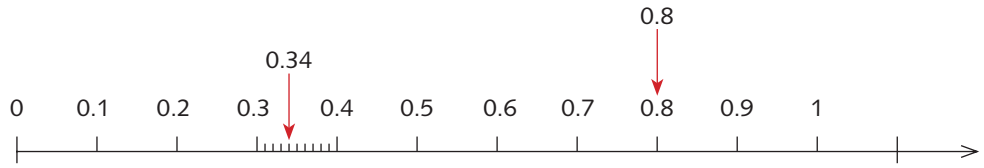
## Comparing decimals

Look at 0.8 and 0.345. In the past, when you worked with whole numbers, you used the 'longer numbers are larger' idea to help you. This does not work with decimal numbers. The number line makes it easy to see which decimal is larger.

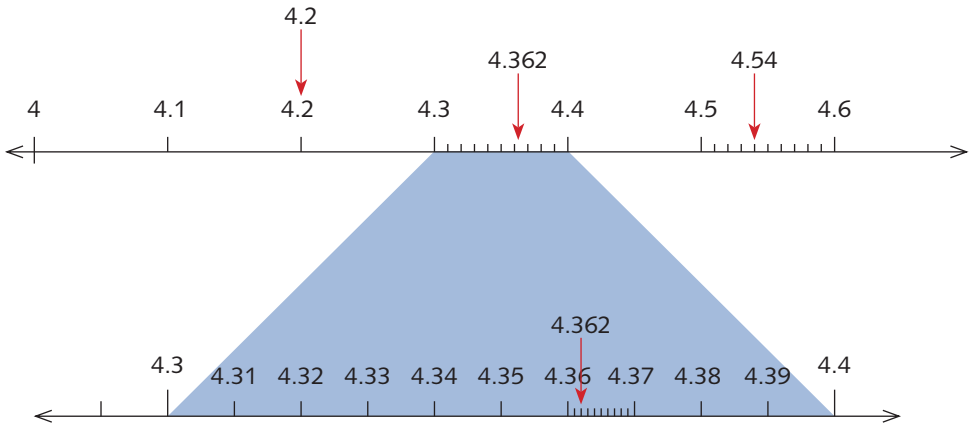
### Comparing decimals using the number line

On the number line, 0.8 and 0.34 are both between 0 and 1.

Because 0.8 is to the right of 0.34, we know that $0.8 > 0.34$.




To make a decision about which of 4.2, 4.54 and 4.362 is the largest, we locate each number on the number line.




We can 'zoom in' on a section of the number line to show that 4.362 is between 4.36 and 4.37.

So we find that $4.2 < 4.362 < 4.54$.


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## Comparing numbers by comparing the digits left to right

When using this method, it is important to write the numbers one underneath the other, with the decimal points lined up.

For example, to compare 3.78 and 3.612, we first write the numbers one beneath the other, aligning the decimal points.

3.78
3.612

Next, we compare the whole-number parts. Both are the same, so we proceed to the next digit or place.

We compare the tenths: 7 tenths is larger than 6 tenths. So 3.78 is larger than 3.612.

This method is the quickest and most straightforward method to use.

### Decimal place value

- The places filled by the digits after the decimal point are called the **decimal places**. For example, 3.146 is a number with three decimal places and 14.1256 has four decimal places.
- Every decimal can be written as a fraction in which the denominator is a power of 10.
- Fractions with denominators that are powers of ten can easily be written as decimals.
- To compare decimals, line up the decimal point and compare the digits from left to right.

### Example 4

Write down the place value of the digit 3 in each decimal.

**a** 231.45 **b** 24.31 **c** 27.031 **d** 2.7503

### Solution

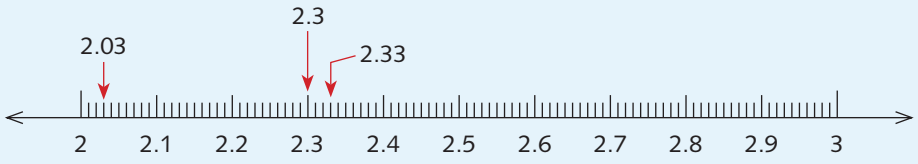
**a** 30 **b** $\frac{3}{10}$ **c** $\frac{3}{100}$ **d** $\frac{3}{10\,000}$

### Example 5

Use a number line to show which is the largest of 2.03, 2.3 and 2.33.

### Solution

All of the numbers are between 2 and 3. Draw the number line:




So 2.33 is the largest.


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### Example 6

Which is larger, 2.52 or 2.574 83? Use left-to-right comparison of digits.

### Solution

Align the digits and the decimal points.

2.52

2.574 83

The whole numbers are the same and the tenths digits are the same.

In the hundredths column, 7 is larger than 2, so 2.574 83 is larger than 2.52.

## Exercise 8A

Example 1 **1** Using the place values of the digits, write the decimal number 276.3507 as a sum of hundreds, tens, units, tenths, hundredths, thousandths and ten-thousandths.

Example 2 **2** Write 4.276 as a fraction whose denominator is a power of 10.

Example 3 **3** Write $\frac{57}{1000}$ as a decimal.

Example 4 **4** Write down the place value of the digit 2 in each decimal.

**a** 32.45 **b** 4.92 **c** 0.21

**d** 23.09 **e** 0.002 **f** 45.978 723

**5** List 10 numbers that can be made from the digits 0, 1, 2, 3 and a decimal point, using each digit only once in each number.

Example 5 **6** Using a number line, find the whole number closest to each decimal.

**a** 3.09 **b** 3.9 **c** 1.284 93

**d** 5.700 001 **e** 4.499 **f** 9.099 99

Example 6 **7** Copy these numbers and circle the number that is the larger of each pair.

**a** 0.4 or 0.32 **b** 1.8 or 1.93

**c** 6.8 or 6.08 **d** 5.63 or 5.064

**e** 7.34 or 7.3412 **f** 5.001 or 5.1

**g** 8.999 78 or 8.342 **h** 3.67 or 3.5

**i** 297.2357 or 297.23 **j** 11.3 or 1.13

**k** 434.897 69 or 434.811 98 **l** 3.56 or 3.499 99


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**8** Order these numbers from smallest to largest.

**a** 2.3, 0.2, 8.153, 7.2, 4.08

**b** 1.09, 1.93, 1.39, 1.9, 1.30

**c** 7.000 023, 7.230 000, 7.99, 7.410 9572, 7.0748

**d** 6.66, 6.4, 6.9234, 6.888, 6.985 74

**9** State whether each number is closer to 4 or closer to 5.

**a** 4.102 534 **b** 4.99 **c** 4.6

**d** 4.52 **e** 4.01 **f** 4.098 795 62

**g** 4.49 **h** 4.000 03 **i** 4.499 989

**j** 4.494 94 **k** 4.831 12 **l** 4.121 09

**10** Complete these statements. (One has been done for you.)

**a** 6 tenths = ____ thousandths

**b** 6 tenths = ____ hundredths

**c** 6 tenths = 0.6 units

**d** 6 tenths = ____ tens

**e** 6 tenths = ____ hundreds

**f** 6 tenths = ____ thousands

**11** Complete these statements. (One has been done for you.)

**a** 89 hundredths = ____ thousandths

**b** 89 hundredths = ____ tenths

**c** 89 hundredths = 0.89 units

**d** 89 hundredths = ____ tens

**e** 89 hundredths = ____ hundreds

**f** 89 hundredths = ____ thousands

**12** Complete these statements. (One has been done for you.)

**a** 723 thousandths = ____ hundredths

**b** 723 thousandths = ____ tenths

**c** 723 thousandths = 0.723 units

**d** 723 thousandths = ____ tens

**e** 723 thousandths = ____ hundreds

**f** 723 thousandths = ____ thousands

# 8B Converting decimals to fractions and fractions to decimals

In this section, you will learn more about how to convert decimals to fractions and fractions to decimals.

## Converting decimals to fractions

To convert a decimal to a fraction or mixed numeral:

- write the decimal part as a fraction with a denominator that is a power of 10
- simplify the fraction, if necessary.


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**Example 7**

**a** Convert 2.36 to a mixed numeral.

**b** Convert 0.085 to a fraction.

**Solution**

**a** $2.36 = 2\frac{36}{100}$ (Now simplify.)

$= 2\frac{9}{25}$

*Note*: $2.36 = 2 + \frac{3}{10} + \frac{6}{100}$

**b** $0.085 = \frac{85}{1000}$ (Now simplify.)

$= \frac{17}{200}$

## Converting fractions to decimals

If we have a fraction or mixed numeral with a denominator that is a power of 10, it is easy to convert it to a decimal, as we saw earlier.

$2\frac{34}{100} = 2.34$

If the denominator is not a power of 10, first try to find an equivalent fraction with a denominator that is a power of 10.

**Example 8**

Convert each fraction to a decimal by making the denominator into a power of 10.

**a** $\frac{3}{5}$ **b** $\frac{3}{4}$ **c** $\frac{17}{20}$ **d** $\frac{7}{8}$

**Solution**

In each case, ask yourself, 'Can I make the denominator a factor of a power of 10?'

**a** $\frac{3}{5} = \frac{3 \times 2}{5 \times 2}$

$= \frac{6}{10}$

$= 0.6$

**b** $\frac{3}{4} = \frac{3 \times 25}{4 \times 25}$

$= \frac{75}{100}$

$= 0.75$

**c** $\frac{17}{20} = \frac{17 \times 5}{20 \times 5}$

$= \frac{85}{100}$

$= 0.85$

**d** $\frac{7}{8} = \frac{7 \times 125}{8 \times 125}$

$= \frac{875}{1000}$

$= 0.875$

The only fractions that are equivalent to fractions with denominators that are a power of 10 are those with denominators that are products of a power of 2 and a power of 5.


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## The division process

An alternative method of converting a fraction to a decimal is to divide the numerator of the fraction by its denominator, as shown in the following example. The algorithm is an extension of the short division algorithm discussed in Section 1I of Chapter 1.

### Example 9

Convert $\frac{743}{8}$ to a decimal, using the division process.

### Solution

Write $\frac{743}{8} = 743 \div 8$.

Set out $743 \div 8$ using the short division algorithm, as shown:

$$8\overline{)7\,4^{2}3.^{7}0^{6}0^{4}0} \quad 9\ 2.\ 8\ 7\ 5$$

So $\frac{743}{8} = 92.875$.

## Basic decimals

You have already seen the fractions equivalent to 0.1, 0.01 and 0.001. You probably already know some of these fractions and their decimal equivalents.

| Decimal number | 0.5 | 0.25 | 0.75 | 0.125 | 0.375 | 0.625 | 0.875 |
|---|---|---|---|---|---|---|---|
| Equivalent fraction | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{3}{4}$ | $\frac{1}{8}$ | $\frac{3}{8}$ | $\frac{5}{8}$ | $\frac{7}{8}$ |

In earlier chapters, you will have found that memorising a good range of number facts is extremely helpful. Knowing the equivalents of a few simple fractions and decimals between 0 and 1, such as those given above, will also be very useful in everyday life.

### Converting between fractions and decimals

- A decimal can be converted to a fraction by writing it as a fraction with a denominator that is a power of ten, and then simplifying.
- A fraction can sometimes be converted to a decimal by finding an equivalent fraction with a denominator that is a power of 10, or by division.


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## Exercise 8B

Example 7

**1** Express each decimal as a fraction or mixed numeral.

**a** 0.75 **b** 15.25 **c** 0.34 **d** 4.125 **e** 0.025

**f** 0.009 **g** 0.806 **h** 8.75 **i** 0.0125 **j** 54.625

**2** Express each fraction or mixed numeral as a decimal.

**a** $\frac{5}{10}$ **b** $\frac{43}{100}$ **c** $\frac{5}{100}$ **d** $\frac{5}{1000}$ **e** $\frac{57}{1000}$

**f** $\frac{30}{1000}$ **g** $\frac{27}{100}$ **h** $\frac{9}{10\,000}$ **i** $\frac{230}{10}$ **j** $2\frac{3}{10}$

**k** $16\frac{96}{100}$ **l** $16\frac{96}{1000}$ **m** $24\frac{35}{10\,000}$ **n** $101\frac{101}{1000}$ **o** $101\frac{101}{10\,000}$

Examples 8

**3** Convert each fraction to a decimal.

**a** $\frac{22}{4}$ **b** $\frac{100}{8}$ **c** $\frac{13}{2}$ **d** $\frac{43}{5}$

**e** $\frac{574}{10}$ **f** $\frac{381}{4}$ **g** $\frac{1095}{20}$ **h** $\frac{27}{5}$

Examples 9

**4** Convert each fraction to a decimal.

**a** $\frac{22}{20}$ **b** $\frac{46}{50}$ **c** $\frac{3}{5}$ **d** $\frac{21}{20}$ **e** $\frac{33}{250}$

**f** $\frac{163}{200}$ **g** $\frac{8}{500}$ **h** $\frac{26}{25}$ **i** $\frac{267}{250}$ **j** $\frac{135}{50}$

**k** $5\frac{2}{5}$ **l** $19\frac{3}{25}$ **m** $2\frac{31}{50}$ **n** $\frac{3}{25}$ **o** $\frac{2}{125}$

**5** Insert the digits 2, 3, 4, 5, 6 and 7 into the boxes to make these statements true.

**a** $\frac{1}{\square} = 0.5$ **b** $\frac{3}{4} = 0.\square 5$ **c** $\frac{8}{5} = 1.\square$

**d** $\frac{1}{\square} = 0.25$ **e** $\frac{\square}{8} = 0.625$ **f** $\frac{5}{16} = 0.\square 125$

**6** Match each decimal to its equivalent fraction.

| Decimal | 0.01 | 0.125 | 0.25 | 0.2 | 0.5 |
|---|---|---|---|---|---|
| Fraction | $\frac{1}{5}$ | $\frac{1}{100}$ | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{8}$ |


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# 8C Addition and subtraction of decimals

Addition and subtraction of decimals can be carried out using the standard algorithms.

## Adding decimals

The vertical addition algorithm is shorthand for adding hundreds to hundreds, tens to tens, ones to ones, tenths to tenths and so on. It is important to line up the place-value columns by lining up the decimal points.

**Example 10**

Add 4.326 and 15.09.

**Solution**

$$\begin{array}{r} 4.3\ 26 \\ +\ 15.0_{1}90 \\ \hline 19.4\ 16 \end{array}$$

(Fill out with zeros to match the places above.)

## Subtracting decimals

When subtracting one decimal from another, write the numbers one under the other, as for whole number subtraction, making sure the decimal points are aligned. Then proceed as for whole numbers. The decimal point in the answer is placed directly under the decimal points in the two numbers.

Below are the two standard subtraction algorithms showing the methods used for subtracting 16.532 from 23.84.

**Method 1**

$$\begin{array}{r} 2^{1}3.84^{1}0 \\ -\ 1_{1}6.53_{1}2 \\ \hline 7.30\ 8 \end{array}$$

**Method 2**

$$\begin{array}{r} {}^{1}2^{1}3.8\cancel{3}4^{1}0 \\ -\ 1\ 6.5\ 3\ 2 \\ \hline 7.3\ 0\ 8 \end{array}$$

Sometimes, the numbers of decimal places are different in the two numbers. It is advisable when doing subtractions with these 'ragged' decimals to add zeros at the end of the 'shorter' number (in the above case, 23.84). This does not change the number; it simply says that, in this case, there are 'no thousandths', and makes the algorithm work.


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## Addition and subtraction of decimals

Addition and subtraction of decimal numbers follows the same procedures as for whole numbers. The decimal points and decimal places should always be aligned one under the other.

### Example 11

Subtract 8.387 57 from 30.102.

**Solution**

$$\begin{array}{r} 3\,{}^{1}0.{}^{1}1\,{}^{1}0\,{}^{1}2\,{}^{1}0\,{}^{1}0 \\ -\ {}_{1}8_{1}.3_{1}8_{1}7_{1}5_{1}7 \\ \hline 2\ \ 1.7\ 1\ 4\ 4\ 3 \end{array}$$

## Exercise 8C

Example 10

**1** Calculate:

**a** $\begin{array}{r} 2.4 \\ +\ 3.5 \\ \hline \end{array}$ **b** $\begin{array}{r} 4.1 \\ +\ 3.2 \\ \hline \end{array}$ **c** $\begin{array}{r} 6.8 \\ +\ 2.1 \\ \hline \end{array}$

**d** 3.12 + 3.2 **e** 6.08 + 3.1 **f** 0.34 + 9.63

**g** 2.312 + 7.25 **h** 7.301 + 2.5987 **i** 3.12 + 4.888

**j** 5.783 + 6.032 **k** 2.967 + 7.2323 **l** 63.8924 + 24.167

**m** 3.692 + 36.195 01 **n** 24.0349 + 102.939 **o** 201.012 + 36.1008

Example 11

**2** Calculate:

**a** $\begin{array}{r} 7.8 \\ -\ 3.4 \\ \hline \end{array}$ **b** $\begin{array}{r} 9.3 \\ -\ 1.2 \\ \hline \end{array}$ **c** $\begin{array}{r} 3.4 \\ -\ 2.3 \\ \hline \end{array}$

**d** 10.8 − 9.03 **e** 12.6 − 7.35 **f** 23.9 − 8.4

**g** 18.6 − 12.7 **h** 35.7 − 23.8 **i** 13.456 − 9.978

**j** 36.24 − 27.985 **k** 819.3407 − 738.9657 **l** 8.345 9038 − 0.948 567 024


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3 Two parcels together have a total weight of 54.7 kg. The first weighs 29.44 kg. Find the weight of the second parcel.

4 A carpenter needs to cut two lengths, 1.04 m and 0.7 m, from a piece of timber 2 m long. How long is the leftover piece?

5 Three girls ran first, second and third in a 100m race, with times of 12.30s, 12.63s and 13.16s, respectively. If the national record time for this race is 11.47s, how far behind the record is each of these girls?

6 Four members of a school walkathon team completed a total of 6 km. The distances walked by three of the members were 1.4 km, 1.24 km and 1.1 km. How far did the fourth person walk?

7 A boy walked 500 m to the bus stop, travelled 1.8 km on the bus to the train station, made two consecutive train trips of 3.5 and 2.3 km, and then walked 700 m to the school gate. How far was his journey to school, in kilometres?

8 Four pieces of string, each of length 2.79 m, were cut from a roll, leaving 1.56 m. How long was the original roll of string?

# 8D Multiplication and division by powers of 10

## Multiplying by 10

When any number is multiplied by 10, we can write the number as a sum and multiply each term by 10.

$$\begin{aligned} 7.92 \times 10 &= 7 \times 10 + 0.9 \times 10 + 0.02 \times 10 \\ &= 70 + \frac{9}{10} \times 10 + \frac{2}{100} \times 10 \\ &= 70 + 9 + \frac{2}{10} \\ &= 79.2 \end{aligned}$$

We see that multiplying by 10 corresponds to moving the decimal point one place to the right.

**Example 12**

Calculate $7.18 \times 10$.

**Solution**

$7.18 \times 10 = 71.8$ (The decimal point is moved one place to the right.)


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## Multiplying by powers of 10

Multiplying by 100 is the same as multiplying by 10 twice.

- Multiplying by $100 = 10^2$ corresponds to moving the decimal point two places to the right and inserting zeros where necessary. For example:

  $8.7 \times 100 = 870$

- Multiplying by $1000 = 10^3$ corresponds to moving the decimal point three places to the right and inserting zeros where necessary. For example:

  $8.7 \times 1000 = 8700$

- Multiplying by $10\,000 = 10^4$ corresponds to moving the decimal point four places to the right and inserting zeros where necessary. For example:

  $8.7 \times 10\,000 = 87\,000$

### Example 13

**a** Calculate $26.235 \times 100$.

**b** Calculate $42.9 \times 1000$.

**Solution**

**a** $26.235 \times 100 = 2623.5$ (Move the decimal point two places to the right.)

**b** $42.9 \times 1000 = 42\,900$ (Move the decimal point three places to the right.)

## Dividing by 10

When any number is divided by 10, we can write the number as a sum and divide each term by 10.

$$\begin{aligned} 4.8 \div 10 &= 4 \div 10 + 0.8 \div 10 \\ &= 0.4 + 0.08 \\ &= 0.48 \end{aligned}$$

Division is the reverse process of multiplication, so dividing by 10 corresponds to moving the decimal point one place to the left and inserting a zero if necessary.

### Example 14

Calculate:

**a** $98.72 \div 10$

**b** $0.982 \div 10$

**c** $0.46 \div 10$

**Solution**

**a** $98.72 \div 10 = 9.872$

**b** $0.982 \div 10 = 0.0982$

**c** $0.46 \div 10 = 0.046$


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## Dividing by powers of 10

Dividing by 100 is the same as dividing by 10 twice.

- Dividing by $100 = 10^2$ corresponds to moving the decimal point two places to the left and inserting zeros if necessary.

  $23.2 \div 100 = 0.232$

- Dividing by $1000 = 10^3$ corresponds to moving the decimal point three places to the left and inserting zeros if necessary.

  $568.2 \div 1000 = 0.5682$

- Dividing by $10\,000 = 10^4$ corresponds to moving the decimal point four places to the left and inserting zeros if necessary.

  $43.21 \div 10\,000 = 0.004\,321$

### Example 15

Calculate:

**a** $3576 \div 100$ **b** $3.576 \div 1000$

### Solution

**a** $3576 \div 100 = 35.76$ **b** $3.576 \div 1000 = 0.003\,576$

### Multiplication and division by powers of 10

- To multiply a number by 10, move the decimal point one place to the right and insert a zero if necessary.
- To multiply by $100 = 10^2$, move the decimal point two places to the right and insert zeros if necessary.
- To multiply by $1000 = 10^3$, move the decimal point three places to the right and insert zeros if necessary.
- To divide a number by 10, move the decimal point one place to the left and insert a zero if necessary.
- To divide by $100 = 10^2$, move the decimal point two places to the left and insert zeros if necessary.
- To divide by $1000 = 10^3$, move the decimal point three places to the left and insert zeros if necessary.

## Exercise 8D

Example 12

**1** Multiply each number by 10.

**a** 0.8 **b** 8.3 **c** 0.08 **d** 0.5

**e** 0.05 **f** 0.005 **g** 92.894 32 **h** 392.001



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Example 13

**2** Multiply each number by 100.

**a** 0.34 **b** 4.78 **c** 0.013 **d** 0.8

**e** 0.004 **f** 28.304 **g** 4.309 387 **h** 201.201 201

Example 14

**3** Divide each number by 10.

**a** 1.2 **b** 0.08 **c** 0.002 **d** 201.201 201

Example 15

**4** Divide each number by 100.

**a** 240.6 **b** 14.06 **c** 55.703 **d** 2.0006

Example 13, 15

**5** Calculate:

**a** $5.07 \times 100$ **b** $789.028 \times 1000$

**c** $230.0001 \div 100$ **d** $23.456 \times 10\,000$

**e** $1.0462 \div 100$ **f** $60 \div 100$

**g** $2.5 \times 10\,000$ **h** $3 \div 0.1$

**i** $12 \times 0.1$ **j** $34.35 \div 10$

**k** $4025.21 \div 10$ **l** $1000 \div 100$

**6** Calculate:

**a** $0.0082 \times 100$ **b** $0.0082 \times 10$

**c** $0.0082 \times 1000$ **d** $85.7 \times 1000$

**e** $85.7 \times 100$ **f** $85.7 \times 10$

**g** $5.02 \div 10$ **h** $5.02 \div 100$

**i** $5.02 \div 1000$ **j** $0.005\,43 \div 1000$

**k** $0.005\,43 \div 100$ **l** $0.005\,43 \div 10$

**7** A $325.80 restaurant bill for dinner for 10 people is shared equally. How much does each person pay?

**8** At a factory, it takes 408 seconds to make 100 chocolates.

**a** How long, in minutes, does it take to make 10 000 chocolates?

**b** How long, in seconds, does it take to make 10 chocolates?

**c** How many chocolates can be made in 4080 seconds?

**9** **a** How many millimetres in a metre?

**b** How many millimetres in a kilometre?

**c** Convert 36.4 km to millimetres.

**d** Convert 276 mm to kilometres.

**e** How many square millimetres in a square kilometre?

**f** Convert 13.06 square kilometres to square millimetres.

**g** Convert 4567.8 square millimetres to square kilometres.


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# 8E Multiplication of one decimal by another

Multiplication of decimals can be done by converting the decimals to fractions. The fractions are multiplied in the usual way (without cancelling) and the result is then converted back to a decimal.

### Multiplication of decimals

We multiply decimals by converting each decimal to a fraction, multiplying the fractions (without cancelling) and converting the result back to a decimal.

### Example 16

Calculate:

**a** $7 \times 0.6$ **b** $0.03 \times 0.18$

### Solution

**a** $7 \times 0.6 = \frac{7}{1} \times \frac{6}{10}$

$= \frac{42}{10}$

$= 4.2$

**b** $0.03 \times 0.18 = \frac{3}{100} \times \frac{18}{100}$

$= \frac{54}{10\,000}$

$= 0.0054$

## Another method for multiplying decimals

There is a method for multiplying whole numbers by decimals without first changing the decimal to a fraction:

- Temporarily ignore the decimal point and multiply the number.
- Note that the number of decimal places in the answer must be the same as the number of decimal places in the original decimal.

We can see that a similar rule for multiplying two decimals together must hold.

From above we saw that:

$7 \times 0.6 = 4.2$

$0.03 \times 0.18 = 0.0054$

Note that the total number of decimal places of the factors being multiplied is equal to the number of decimal places in the answer. Care must be taken with zeros.

We can describe the procedure as follows:

- Ignore the decimal points and carry out the multiplication as if the factors were whole numbers.
- Place the decimal point so that the number of decimal places in the answer is equal to the total number of decimal places in the factors.


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For example, for the product:

$1.5 \times 0.03$

there are a total of three decimal places in the factors.

We multiply:

$15 \times 3 = 45$

The answer must have three decimal places.

Hence:

$1.5 \times 0.03 = 0.045$

### Example 17

Calculate:

**a** $7 \times 0.2$ **b** $0.9 \times 0.6$ **c** $0.07 \times 0.002$ **d** $1.01 \times 0.08$

### Solution

**a** $7 \times 0.2 = 1.4$ ($7 \times 2 = 14$. There is one decimal place.)

**b** $0.9 \times 0.6 = 0.54$ ($9 \times 6 = 54$. There are two decimal places.)

**c** $0.07 \times 0.002 = 0.000\,14$ ($9 \times 6 = 54$. There are five decimal places.)

**d** $1.01 \times 0.08 = 0.0808$ ($101 \times 8 = 808$. There are four decimal places.)

## Exercise 8E

Example 16

**1** Evaluate by first changing each decimal to a fraction:

**a** $0.3 \times 0.1$ **b** $0.8 \times 0.2$ **c** $0.7 \times 0.3$ **d** $0.8 \times 0.03$

**e** $1.2 \times 0.1$ **f** $1.7 \times 0.2$ **g** $1.6 \times 0.03$ **h** $2.2 \times 0.05$

**i** $0.6 \times 0.111$ **j** $4.4 \times 0.01$ **k** $0.55 \times 0.02$ **l** $0.6 \times 1.08$

Example 17a

**2** Evaluate:

**a** $7 \times 0.1$ **b** $8 \times 0.01$ **c** $7 \times 0.2$ **d** $8 \times 0.003$

**e** $12 \times 0.1$ **f** $15 \times 0.2$ **g** $16 \times 0.03$ **h** $20 \times 0.08$

**i** $6 \times 0.111$ **j** $44 \times 0.0001$ **k** $55 \times 0.0002$ **l** $6 \times 1.08$

Example 17

**3** Evaluate:

**a** $1.2 \times 1.5$ **b** $0.2 \times 0.4$ **c** $0.02 \times 0.04$ **d** $0.7 \times 1.4$

**e** $0.8 \times 0.12$ **f** $2.5 \times 4$ **g** $0.6 \times 0.2$ **h** $0.3 \times 1.2$

**i** $0.01 \times 0.09$ **j** $0.6 \times 0.07$ **k** $3.92 \times 4.3$ **l** $7.93 \times 7.8$


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**4** Complete the table.

| × | 0.009 | 0.09 | 0.9 | 9 | 90 | 900 | 9000 |
|---|---|---|---|---|---|---|---|
| 0.008 | | | | | | | |
| 0.08 | | | | | | | |
| 0.8 | | | | | | | |
| 8 | | | | 72 | | | |
| 80 | | | | | | | |
| 800 | | | | | | | |
| 8000 | | | | | | | |

**5** Complete these multiplications.

**a** $0.2 \times 1.5$ **b** $3 \times 0.6$ **c** $2.1 \times 3$

**d** $1.03 \times 2$ **e** $0.14 \times 0.03$ **f** $0.08 \times 1.9$

**g** $1.23 \times 0.12$ **h** $1.01 \times 0.06$ **i** $2.15 \times 0.02$

**6** Multiply each number by 0.1.

**a** 12 **b** 10 **c** 43 **d** 0.5 **e** 2.6 **f** 1.08

**7** Multiply each number by 0.01.

**a** 15 **b** 10 **c** 63 **d** 0.5 **e** 4.3 **f** 2.06

**8** Multiply each number by 0.001.

**a** 23 **b** 100 **c** 10 **d** 0.05 **e** 0.5 **f** 4.07

**9** Calculate the total cost of buying:

**a** 4.5 m of rope at \$2.70 per metre

**b** 2.01 minutes of mobile phone talk time at \$0.35 per minute

**c** 4.32 m of PVC pipe at \$3.45 per metre

**d** 1.97 kg of breadcrumbs at \$0.95 per kilogram

**e** 2.2 L of ice cream at \$4.95 per litre

# 8F Division of decimals

## Dividing decimals by whole numbers

Suppose that we have a piece of rope that is 9.6 m long and we want to cut it into 5 equal pieces. We need to divide 9.6 by 5. This can be done using the division algorithm.

$$5\overline{)9.^{4}6^{1}0}\quad 1.92$$

(Notice that a zero has been placed to the right of the 6.)

Thus, each piece is 1.92 metres long.


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**Example 18**

Calculate:

**a** $12.312 \div 3$ **b** $2.835 \div 2$ **c** $13.14 \div 7$

**Solution**

**a**
$$3\overline{)12.312} = 4.104$$

**b**
$$2\overline{)2.835} = 1.4175$$
(Notice that a zero has been placed to the right.)

**c**
$$7\overline{)13.6150301030206040501030} = 1.877142857142\ldots$$

In part **c** of the example, we see that the division algorithm does not stop. We have continued the algorithm until the process starts to repeat. Divisions like this will be covered in the following section.

## Dividing decimals by decimals

Suppose that we have 1.8 m of ribbon and we want to cut it into lengths of 0.3 m. To find out how many such lengths we can make, we need to divide 1.8 by 0.3.

Below we show you two methods that can be used to divide 1.8 by 0.3.

**Method 1**
Write the division as a quotient (write one decimal over the other) and multiply top and bottom by a power of 10 so that the denominator is a whole number.

$$\begin{aligned} 1.8 \div 0.3 &= \frac{1.8}{0.3} \\ &= \frac{1.8 \times 10}{0.3 \times 10} \\ &= \frac{18}{3} \\ &= 6 \end{aligned}$$

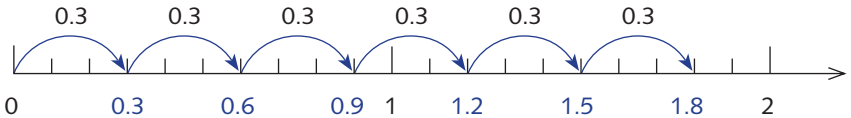
**Method 2**
Convert both decimals to fractions.

$$\begin{aligned} 1.8 \div 0.3 &= \frac{18}{10} \div \frac{3}{10} \\ &= \frac{18}{\cancel{10}^{1}} \times \frac{\cancel{10}^{1}}{3} \\ &= \frac{18}{3} \\ &= 6 \end{aligned}$$

We can cut 6 pieces of ribbon 0.3 m in length from 1.8 m of ribbon.

Sometimes we can perform such a calculation mentally. We can do this here. We know that $6 \times 3 = 18$, so $6 \times 0.3 = 1.8$ and $1.8 \div 0.3 = 6$.

The division $1.8 \div 0.3$ can also be visualised on a number line. Starting at 0, we take jumps of 0.3 until we get to 1.8. It takes 6 jumps of 0.3 to get from 0 to 1.8 because $1.8 \div 3 = 6$.





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## Example 19

Use method 1 and method 2 as shown on the previous page to divide 0.427 by 0.07.

### Solution

**Method 1**

$$0.427 \div 0.07 = \frac{0.427}{0.07}$$
$$= \frac{0.427 \times 100}{0.7 \times 100}$$
$$= \frac{42.7}{7}$$
$$= 6.1$$

**Method 2**

$$0.427 \div 0.07 = \frac{427}{1000} \div \frac{7}{100}$$
$$= \frac{\cancel{427}^{61}}{\cancel{1000}} \times \frac{\cancel{100}^{1}}{\cancel{7}^{1}}$$
$$= \frac{61}{10}$$
$$= 6.1$$

## Division of decimals

- To divide a decimal by a whole number, follow the same algorithm as for whole number division.
- To divide a decimal by another decimal:
  - Write the division as a quotient, and multiply top and bottom by a power of 10 so as to make the denominator a whole number. Then do the division.

or

- Convert each decimal to a fraction, perform the division and then convert back to a decimal.

The first method can also be set out as shown in Example 20.

## Example 20

Calculate:

**a** $0.42 \div 0.7$ **b** $421.5 \div 0.03$ **c** $41.05 \div 0.005$

### Solution

**a** $0.42 \div 0.7 = 4.2 \div 7$
$= 0.6$ (Multiply the dividend and divisor by 10.)

**b** $421.5 \div 0.03 = 42\,150 \div 3$ (Multiply the dividend and divisor by 100.)

$$3\overline{)4^{1}2\,1^{1}5\,0}\quad 1\;4\,0\;5\,0$$

$421.5 \div 0.03 = 14\,050$

**c** $41.05 \div 0.005 = 41\,050 \div 5$ (Multiply the dividend and divisor by 1000.)
$= 8210$


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## Exercise 8F

Example 18

**1** Calculate:

**a** $4.5 \div 9$ **b** $6.3 \div 7$ **c** $45.5 \div 5$ **d** $0.12 \div 6$

**e** $2.4 \div 12$ **f** $5.6 \div 7$ **g** $9.3 \div 3$ **h** $0.256 \div 8$

**i** $1.32 \div 4$ **j** $0.0512 \div 8$ **k** $0.288 \div 6$ **l** $24.7 \div 5$

**m** $67.7 \div 2$ **n** $0.029 \div 5$ **o** $0.00276 \div 8$ **p** $0.00677 \div 5$

Example 20

**2** Calculate:

**a** $0.12 \div 0.3$ **b** $0.36 \div 1.2$ **c** $0.36 \div 0.12$ **d** $0.004 \div 0.02$

**e** $6.03 \div 0.03$ **f** $0.06 \div 0.02$ **g** $0.025 \div 0.05$ **h** $0.016 \div 0.04$

Example 18, 19, 20

**3** Calculate (mentally if possible):

**a** $2.4 \div 6$ **b** $1.8 \div 9$ **c** $7.2 \div 8$ **d** $14.4 \div 12$

**e** $2.4 \div 0.2$ **f** $9.6 \div 0.4$ **g** $1.28 \div 0.4$ **h** $0.2 \div 0.8$

**i** $2.17 \div 0.7$ **j** $45.81 \div 0.3$ **k** $1.297 \div 4$ **l** $286.395 \div 0.08$

**4** Divide each number below by 0.1, then by 0.01 and finally by 0.001.

**a** 12 **b** 10 **c** 43 **d** 0.5 **e** 2.6 **f** 100

**5** You know that $24 \div 6 = 4$. From this you can calculate many related facts. Complete this division grid by dividing each number in the cells in the left most column by each number in the cells in the top row.

| ÷ | 0.006 | 0.06 | 0.6 | 6 | 60 | 600 | 6000 |
|---|---|---|---|---|---|---|---|
| 0.024 | 4 | | | | | | |
| 0.24 | | 4 | | | | | |
| 2.4 | | | 4 | | | | |
| 24 | | | | 4 | | | |
| 240 | | | | | 4 | | |
| 2400 | | | | | | 4 | |
| 24 000 | | | | | | | 4 |

**6** Sharon paid \$13.56 to fill her lawn mower with 12 L of fuel. What was the price per litre of the fuel she purchased?

**7** Alex buys 96.8 m of timber to make picture frames requiring 0.8 m of timber each. How many frames can he make?


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# 8G Recurring decimals

When some fractions are converted to decimals, they produce **terminating** decimals. That is, the decimal has a finite number of digits after the decimal point. For example:

$$\frac{1}{2} = 0.5,\ \frac{1}{4} = 0.25 \text{ and } \frac{1}{8} = 0.12$$

Fractions convert to decimals that terminate when they have denominators that factor into a product of powers of 2 and 5. For example, $40 = 2^3 \times 5$ and:

$$\frac{1}{40} = \frac{25}{1000} \qquad \text{(Multiply numerator and denominator by 25.)}$$
$$= 0.025$$

or

$$40\overline{)1.0^{2}0^{0}0} \quad 0.025$$

In most cases, a fraction cannot be converted to a terminating decimal, because the division algorithm produces an unending string of digits. This infinite string will always contain a repeating cycle, so the decimal is called a **recurring** (or **repeating**) **decimal**.

**Example 21**

Convert $\frac{1}{3}$ to a recurring decimal.

**Solution**

$$3\overline{)1.{}^{1}0{}^{1}0{}^{1}0{}^{1}0{}^{1}0\ldots} \quad 0.3\,3\,3\,3\,3\ldots$$

Hence, $\frac{1}{3} = 0.33333\ldots = 0.\dot{3}$

We write $\frac{1}{3} = 0.\dot{3}$, with the dot above the 3 indicating that '3' is the repeated digit.

From 0.3333… we look at:

$$0.3 = \frac{3}{10},\ 0.33 = \frac{33}{100},\ 0.333 = \frac{333}{1000}, \ldots$$

These numbers are increasing and getting closer to $\frac{1}{3}$. In fact, they get as close as you like to $\frac{1}{3}$.


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### Example 22

Convert $\frac{3}{11}$ to a recurring decimal.

**Solution**

$$11\overline{)3.^{3}0^{8}0^{3}0^{8}0\ldots}\quad 0.\ 2\ \ 7\ \ 2\ \ 7\ldots$$

The block 27 repeats indefinitely and we get

$$\frac{3}{11} = 0.272727\ldots$$
$$= 0.\dot{2}\dot{7}$$

$0.\dot{2}\dot{7}$ is a repeating decimal; the dots above the 2 and the 7 indicate that '27' is the **recurring cycle**. Other fractions can give longer repeating cycles. The decimals 0.27, 0.2727… are increasing and get as close as we like to $\frac{3}{11}$.

### Example 23

Convert $\frac{3}{7}$ to a decimal.

**Solution**

To convert $\frac{3}{7}$ to a decimal, divide 3 by 7.

$$7\overline{)3.^{3}0^{2}0^{6}0^{4}0^{5}0^{1}0^{3}0^{2}0^{6}0^{4}0^{5}0^{1}0^{3}0\ldots}\quad 0.\ 4\ \ 2\ \ 8\ \ 5\ \ 7\ \ 1\ \ 4\ \ 2\ \ 8\ \ 5\ \ 7\ \ 1\ \ 4\ldots$$

So $\frac{3}{7} = 0.\dot{4}28571\dot{4}$

Notice that the cycle repeats when the remainder occurs again.

### Terminating and repeating decimals

- A **terminating decimal** is one with a finite number of digits after the decimal point (apart from zeroes).
- If the denominator of a fraction is a product of powers of 2 and 5, then the equivalent decimal will terminate.
- Decimal numbers with recurring digits that cycle indefinitely are called **recurring** (or **repeating**) **decimals**. A recurring decimal can be written using a dot above the first and last repeating digits. For example:

$$\frac{1}{3} = 0.333333333\ldots \qquad \frac{1}{11} = 0.09090909\ldots$$
$$= 0.\dot{3} \qquad = 0.\dot{0}\dot{9}$$

The decimals 0.3, 033, 0.333,… are increasing and get as close as we like to $\frac{1}{3}$.

The decimals 0.09, 0.0909, 0.090909,… are increasing and get as close as we like to $\frac{1}{11}$.


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## Exercise 8G

**1** Express these fractions as decimals.

**a** $\frac{3}{8}$ **b** $\frac{7}{4}$ **c** $\frac{1}{20}$ **d** $\frac{7}{20}$ **e** $\frac{1}{40}$

Example 21

**2** Express these fractions as decimals.

**a** $\frac{1}{3}$ **b** $\frac{2}{3}$ **c** $\frac{1}{9}$ **d** $\frac{2}{9}$ **e** $\frac{3}{9}$

**f** $\frac{4}{9}$ **g** $\frac{5}{9}$ **h** $\frac{6}{9}$ **i** $\frac{7}{9}$ **j** $\frac{8}{9}$

Example 22

**3** Express these fractions as decimals.

**a** $\frac{1}{11}$ **b** $\frac{2}{11}$ **c** $\frac{3}{11}$ **d** $\frac{4}{11}$ **e** $\frac{5}{11}$

**f** $\frac{6}{11}$ **g** $\frac{7}{11}$ **h** $\frac{8}{11}$ **i** $\frac{9}{11}$ **j** $\frac{10}{11}$

**4** Express these fractions as decimals.

**a** $\frac{1}{6}$ **b** $\frac{2}{6}$ **c** $\frac{3}{6}$ **d** $\frac{4}{6}$ **e** $\frac{5}{6}$ **f** $\frac{1}{12}$

**g** $\frac{3}{12}$ **h** $\frac{5}{12}$ **i** $\frac{7}{12}$ **j** $\frac{9}{12}$ **k** $\frac{11}{12}$

**5** Express as decimals the fractions between 0 and 1 that have 7 as a denominator. Can you see a pattern?

Example 23

**6** Perform these divisions. (They will result in repeating decimals.)

**a** $4.8 \div 7$ **b** $2.9 \div 3$ **c** $50 \div 11$ **d** $7.9 \div 6$

**e** $6.47 \div 7$ **f** $2.58 \div 7$ **g** $62 \div 12$ **h** $59 \div 11$

**i** $52.6 \div 11$ **j** $42.2 \div 9$ **k** $472 \div 3$ **l** $2.36 \div 6$

# 8H Rounding of decimals

The last few decimal places of a decimal such as 3.141 5927 may have no practical value. For example, if 3.141 5927 represents the number of kilometres between two farmyard gates, then the final digit 7 represents $\frac{7}{10}$ of a millimetre, which is completely irrelevant.

This is why we use **rounding**. When we round a number, we write in out in full, then correct it to a certain number of decimal places.


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## Writing a decimal correct to a number of decimal places

The rules for rounding are as follows. Suppose that we want to round 10.125 89 correct to two decimal places.

- Identify the rounding digit in the second decimal place – in this case it is 2.
- Look at the next digit to the right of the rounding digit – in this case it is 5.
  - If this next digit is 0, 1, 2, 3 or 4, leave the rounding digit alone.
  - If this next digit is 5, 6, 7, 8 or 9, increase the rounding digit by 1.

  In this case, the next digit is 5, so the rounding digit increases from 2 to 3.
- Now discard all the digits after the rounding digit.

Thus, we write 10.125 89 as 10.13, correct to two decimal places.

### Example 24

Write 45.7456 correct to:

**a** one decimal place **b** two decimal places **c** three decimal places

### Solution

**a** $45.7456 \approx 45.7$ correct to one decimal place (The rounding digit is 7 and the digit to the right is smaller than 5.)

**b** $45.7456 \approx 45.75$ correct to two decimal places (The rounding digit is 4 and the digit to the right is 5.)

**c** $45.7456 \approx 45.746$ correct to three decimal places (The rounding digit is 5 and the digit to the right is 6.)

The rounding procedure can also be used with recurring decimals.

### Example 25

Write $0.\dot{4}\dot{6}$ correct to:

**a** two decimal places **b** three decimal places **c** four decimal places

### Solution

**a** $0.\dot{4}\dot{6} = 0.4646\ldots \approx 0.46$ correct to two decimal places

**b** $0.\dot{4}\dot{6} = 0.4646\ldots \approx 0.465$ correct to three decimal places

**c** $0.\dot{4}\dot{6} = 0.464\,64\ldots \approx 0.4646$ correct to four decimal places


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## Exercise 8H

Example 24

**1** Round these decimals correct to the given number of decimal places.

**a** 3.1529 (2 places) **b** 9.497 (2 places) **c** 7.999 (2 places)

**d** 2.1537 (2 places) **e** 6.454 (2 places) **f** 7.991 (2 places)

**g** 12.3547 (3 places) **h** 6.454 12 (3 places) **i** 7.9919 (3 places)

**j** 12.354 78 (4 places) **k** 6.454 1212 (4 places) **l** 7.991 99 (4 places)

Example 25

**2** Round these repeating decimals correct to the given number of decimal places.

**a** $0.\dot{5}\dot{7}$ (2 places) **b** $0.\dot{6}\dot{3}$ (2 places) **c** $0.\dot{5}\dot{7}$ (3 places)

**d** $0.\dot{6}\dot{3}$ (3 places) **e** $0.\dot{5}\dot{7}$ (4 places) **f** $0.\dot{6}\dot{3}$ (4 places)

**g** $0.\dot{3}6\dot{8}$ (2 places) **h** $0.\dot{7}7\dot{4}$ (4 places) **i** $0.456\dot{6}\,7\dot{4}$ (2 places)

**j** $0.47\dot{7}\,7\dot{4}$ (4 places) **k** $0.2\dot{3}7\dot{8}$ (5 places) **l** $0.1\dot{7}5\dot{4}$ (4 places)

# Review exercise

**1** Write each fraction as a decimal.

**a** $\frac{1}{10}$ **b** $\frac{9}{10}$ **c** $\frac{1}{100}$ **d** $\frac{9}{100}$ **e** $\frac{39}{100}$ **f** $\frac{1}{1000}$ **g** $\frac{4}{1000}$

**h** $\frac{28}{1000}$ **i** $\frac{298}{1000}$ **j** $\frac{1}{10\,000}$ **k** $\frac{9}{10\,000}$ **l** $\frac{19}{10\,000}$ **m** $\frac{241}{10\,000}$ **n** $\frac{6939}{10\,000}$

**2** Write down the place value of the digit 8 in each number.

**a** 3.8 **b** 1.08 **c** 8.45 **d** 4.998 **e** 6.0008 **f** 80.903 8275

**3** Write down the place value of the digit 4 in each number.

**a** 1.4 **b** 2.04 **c** 2.189 42 **d** 0.009 74

**4** Choose the larger of each pair of decimal numbers.

**a** 2.9 and 2.3 **b** 1.91 and 1.87 **c** 3.09 and 3.11

**d** 2.6832 and 2.8 **e** 4.2396 and 4.999 **f** 5.501 and 5.4999

**g** 5.000 03 and 5.12 **h** 1.098 798 and 1.231 **i** 4.5 and 4.51

**5** Arrange each set of numbers from largest to smallest.

**a** 1.5, 1.28, 1.09, 1.4732 **b** 1.2, 1.36, 1.928, 1.2849

**c** 9.09, 9.909, 9.9, 9.999 **d** 23.5, 23.451, 23.0001, 23.09


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**6** Write down the whole number that is closest to:

**a** 1.3 **b** 2.9 **c** 1.8 **d** 4.03

**e** 27.009 29 **f** 43.92 **g** 68.010 101 **h** 299.9

**i** 0.0019 **j** 187.409 **k** 213.5002 **l** 627.099 999

**7** Express each decimal as a fraction and simplify where possible.

**a** 0.3 **b** 3.9 **c** 1.4 **d** 0.32 **e** 0.98 **f** 0.04

**g** 0.111 **h** 0.204 **i** 0.099 **j** 0.75 **k** 0.125 **l** 2.848

**8** Express each fraction or mixed numeral as a terminating decimal.

**a** $\frac{2}{10}$ **b** $\frac{4}{8}$ **c** $1\frac{2}{100}$ **d** $\frac{4}{50}$ **e** $10\frac{9}{30}$ **f** $\frac{2}{25}$

**g** $4\frac{3}{75}$ **h** $\frac{2}{500}$ **i** $\frac{47}{10}$ **j** $\frac{101}{10}$ **k** $\frac{48}{20}$ **l** $1\frac{1}{16}$

**m** $\frac{3}{125}$ **n** $\frac{196}{200}$ **o** $\frac{33}{40}$ **p** $\frac{318}{400}$ **q** $\frac{104}{400}$ **r** $\frac{18}{125}$

**9** Match each fraction to its decimal equivalent.

| **Decimal** | $1.\dot{3}$ | 1.25 | 0.4 | $4.\dot{2}$ | 0.50 | 0.09 | 0.625 |
|---|---|---|---|---|---|---|---|
| **Fraction** | $4\frac{2}{9}$ | $\frac{1}{2}$ | $1\frac{1}{3}$ | $\frac{9}{100}$ | $\frac{2}{5}$ | $\frac{5}{8}$ | $1\frac{1}{4}$ |

**10** Which of these fractions is closest to 1.45?

$1\frac{1}{2}$, $1\frac{33}{50}$, $1\frac{2}{5}$, $\frac{147}{100}$

**11** Express each fraction as a decimal.

**a** $\frac{3}{7}$ **b** $\frac{2}{11}$ **c** $\frac{5}{12}$ **d** $\frac{6}{13}$ **e** $\frac{5}{6}$ **f** $\frac{7}{12}$

**12** Arrange each set of numbers from smallest to largest.

**a** $\frac{1}{5}$, 0.5, 1, $\frac{3}{4}$, $\frac{5}{8}$, 0.1

**b** 2.4, $\frac{1}{4}$, 4.1, 0.24, 0.4, $\frac{1}{24}$

**c** 0.08, $\frac{5}{8}$, 0, $\frac{1}{5}$, 0.9, $\frac{1}{25}$

**d** 2, $\frac{290}{100}$, 2.07, $\frac{23}{11}$, 2.7, $2\frac{1}{5}$

**13** Calculate:

**a** 0.4 + 3.5 **b** 3.7 + 5.2 **c** 8.15 – 1.2

**d** 5.34 + 3.63 **e** 4.75 – 4.012 **f** 17.8 – 3.4

**g** 7.3 + 15.8 **h** 12.6 – 7.3 **i** 18.6 – 12.7

**j** 25.3 – 23.8 **k** 36.204 + 27.985 **l** 247.4967 – 138.968

**14** Write down the result when each number is multiplied by 10, 100 and 1000.

**a** 4.5 **b** 0.3 **c** 0.06 **d** 1.396 **e** 100.485 **f** 0.0002


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**15** Write down the result when each number is divided by 10, 100 and 1000.

**a** 4.5 **b** 0.3 **c** 0.06 **d** 1.396 **e** 43.8 **f** 100.485

**16** Complete the grid.

| × | 0.007 | 0.07 | 0.7 | 7 | 70 | 700 | 7000 |
|---|---|---|---|---|---|---|---|
| 0.003 | | | | | | | |
| 0.03 | | | | | | | |
| 0.3 | | | | | | | |
| 3 | | | | 21 | | | |
| 30 | | | | | | | |
| 300 | | | | | | | |
| 3000 | | | | | | | |

**17** Calculate:

**a** $6.03 \times 7$ **b** $4.8 \times 8$ **c** $4.92 \times 3$ **d** $5.9 \times 3$

**e** $23.8 \times 12$ **f** $2.486 \times 8$ **g** $1.2 \times 3.4$ **h** $5.947 \times 2.6$

**18** Complete the grid.

| × | 0.001 | 0.01 | 0.1 | 1 | 10 | 100 | 1000 |
|---|---|---|---|---|---|---|---|
| 0.052 | | | | | | | |
| 0.52 | | | | | | | |
| 5.2 | | | | | | | |
| 52 | | | | | | | |
| 520 | | | | | | | |
| 5200 | | | | | | | |
| 52 000 | | | | | | | |

**19** Calculate:

**a** $2.4 \div 0.8$ **b** $1 \div 0.2$ **c** $3.6 \div 0.06$ **d** $0.06 \div 1.2$

**e** $5.3 \div 7$ **f** $12.256 \div 1.2$ **g** $10.4 \div 1.25$ **h** $28.2 \div 0.008$

**20** Judy bought 0.5 m of material at \$6.30 per metre. What was the cost?

**21** Find the mass of 0.8 L of water if one litre weighs 1.0005 kg.

**22** Find the cost of 5.2 kg of meat if 1 kg costs \$4.60.

**23** Mrs Tang paid \$15.45 for three kilograms of beans. How much did one kilogram cost?

**24** Frank bought 2.75 m of wire at 68 cents per metre. What was the cost?

**25** Nisha paid \$63.14 for 2.2 m of fabric.

**a** What was the cost per metre? **b** What was the cost of 10 m?


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# Challenge exercise

**1** Write down the next five numbers, continuing the obvious pattern in each case.

**a** 2.6, 2.7, 2.8, __, __, __, __, __

**b** 3.1, 3.4, 3.7, __, __, __, __, __

**c** 12.5, 12.4, 12.3, __, __, __, __, __

**d** 12.4, 11.3, 10.2, __, __, __, __, __

**2** Using each of the digits 5, 6, 7 and 8 once only, fill in the spaces to make these products as large as possible.

**a** 0.__ $\times$ 0.__ __ __

**b** 0.__ __ $\times$ 0.__ __

**3** Jessica was preparing for her birthday party. She spent \$3 less than $\frac{3}{5}$ of her money on soft drink and \$3 more than $\frac{3}{4}$ of her remaining money on food. She still had \$3 left. How much did she start with?

**4** To write the number $0.\dot{5}$ we let $x = 0.555...$

Therefore, $10x = 5.555...$

$$\begin{array}{r} -10x = 0.555... \\ \hline 9x = 5 \end{array}$$

$$x = \frac{5}{9}$$

Therefore $0.\dot{5} = \frac{5}{9}$.

**a** Use the same method to write $0.\dot{4}$ as a fraction.

**b** Complete following to write $0.\dot{1}\dot{2}$ as a fraction.

Let $x = 0.121\,212...$

$$\begin{array}{r} 100x = 12.121\,212... \\ -\ x = 0.121\,212... \\ \hline 99x = ? \end{array}$$

**c** Use a similar method to write $0.\dot{1}2\dot{3}$ as a fraction.

**5** Recall that terminating decimals are produced from fractions with a denominator that is a product of powers of 2 and 5. List all the fractions with numerator 1 and denominator less than 100 that produce terminating decimals.

**6** Explain why the cycle length of a recurring decimal corresponding to a fraction is always less than the denominator of the fraction.


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CHAPTER

# 9

Measurement and Geometry

# Measurement

When we are building a table, hanging a picture on the wall, taking some cough mixture, timing a race and so on, we need to be able to make measurements. Measuring allows us to determine how big, how long, how deep or how heavy things are. Sometimes all we need is a rough idea of a measurement. For example, we might walk along the edge of a garden to measure its length in paces or use a handspan to decide if a table will fit in a certain place in our home. Usually, though, we need a more accurate idea of a measurement. We need to be able to select the right tool and the correct unit to make a measurement, read the scale accurately, and make calculations involving measurements.

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# 9A Units of measurement

In order to make a meaningful measurement of some quantity, we need an appropriate **unit of measurement**. For example, when measuring a person's mass, we normally use the kilogram as the unit. On the other hand, if we needed to weigh a mosquito, we would generally use a smaller unit, such as the gram or milligram.

In Australia, we use the **metric system** of measurement. In this system, the basic units of length, mass and time are the **metre** (m), the **kilogram** (kg) and the **second** (s), respectively.

Apart from some aspects of time measurement, the metric system is a decimal system. (We will discuss time measurement in Section 9I.) Other metric units of length and mass are derived from the basic ones by multiplying or dividing by powers of 10. We indicate the particular power of 10 that we are using by putting a **prefix** in front of the basic unit. The prefixes most commonly used are: **kilo-**, denoting 1000; **centi-**, denoting $\frac{1}{100}$; and **milli-**, denoting $\frac{1}{1000}$. So, for example, as well as measuring lengths in metres, we also commonly use kilometres, centimetres and millimetres.

When making a measurement, it is important to choose a suitable unit so that we can clearly picture what we are measuring. For example, it is not very helpful to be told that a person weighs 0.076 tonnes, but we can easily picture what is meant when we are told that he or she weighs 76 kg.

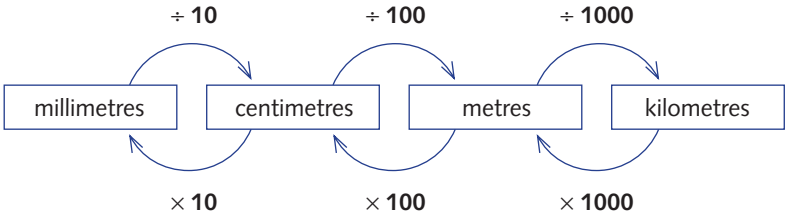
Here is a table of the commonly used units for measuring length, mass, time and liquid volume in the metric system.

| | | |
|---|---|---|
| **Units of length** | | |
| 10 millimetres (mm) | = | 1 centimetre (cm) |
| 100 centimetres (cm) | = | 1 metre (m) |
| 1000 millimetres (mm) | = | 1 metre (m) |
| 1000 metres (m) | = | 1 kilometre (km) |
| **Units of mass** | | |
| 1000 milligrams (mg) | = | 1 gram (g) |
| 1000 grams (g) | = | 1 kilogram (kg) |
| 1000 kilograms (kg) | = | 1 tonne (t) |
| **Units of time** | | |
| 60 seconds (s) | = | 1 minute (min) |
| 60 minutes (min) | = | 1 hour (h) |
| 24 hours (h) | = | 1 day (d) |
| **Units of liquid volume** | | |
| 1000 millimetres (mL) | = | 1 litre (L) |
| 1000 litres (L) | = | 1 kilolitre (kL) |
| 1 000 000 litres (L) | = | 1 megalitre (ML) |

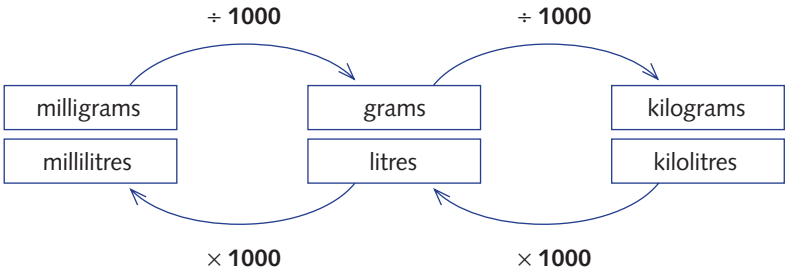
To convert between units, it is helpful to think about how many of one unit are equal to one of another.


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When converting from one length of measurement to a different unit, you need to know the following conversions.




When dealing with mass and volume measurements, the following conversions are useful.




## Units of measurement

- We use the **metric system** of measurement.
- The basic units of **length**, **mass** and **time** are the **metre** (m), the **kilogram** (kg) and the **second** (s), respectively.
- Other useful units of length and mass are derived from these by multiplying or dividing by powers of 10. We indicate which particular power of 10 we are using by adding a **prefix** to the basic unit.

### Example 1

**a** Express 1993 mm in:

**i** centimetres **ii** metres

**b** Express 1.2 km in:

**i** metres **ii** centimetres

### Solution

**a i** $10\,\text{mm} = 1\,\text{cm}$

Measurements in millimetres are converted to centimetres by dividing by 10.

Hence, $1993\text{ mm} = \dfrac{1993}{10}\,\text{cm}$

$= 199.3\,\text{cm}$

**ii** $1000\,\text{mm} = 1\,\text{m}$

Measurements in millimetres are converted to metres by dividing by 1000.

Hence, $1993\text{ mm} = \dfrac{1993}{1000}\,\text{m}$

$= 1.993\text{ m}$

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**b i** 1 km = 1000 m

Measurements in kilometres are converted to metres by multiplying by 1000.

Hence, 1.2 km = 1.2 × 1000 m
= 1200 m

**ii** 1 km = 1000 m, and 1 m = 100 cm

Hence, 1.2 km = 1.2 × 1000 m
= 1200 m

and 1200 m = 1200 × 100 cm
= 120 000 cm

so 1.2 km = 120 000 cm

## Example 2

Copy and complete these statements.

**a** 7 cm 6 mm = _____mm

**b** 4 m 65 cm = _____cm

**c** 10 km 380 m = _____m

### Solution

**a** 7 cm 6 mm = 76 mm

**b** 4 m 65 cm = 465 cm

**c** 10 km 380 m = 10 380 m

## Example 3

**a** Express 2689 g in kilograms.

**b** Express 36.5 kg in grams.

### Solution

**a** Measurements in grams are converted to kilograms by dividing by 1000.

Hence, $2689 \text{ g} = \frac{2689}{1000} \text{ kg}$
$= 2.689 \text{ kg}$

**b** Measurements in kilograms are converted to grams by multiplying by 1000.

36.5 × 1000 = 36 500

Hence, 36.5 kg = 36 500 g


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**Example 4**

**a** Express 4 km 83 m in metres.

**b** Express 2 t 15 kg in kilograms.

**Solution**

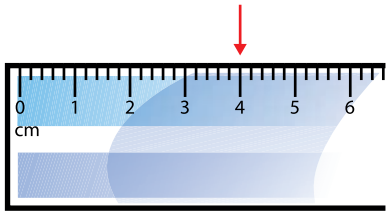
**a** 4 km = 4000 m
Hence, 4 km 83 m = 4000 + 83
= 4083 m

**b** 2 t = 2000 kg
Hence, 2 t 15 kg = 2000 + 15
= 2015 kg

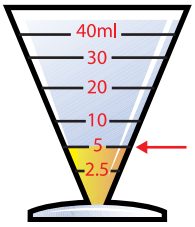
## Exercise 9A

**1** The diagrams below show the readings for measurements taken using a range of different measuring tools. In each diagram, the arrow indicates what the measurement was. Read the scale and write down the reading. Include the unit of measurement in each case.

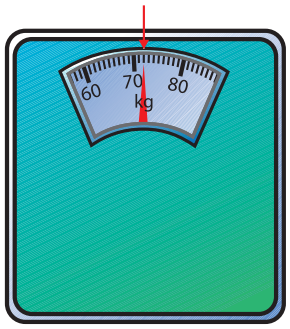
**a**



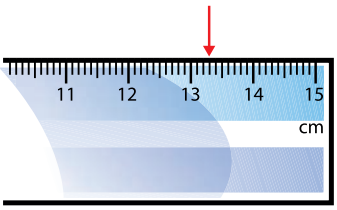

**b**



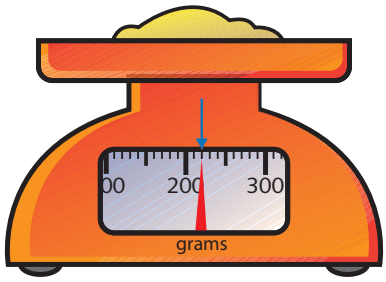

**c**



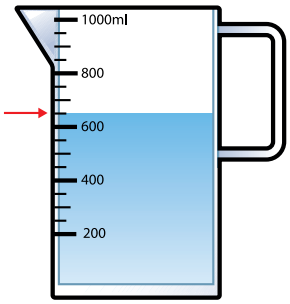

**d**




**e**




**f**




Example 1

**2** Copy and complete these statements.

**a** 6 cm = ____ mm

**b** 180 mm = ____ cm

**c** 18 cm = ____ mm

**d** 4.3 cm = ____ mm

**e** 27 mm = ____ cm

**f** 583 mm = ____ cm

**g** 3 m = ____ cm

**h** 1.9 m = ____ cm

**i** 2 km = ____ m

**j** 1 km = ____ cm



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**k** 6.34 km = ____ m

**l** 200 000 mm = ____ km

**m** 9000 cm = ____ km

**n** 2901 m = ___ km

**o** $\frac{1}{2}$ km = ____ m

**p** $\frac{1}{4}$ m = ____ cm

**q** $\frac{1}{5}$ m = ____ mm

**r** $\frac{7}{10}$ cm = ____ mm

**s** $\frac{5}{8}$ cm = ____ mm

**t** $\frac{7}{8}$ cm = ____ mm

Example 2

**3** Copy and complete these statements.

**a** 8 cm 7 mm = ____ cm

**b** 5 cm 9 mm = ____ mm

**c** 6 cm 7 mm = ____ cm

**d** 3 m 65 cm = ____ cm

**e** 45 km 800 m = ____ m

**f** 14 km 38 m = ____ km

Example 3

**4** Copy and complete these statements.

**a** 493 g = ____ kg

**b** 2.3 kg = ____ g

**c** 3.4 t = ____ kg

**d** $\frac{1}{4}$ kg = ____ g

**e** 45.3 kg = ____ g

**f** 480 mg = ____ g

**g** 290 mg = ____ g

**h** $\frac{1}{10}$ t = ____ kg

Example 4

**5** **a** Express 5 km 23 m in metres.

**b** Express 15 km 20 m in metres.

**c** Express 3 t 20 kg in kilograms.

**d** Express 24 t 15 kg in kilograms.

**6** Copy and complete these statements.

**a** 800 millilitres = ____ litres

**b** 1.5 litres = ___ millilitres

**c** 4002 millilitres = ____ litres

**d** 3968 litres = ____ kilolitres

**7** Each lap of a swimming pool is 50 m. How many laps must I swim to cover 1 km.

**8** A small bag of potatoes weighs 500 g and a large bag weighs 1.5 kg. What is the total weight if I buy 2 small and 3 large bags of potatoes? (Give your answer first in grams, and then in kilograms.)

**9** Markers on the road are placed 20 m apart. If the distance from the first marker to the last one is 1 km, how many markers are there in total?

**10** A picture, which is 60 cm wide, is to be placed in the centre of a wall. The length of the wall from left to right is 5 m. What is the distance between the left-hand edge of the picture and the left-hand edge of the wall?

**11** Calculate:

**a** 3.05 kg × 5

**b** 4.405 m × 3

**c** 11.6 cm × 16

**d** 124 kg × 16

**e** 3.106 t × 34

**f** 68 m ÷ 4

**g** 33.12 t ÷ 24

**h** 72.352 kg ÷ 8

**i** 5.6256 km ÷ 4


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**12** Find the cost of:

**a** 6 m of string at 99 c a metre

**b** 500 mm of ribbon at 84 c a metre

**c** 4 L of oil at $1.50 per litre

**d** 100 g of fudge at $2.80 a kilogram

**e** 8 kg of rice at $0.85 a kilogram

**f** 250 m of rope at $120 a kilometre

**g** carting 5 t of rubbish at $24 a tonne

**h** 200 mL of shampoo at $6.30 a litre

**i** 600 kg of sand at $20 a tonne

# 9B Other units

In addition to the prefixes mentioned in the previous section, other measurement prefixes are also in common use. For example, a **micrometre** (written as μm) is one millionth of a metre. (The Greek letter μ, pronounced *mu*, corresponds to our letter *m*.)

Here is a table of some prefixes and the corresponding factors.

| Multiplying factor | Prefix | Symbol |
|---|---|---|
| one million $= 10^6$ | mega- | M |
| one thousand $= 10^3$ | kilo- | k |
| one tenth $= \frac{1}{10}$ | deci- | d |
| one hundredth $= \frac{1}{10^2}$ | centi- | c |
| one thousandth $= \frac{1}{10^3}$ | milli- | m |
| one millionth $= \frac{1}{10^6}$ | micro- | μ |

## Example 5

How many millilitres are there in an Olympic-sized pool that holds 2.5 megalitres (ML) of water?

### Solution

First convert to litres.

Volume = 2.5 ML
$= 2.5 \times 1\,000\,000$ L (1 ML = 1 000 000 L)
$= 2\,500\,000$ L

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Now convert to millilitres.

Volume = 2 500 000 × 1000 mL (1 L = 1000 mL)
= 2 500 000 000 mL

There are 2 500 000 000 mL in a 2.5 ML Olympic-sized pool.

## Exercise 9B

Example 5

**1** How many litres are there in a megalitre?

**2** How many micrometres are there in a centimetre?

**3** Dartmouth Dam has a capacity of 4 million megalitres. If the dam is currently holding half of its capacity, how many litres is this?

**4** Peter walks 5.1 km each day. How many micrometres is this?

**5** Convert 9.85 megatonnes into milligrams.

**6** If you are 158 cm tall, how many decimetres is that?

**7** If 1 litre is equivalent to 1 cubic decimetre (1 $dm^3$), what is 890 mL in cubic decimetres?

**8** How many minutes are there in a micro-century?
(Assume there are 36 525 days in a century.)

# 9C The unitary method

If 5 apples cost $2.50, how much do 3 apples cost?

The easiest way to solve this is to work out the cost of 1 apple and multiply by 3.

5 apples cost $2.50
1 apple costs $0.50 ÷ 5
3 apples cost $1.50 × 3

This method of solving problems is known as the **unitary method**, because we work out the cost of one item (or unit) first.


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### Example 6

If 12 cans of soft drink cost $9.36, how much do 7 cans cost?

### Solution

12 cans of drink cost $9.36
1 can of drink costs $0.78 ÷ 12
7 cans of drink cost $5.46 × 7

## Exercise 9C

Example 6

**1** If 5 blocks of chocolate weigh 1875 g, how much do 3 blocks of chocolate weigh?

**2** If 3 kg of tea cost $27.33, what is the cost of 4 kg?

**3** If 7 tins can hold 1421 g of coffee, how much, in kilograms, can 3 tins hold?

**4** A car travels 77 km on 11 litres of fuel. How far can it travel on 130 litres?

**5** If a train travels 210 km in 3 hours, how far will it go in 8 hours?

**6** Which is cheaper, 12 oranges for $4.08 or 30 oranges for $12.96?

**7** Tim can ride 100 m in 15 seconds on his bike. Assuming he continues at the same speed, how many kilometres can he travel in 30 minutes?

**8** A man walks 200 m in 2 minutes. How far will he walk in an hour?

**9** A particular type of rope costs $2.50 for 25 cm. How much does 2 m of rope cost?

**10** A car travels 200 metres in 10 seconds. How far will it go in:

**a** a minute? **b** an hour?

**11** It is found that 3 apricots weight 300 g. How many kilograms would 20 apricots weigh?

**12** If 9 men can build a wall in 63 days, how long would it take 40 men to build the same wall if they worked at the same pace?

**13** **a** If 7 seedlings cost $5.60, find the cost of:

**i** 10 seedlings **ii** 1000 seedlings **iii** 12 seedlings

**iv** 1200 seedlings **v** 25 seedlings **vi** 125 seedlings

**b** Find the cost of 7 seedlings if:

**i** 10 seedlings cost $4.00 **ii** 12 seedlings cost $36.00

**iii** 50 seedlings cost $60.00 **iv** 200 seedlings cost $24.00

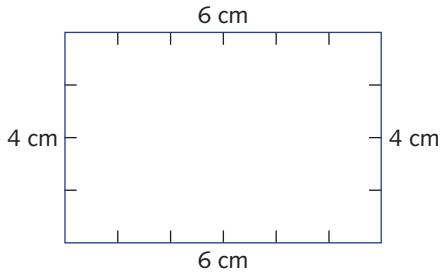
**v** 9 seedlings cost $3.15 **vi** 11 seedlings cost $112.20


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# 9D Perimeter

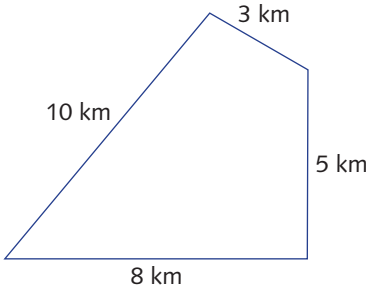
The word **perimeter** comes from two Greek words: *peri*, meaning 'around', and *metron*, meaning 'measure'. Thus, the word *perimeter* means 'measure around'. We use the word *perimeter* to mean the length of the boundary of a two-dimensional figure.

Imagine walking around the edge of the rectangle shown below and counting the number of intervals, each of length 1 cm. This measurement is the perimeter of the rectangle. The perimeter of the rectangle is the sum of the lengths of its sides.



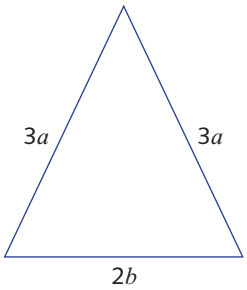

$$\begin{aligned}\text{Perimeter} &= 6 + 4 + 6 + 4\\ &= 20 \text{ cm}\end{aligned}$$

The perimeter of the figure shown below is the sum of the lengths of its sides.




$$\begin{aligned}\text{Perimeter} &= 3 + 5 + 8 + 10\\ &= 26 \text{ km}\end{aligned}$$

The perimeter of the triangle shown below is the sum of the lengths of its sides.




$$\begin{aligned}\text{Perimeter} &= 3a + 3a + 2b\\ &= 6a + 2b\end{aligned}$$

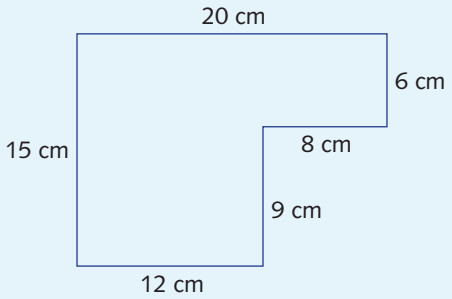
## Perimeter

The **perimeter** of a figure bounded by straight sides is the length of its boundary, and is calculated by finding the sum of the lengths of its sides.


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## Example 7

Find the perimeter of this figure.




### Solution

To find the perimeter, add the lengths of the sides.

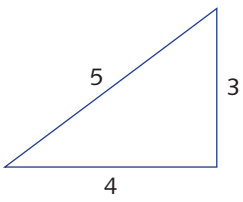
Perimeter = 15 + 20 + 6 + 8 + 9 + 12
= 70 cm

# Exercise 9D

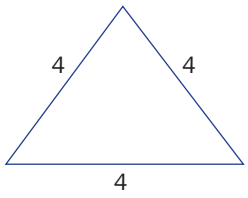
Example 7

**1** Find the perimeter of each of these figures. Sides that are equal are marked with the same symbol.

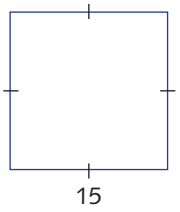
**a**



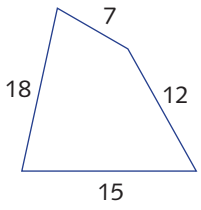

**b**



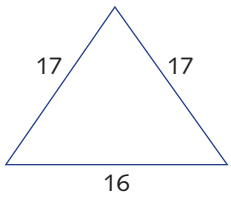

**c**



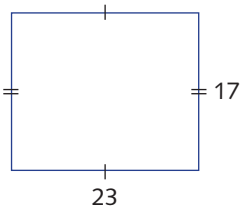

**d**




**e**

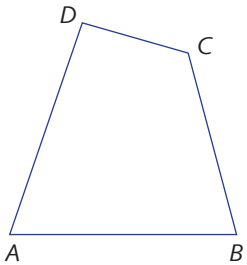



**f**

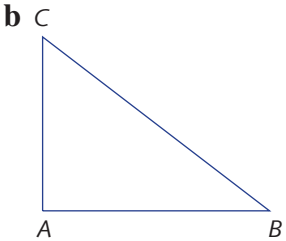



**2** In each figure below, use your ruler to measure the length of each side in millimetres (as accurately as you can) and then calculate the perimeter.

**a**




**b**

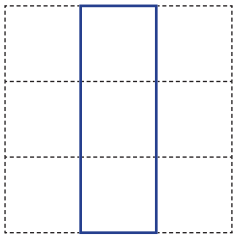




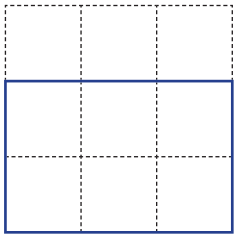
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**3** The figures shown below are drawn on a 1 cm grid. Find the perimeter of each figure, in centimetres.

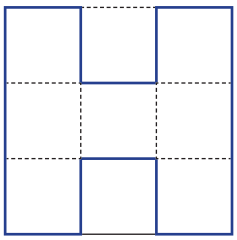
**a**



**b**



**c**

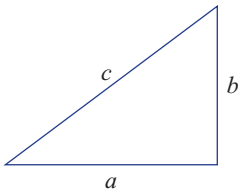


**4** Calculate the perimeter of a rectangular room with dimensions of:

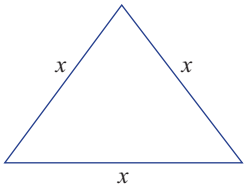
**a** length 2.3 m and width 3.6 m **b** length 9.8 m and width 6.4 m

**5** Write an algebraic expression for the perimeter of each figure.

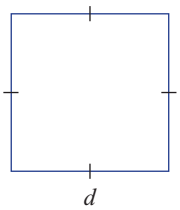
**a**



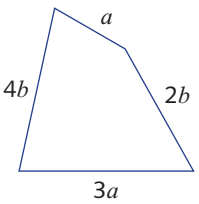

**b**



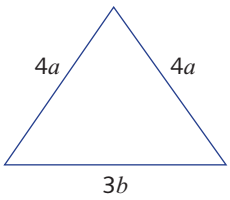

**c**



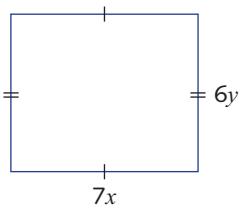

**d**




**e**




**f**



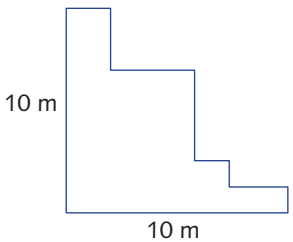

**6** For each part below, draw two different rectangles with the given perimeter.

**a** 20 cm **b** 14 cm **c** 9 cm

**7** For each part below, draw three different rectangles with the given perimeter.

**a** 18 cm **b** 8 cm **c** 14 cm

**8** Find the perimeter of the figure below. All angles are right angles.

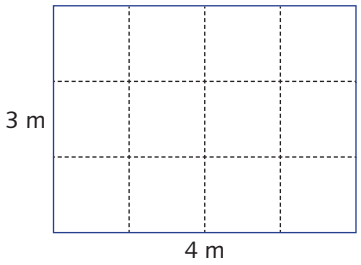




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# 9E Area

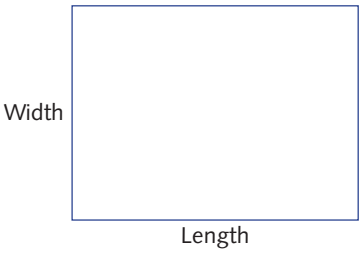
The **area** of a rectangle is the size of the region inside it. We measure the area of a rectangle by counting the number of unit squares inside it.




The area of the above rectangle is 12 square metres, which we write as 12 $m^2$. We can also find the area of this rectangle by multiplying the lengths of the sides.

$$\begin{aligned}\text{Area} &= 3 \times 4 \\ &= 12 \text{ m}^2\end{aligned}$$

We calculate the **area of a rectangle** by taking *the product of the lengths of two adjacent sides*. These two sides are called the **length** and **width** of the rectangle. It does not really matter which is which, because if we turn the rectangle through 90° and so interchange the length and width, we still have the same area.




If we use the letter $L$ to represent the length, and $W$ to represent the width, then we write the area as:

$$\begin{aligned}\text{Area} &= L \times W \\ &= LW\end{aligned}$$

This is called the **formula for the area of a rectangle** because it applies to all rectangles.

When using this formula to calculate the area of a rectangle, we have to make sure that we use the same units for length and width. For example, if we measure the length and width of a rectangle in centimetres (cm), the unit of area is the square centimetre, which is written as $cm^2$ because we are multiplying cm by cm.

**Example 8**

Find the area of a rectangle with length 12 cm and width 14 cm.

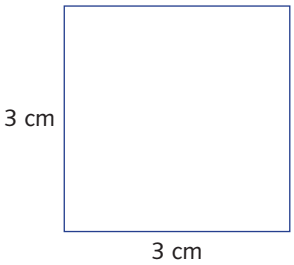
**Solution**

$$\begin{aligned}\text{Area} &= L \times W \\ &= 12 \times 14 \\ &= 168 \text{ cm}^2\end{aligned}$$


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Note that we can use the formula for the area of a rectangle to calculate the area of a square, because a square is just a rectangle with equal side lengths.

The length of this square is 3 cm and its width is 3 cm. The area of the square is given by the formula as:




$$\begin{aligned}\text{Area} &= L \times W \\ &= 3 \times 3 \\ &= 9 \text{ cm}^2\end{aligned}$$

For a square, the length and width are equal, so we can write the formula for the area of a square as:

$$\begin{aligned}\text{Area} &= L \times L \\ &= L^2\end{aligned}$$

## Example 9

Find the area of a square with side length 3.2 cm.

### Solution

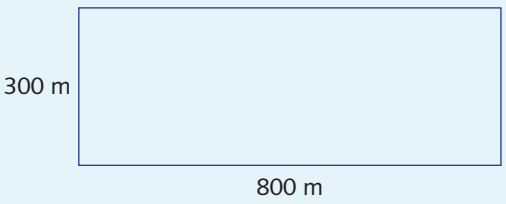
$$\begin{aligned}\text{Area} &= L^2 \\ &= 3.2 \times 3.2 \\ &= 10.24 \text{ cm}^2\end{aligned}$$

Commonly used units of area are:

- the square centimetre ($\text{cm}^2$), which is the area of a 1 cm by 1 cm square
- the square millimetre ($\text{mm}^2$), which is the area of a 1 mm by 1 mm square
- the square metre ($\text{m}^2$), which is the area of a 1 m by 1 m square
- the square kilometre ($\text{km}^2$), which is the area of a 1 km by 1 km square, and
- the hectare (ha), which is the area of a 100 metre by 100 metre square. This is equivalent to 10 000 $\text{m}^2$. There are 100 ha in a square kilometre.

## Example 10

A farmer has a rectangular paddock as shown. Find its area in hectares.





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**Solution**

$$\begin{aligned}\text{Area} &= L \times W \\ &= 800 \times 300 \\ &= 240\,000 \text{ m}^2 \\ &= \frac{240\,000}{10\,000} \\ &= 24 \text{ ha}\end{aligned}$$

## Area

- The **area** of a rectangle is the size of the region inside it.
- The area of a rectangle is found by multiplying the length by the width:

  $\text{Area} = L \times W$
- The area of a square is the square of its side length:

  $\text{Area} = L^2$
- When using either of these formulas, all measurements must be in the same units.
- One hectare = 10 000 square metres and 100 hectares = 1 km$^2$.

**Example 11**

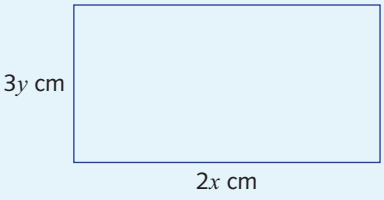
Find the area of a rectangle with length 3 m and width 86 cm.

**Solution**

$$\begin{aligned}\text{Area} &= L \times W \\ &= 3 \times 0.86 \quad \text{(Convert centimetres to metres.)} \\ &= 2.58 \text{ m}^2\end{aligned}$$

**Example 12**

A rectangle has length $2x$ cm and width $3y$ cm. Find its area.



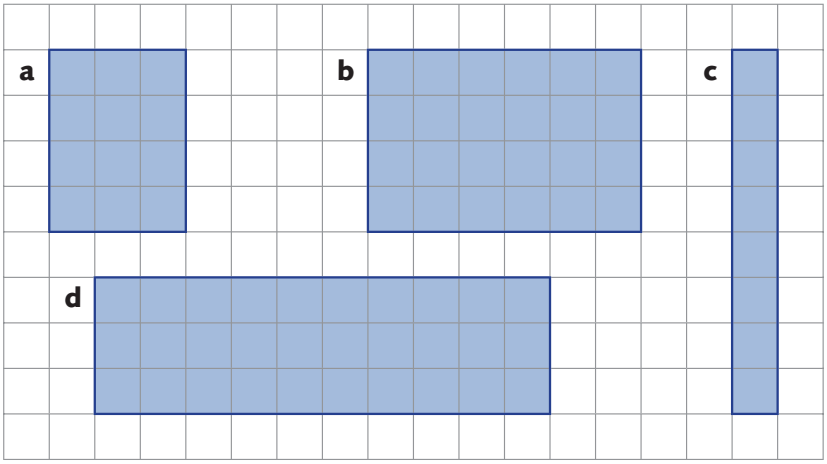

**Solution**

$$\begin{aligned}\text{Area} &= L \times W \\ &= 2x \times 3y \\ &= 6xy \text{ cm}^2\end{aligned}$$


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## Exercise 9E

**1** Find the area of each of the rectangles on the grid below by counting the number of squares. Imaging that the figures below have been drawn on 1 $cm^2$ grid paper.




**2** Find the areas of these figures.

Example 8, 9

**a** A square with side length 3 m

Example 10

**b** A rectangle with length 9 cm and width 7 cm

**c** A square with side length 13 mm

**d** A rectangle with length 1.1 cm and width 3.2 cm

**3** Calculate:

**a** the number of square centimetres in a square metre

**b** the number of hectares in a square kilometre

**c** the number of square metres in a square kilometre

Example 11

**4** **a** Find the area in square metres of a rectangle with length 5 m and width 85 cm.

**b** Find the area in square kilometres of a rectangular region of land with length 800 m and width 2 km.

Example 10

**5** Calculate the area of a rectangular field with length 4 km and width 0.5 km. Give your answer in hectares.

**6** Calculate the area of a rectangular field with length 150 m and width 0.6 km. Give your answer in square metres.

**7** The table below gives incomplete information for four rectangles. Copy the table and fill in the gaps.

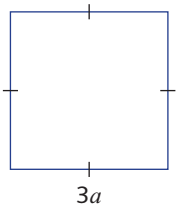
| | Length | Width | Area |
|---|---|---|---|
| **a** | 7 cm | 6 cm | |
| **b** | 9 cm | | 27 $cm^2$ |
| **c** | | 17 m | 85 $m^2$ |
| **d** | 12 km | | 144 $km^2$ |


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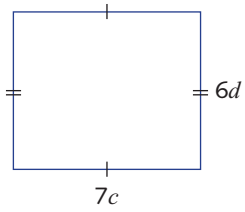
Example 12

**8** Write down algebraic expressions for the areas of the square and rectangle shown.

**a**




**b**




**9** A rectangular deck has length 6 m and width 4 m. It costs \$90 per square metre to tile the deck. How much will the tiling cost?

**10** A wall that is to be painted on one side has length 5 m and height 3.5 m. It requires two coats of paint. What is the total area to be painted?

**11** Glass costs \$25 per square metre. Find the cost of glazing a window with length 225 cm and width 150 cm.

**12** It costs \$225 to put grass on a rectangular piece of ground measuring 3 m by 5 m. How much will it cost to put the same kind of grass on a piece of ground that is 7 m by 4 m?

**13** Calculate the area of a road that is 10 m in width and 3.8 km in length. Give your answer in hectares.

**14** Calculate the area in square metres of a square table top with side length 130 cm.

**15** The floors of three rooms of a palace are to be covered in tiles. Each tile is one centimetre square and costs \$4.50. The rooms measure 8 m by 5.4 m, 2.7 m by 8.9 m and 300 cm by 13.3 m, respectively. Calculate the total cost.

**16** For each part below, draw two different rectangles with the given area.

**a** 9 $cm^2$ **b** 25 $cm^2$ **c** 36 $cm^2$

**17** For each part below, draw three different rectangles with the given area.

**a** 12 $cm^2$ **b** 15 $cm^2$ **c** 24 $cm^2$

# 9F Areas by addition and subtraction

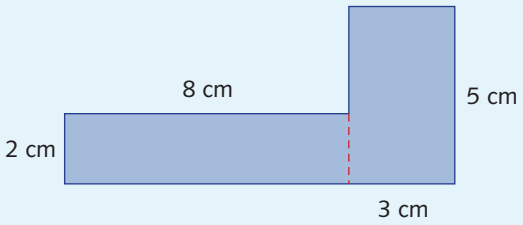
Sometimes a figure can be broken up into a number of separate rectangles. We can calculate its area by adding up the areas of these rectangles. We can also find areas by subtracting a smaller area from a larger one.


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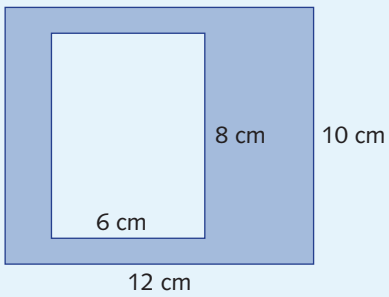
## Example 13

Calculate the area of the shaded regions below.

**a**




**b**



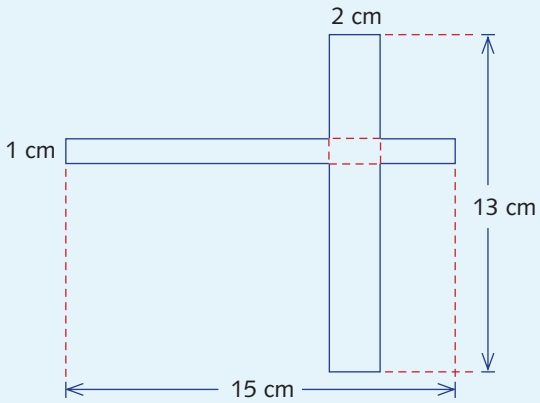

### Solution

**a** Area $= 8 \times 2 + 3 \times 5$
$= 16 + 15$
$= 31 \text{ cm}^2$

**b** Area $= 12 \times 10 - 8 \times 6$
$= 120 - 48$
$= 72 \text{ cm}^2$

## Example 14

Find the area of the figure shown.




### Solution

In this figure, we have two rectangles intersecting in a small rectangle. Hence, we add the areas of the two rectangles and subtract the area of the intersection so that it is not counted twice.

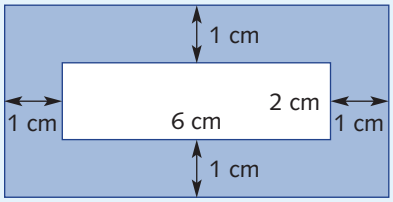
Area $= 15 \times 1 + 2 \times 13 - 2 \times 1$
$= 15 + 26 - 2$
$= 39 \text{ cm}^2$



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## Example 15

Find the area of the shaded region in the figure shown.




### Solution

Length of outer rectangle $= 6 + 1 + 1$
$= 8$ cm

Width of outer rectangle $= 2 + 1 + 1$
$= 4$ cm

Area of outer rectangle $= 8 \times 4$
$= 32$ cm$^2$

Area of inner rectangle $= 6 \times 2$
$= 12$ cm$^2$

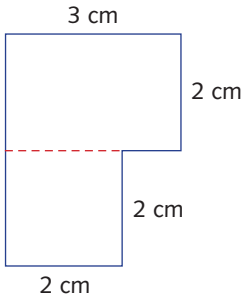
Area of shaded region $= 32 - 12$
$= 20$ cm$^2$

## Exercise 9F

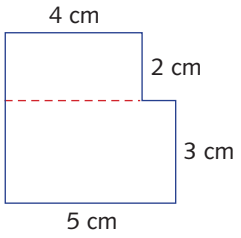
Example 13a

**1** Find the area of each figure by adding areas of squares and rectangles.

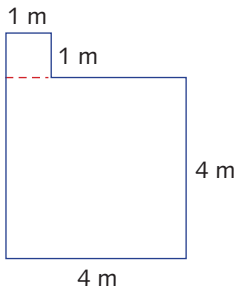
**a**




**b**



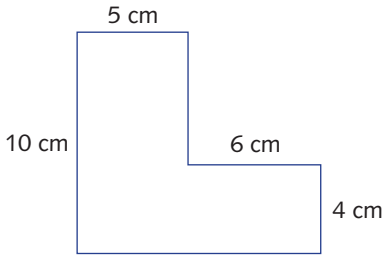

**c**



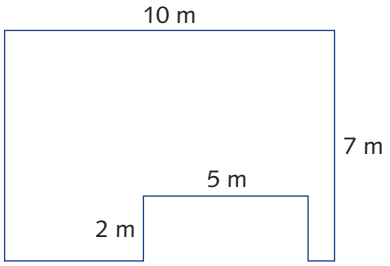

Example 13b

**2** Calculate the area of each figure. (In part **c**, find the shaded area.)

**a**



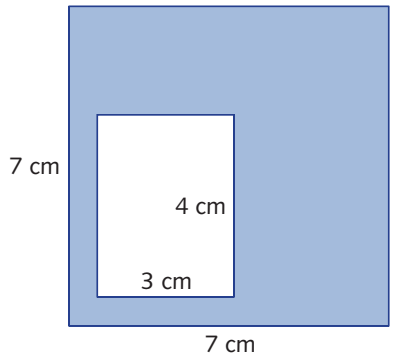

**b**



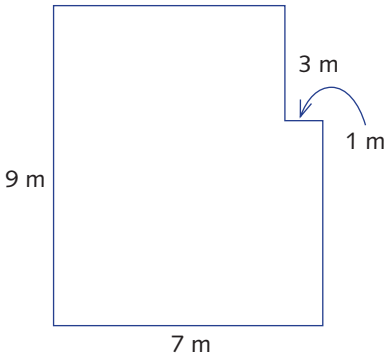



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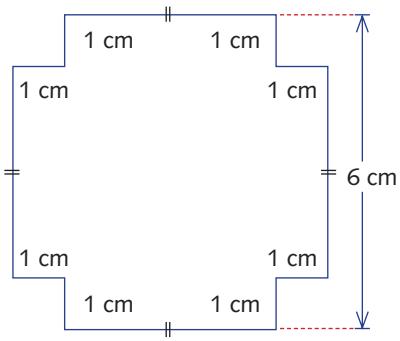
**c**



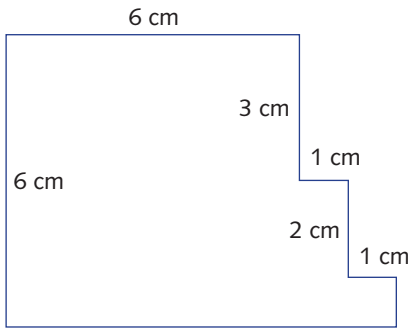

**d**




**e**

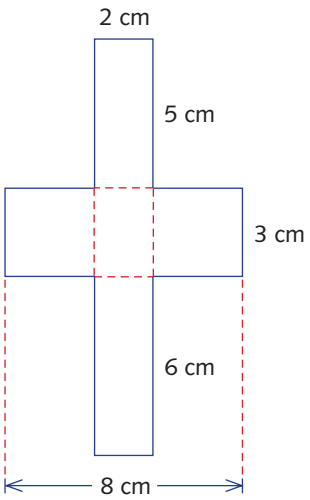



**f**

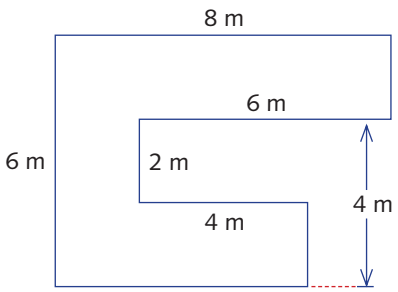



Example 14

**g**

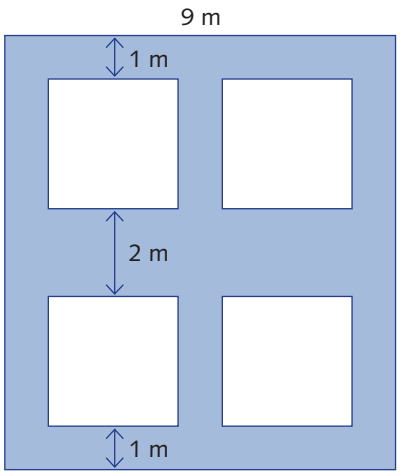



**h**




Example 15

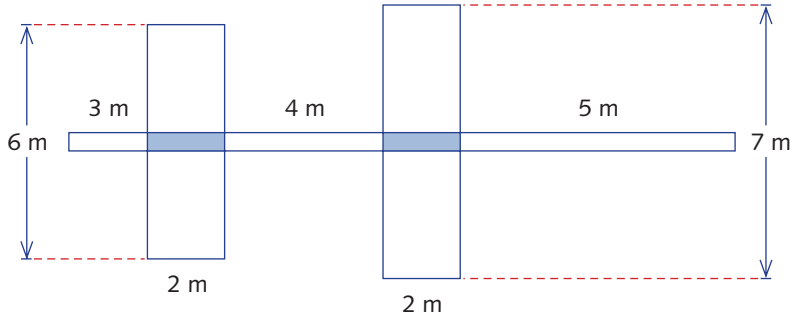
**3** Calculate the area of the shaded region. Each small square has side length 3 m.



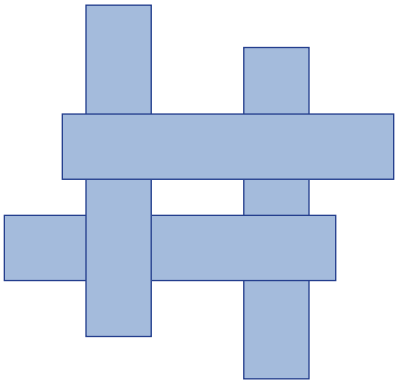


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**4** The area of each shaded region is 1 $m^2$. What is the total area of the figure?



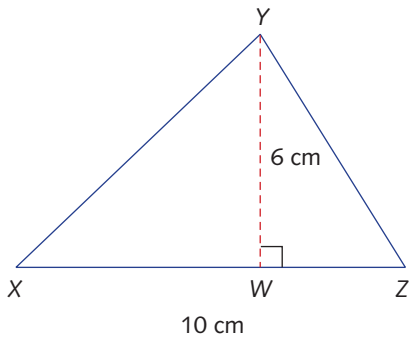

**5** Four rectangles, each with length 5 m and width 1 m, overlap to form the figure shown. Calculate its area.

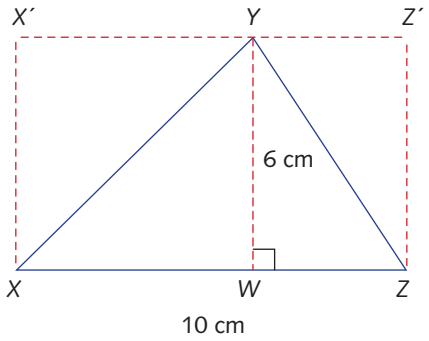


# 9G Areas of triangles and parallelograms

## Triangles

Take triangle $XYZ$, shown below, with base $XZ = 10$ cm and height $WY = 6$ cm. We can complete a rectangle around the triangle as shown.




The rectangle $XX´Z´Z$ has length 10 cm and width 6 cm, so its area is 60 $cm^2$.

The diagram shows that the area of each smaller triangle is equal to half the area of the corresponding enclosing rectangle. Hence, the area of the large triangle is equal to half the area of the large rectangle.


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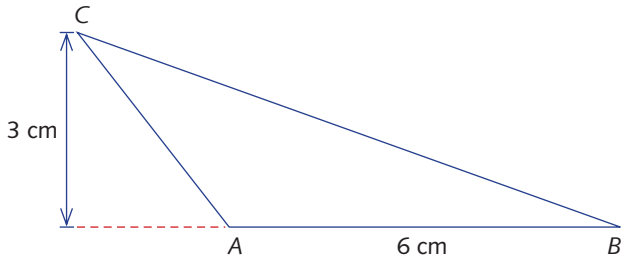
The area of triangle $XYZ$ is:

$$\begin{aligned}\text{Area of triangle } XYZ &= \frac{1}{2} \times 10 \times 6 \\ &= 30 \text{ cm}^2\end{aligned}$$

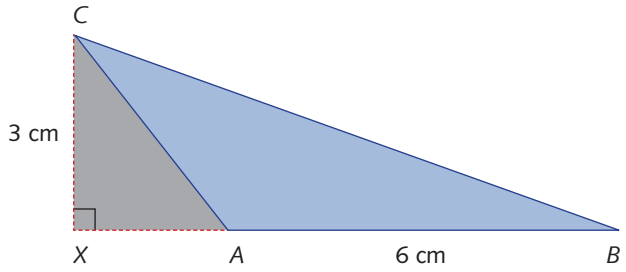
Is it always the case that the area of a triangle is equal to half its base multiplied by its height? We have shown above that this is true for triangles with angles that are no more than 90º, so finally we consider the case of an obtuse-angled triangle.

## Obtuse-angled triangles

Let $ABC$ be an obtuse-angled triangle with base length 6 cm and height 3 cm.




Make a right-angled triangle as shown below. Let the length of $XA$ be $\ell$ cm.




The area of the blue shaded triangle is found by subtracting the grey shaded area from the area of the right-angled triangle $CXB$:

$$\begin{aligned}\text{Area} &= \frac{1}{2} \times 3 \times (\ell + 6) - \frac{1}{2} \times 3 \times \ell \\ &= \frac{1}{2} \times 3 \times (\ell + 6 - \ell) \\ &= \frac{1}{2} \times 3 \times 6 \\ &= \frac{1}{2} \times \text{height} \times \text{base} \\ &= 9 \text{ cm}^2\end{aligned}$$

So we see that, in all cases, the **formula for the area of a triangle** with and perpendicular height $h$ base $b$ is:

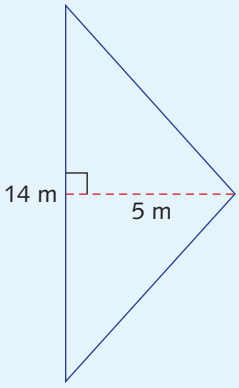
$$\begin{aligned}\text{Area} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times b \times h \\ &= \frac{1}{2} bh\end{aligned}$$


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Although we use the words 'height' and 'base', we sometimes have to turn the figure around to work with a different base and its corresponding height. The height is always taken to be the length of the perpendicular line from the vertex opposite to the chosen base.

### Example 16

Calculate the area of the triangle shown.




### Solution

$$\begin{aligned}\text{Area} &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 5 \times 14 \\ &= 7 \times 5 \\ &= 35 \text{ m}^2\end{aligned}$$

### Area of a triangle

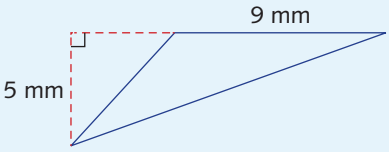
- The area of a triangle is found by multiplying half the length of the base by the height of the triangle:

$$\text{Area} = \frac{1}{2}bh$$

- It does not matter which side is chosen as the base, as long as the height is taken as the length of the perpendicular line from the vertex opposite to the chosen base.

### Example 17

Calculate the area of the triangle shown.





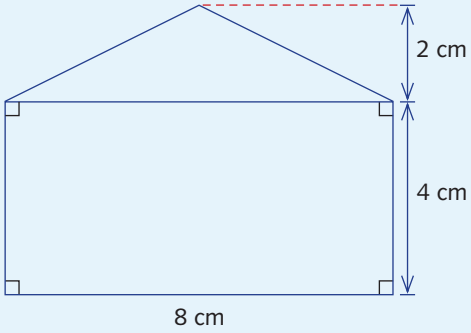
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**Solution**

$$\text{Area} = \frac{1}{2} \times 9 \times 5$$
$$= \frac{1}{2} \times 45$$
$$= 22\tfrac{1}{2} \text{ mm}^2$$

**Example 18**

Find the area of the figure shown.



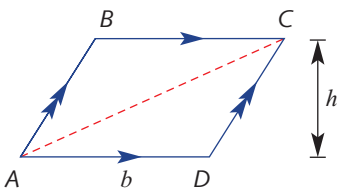

**Solution**

$$\text{Area of rectangle} = 8 \times 4$$
$$= 32 \text{ cm}^2$$
$$\text{Area of triangle} = \frac{1}{2} \times 8 \times 2$$
$$= 8 \text{ cm}^2$$
$$\text{Area of figure} = 32 + 8$$
$$= 40 \text{ cm}^2$$

## Parallelograms

### Area of a parallelogram

A **parallelogram** has each pair of its opposite sides parallel, as shown. To find the area of this figure, first draw in the diagonal $AC$ to form triangles $ABC$ and $ADC$. The area of $\Delta ABC = \frac{1}{2} \times b \times h$. Similarly, the area of $\Delta ADC = \frac{1}{2} \times b \times h$.

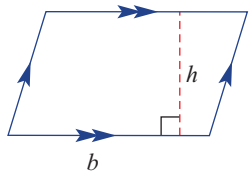



$$\text{Area of the parallelogram } ABCD = \text{ Area of } \Delta ADC + \text{ Area of } \Delta ABC$$
$$= \frac{1}{2} bh + \frac{1}{2} bh$$
$$= bh$$


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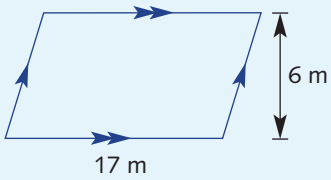
## Area of a parallelogram

Area of a parallelogram = $bh$




### Example 19

Find the area of the parallelogram shown.




### Solution

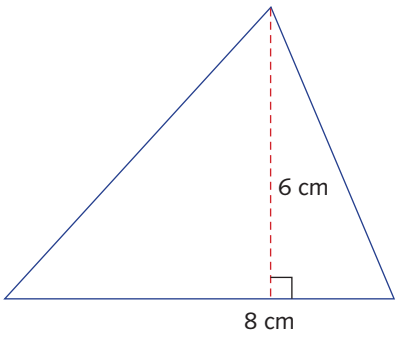
Area = base × height
= 17 × 6
= 102 m$^2$

## Exercise 9G

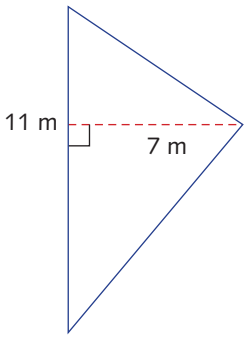
Example 16

**1** Find the area of each triangle.

**a**

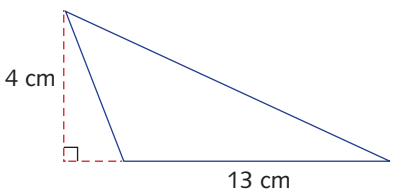



**b**

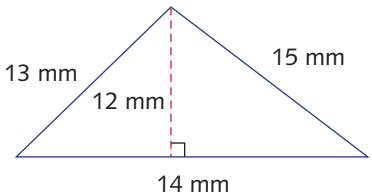



Example 17

**c**



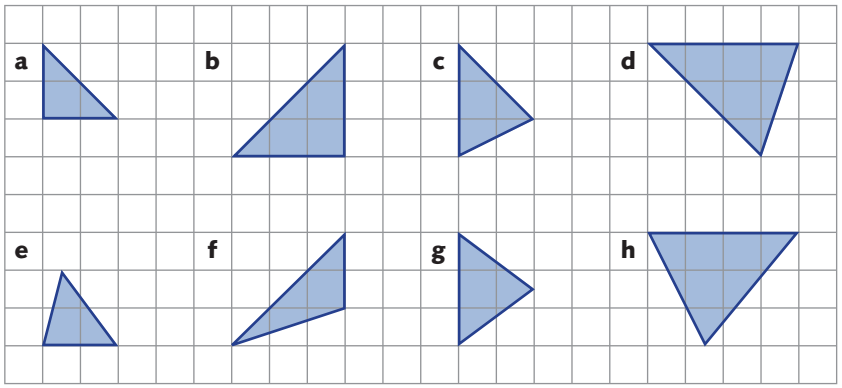

**d**





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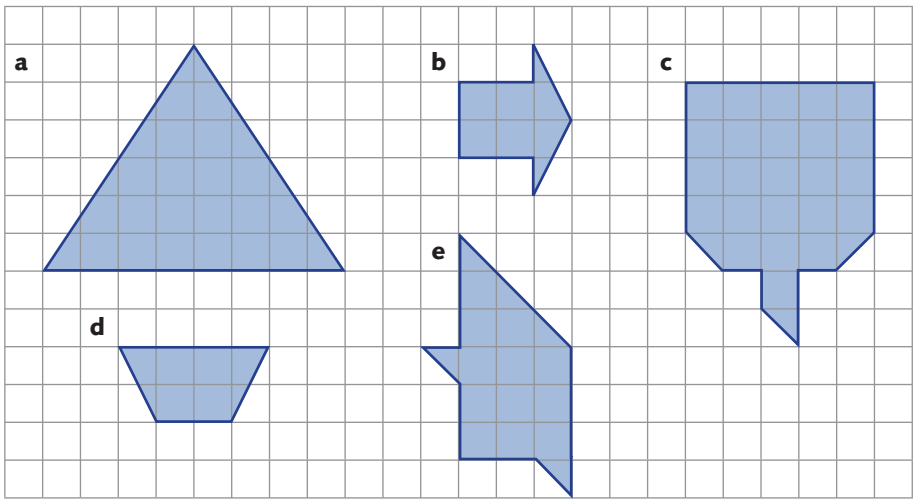
**2** Find the area of each of these triangles. Imagine that they are drawn on 1 $cm^2$ grid paper.




**3** Copy and complete the areas of triangles **a** to **d**.

| | Base | Height | Area |
|---|---|---|---|
| **a** | 7 cm | 6 cm | |
| **b** | 9 cm | | 27 $cm^2$ |
| **c** | | 17 m | 85 $m^2$ |
| **d** | 12 km | | 144 $km^2$ |

**4** Imagine that the figures below have been drawn on 1 $cm^2$ grid paper. Find the area of each.



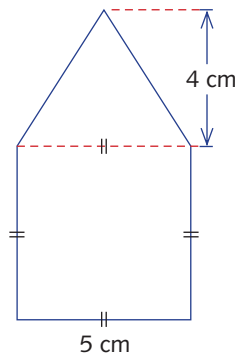

**5** Draw three different triangles with the same base and the same area.

**6** How many triangles such as those in question **5** are there? Explain how you could draw 15 such triangles.

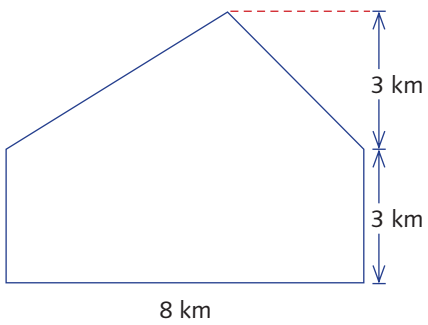
Example 18

**7** Find the area of each figure below by adding the areas of squares, rectangles and triangles as necessary.

**a**



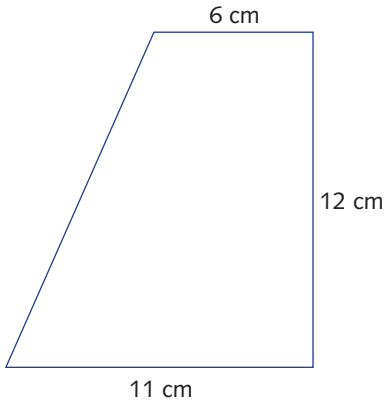

**b**



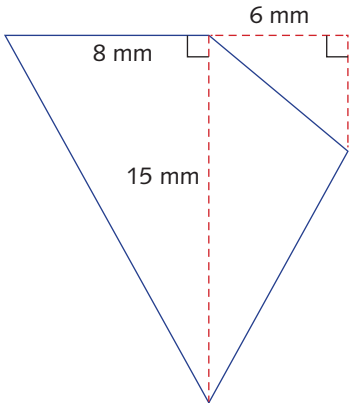


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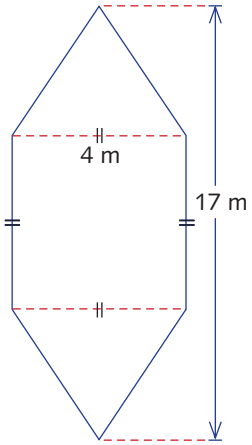
**c**



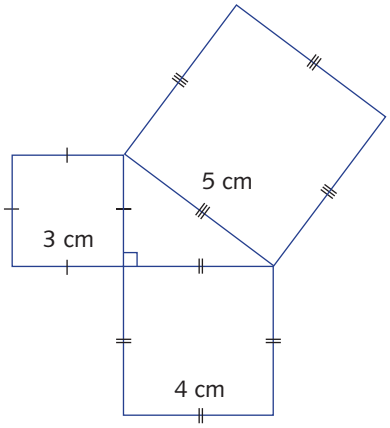

**d**




**e**

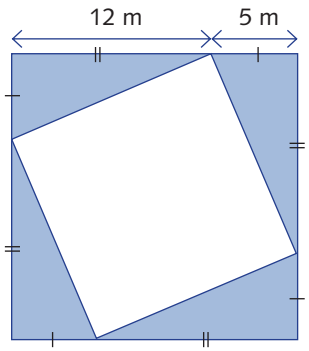



**f**

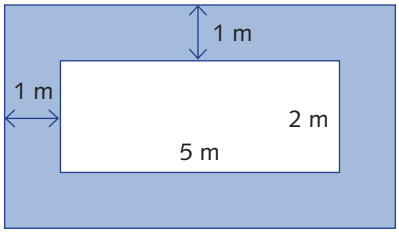



**8** Find the area of the shaded region in each figure below by adding and subtracting the areas of squares, rectangles and triangles as necessary.

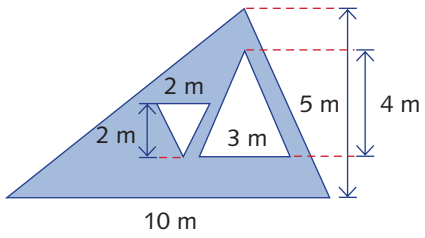
**a**



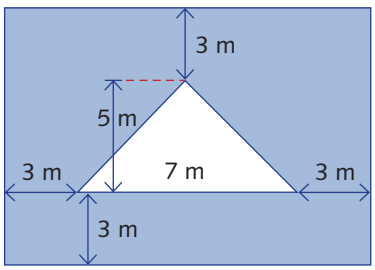

**b**




**c**



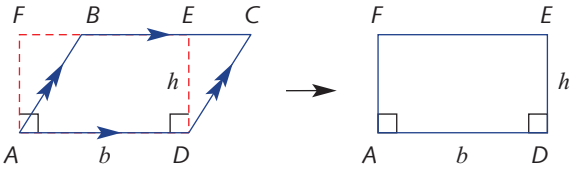

**d**





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**9** Prove that the area of a parallelogram is $bh$ by a method suggested by this diagram.



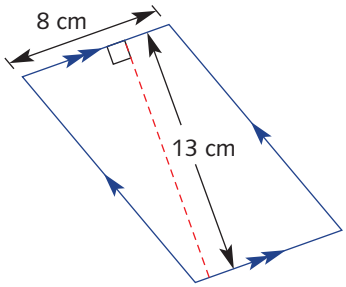

Example 19

**10** The table below gives the bases, heights and areas of various parallelograms. Copy the table and fill in the missing entries.

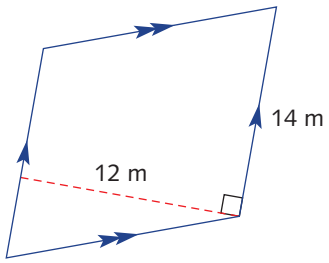
| | Base | Height | Area |
|---|---|---|---|
| **a** | 7 cm | 6 cm | |
| **b** | 9 cm | | 27 cm$^2$ |
| **c** | | 17 m | 85 m$^2$ |
| **d** | 12 km | | 144 km$^2$ |

**11** Find the areas of these regions.

**a**




**b**

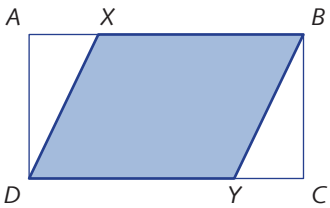



**12** A parallelogram has area 56 cm$^2$ and base 14 cm. What is the height of the parallelogram?

**13** A parallelogram has the same area as a rectangle of width 4 cm and length 6 cm. If the height of the parallelogram is 10 cm, what is the length of the base?

**14** A parallelogram has a height that is half of its base. The area of the parallelogram is 18 cm$^2$. What is the height and base of the parallelogram?

**15** Rectangle $ABCD$ has length 8 cm and height 4 cm.



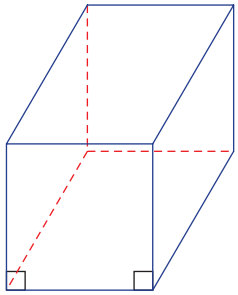

The two triangles $AXD$ and $BYC$ have height 4 cm and base 2 cm. Find the area of the parallelogram $DXBY$.



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# 9H Volume of rectangular prisms

The diagram on the right shows a **right rectangular prism**. Note that:



- the base is a rectangle
- if we slice it through any horizontal plane, the cross-section is a rectangle exactly the same as the base
- there are three right angles at each corner.

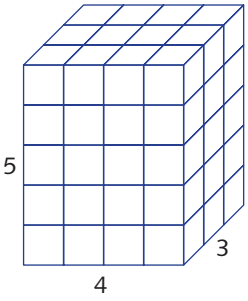
(The word 'right' is used here as a compact and convenient way of saying 'the walls are vertical'.)

An example of such a solid is a shoe box.

If the faces are all exactly the same, we call the solid a **cube**.

The **volume** of a rectangular prism is a measure of the space inside the prism.

Consider a rectangular prism of side lengths 5, 4 and 3.




We call these numbers the **dimensions** of the prism.

(They could also be called the **length**, **width** and **height** of the prism, but since we can rotate the prism, these words are used rather loosely, so it is often better to talk about the dimensions.)

We can cut the prism up into cubes, each of side length 1, as shown. We say that the volume of each of these cubes is 1, which we read as 'one cubic unit'.

Altogether there are $5 \times 3 \times 4 = 60$ cubes, each of volume 1, so we say that the volume of the prism is 60.

In practice, the dimensions could be 3 cm $\times$ 4 cm $\times$ 5 cm. In this case the unit cube will have volume 1 cm$^3$ and the box will have volume 60 cm$^3$. We would therefore say that the box has volume 60 cubic centimetres.

In general, the volume of a rectangular prism is given by:

Volume of a rectangular prism = length $\times$ width $\times$ height = $\ell wh$

**Example 20**

Find the volume of a rectangular prism with dimensions 5 cm, 6 cm and 12 cm.

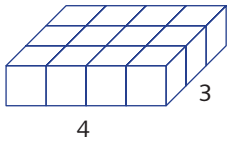
**Solution**

Volume $= \ell wh$
So $V = 5 \times 6 \times 12$
$= 360$ cm$^3$


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## Slices and area

We can see that the rectangular prism is made up of five copies of the $4 \times 3 \times 1$ slice shown. The base area of the slice is $4 \times 3 = 12$ and the height is 1. There are 5 such slices in the original prism, so we obtain:




$$\begin{aligned}\text{Volume} &= 12 \times 5 \\ &= 60, \text{ as before.}\end{aligned}$$

Thus:

$$\text{Volume} = Ah,$$

where $A$ is the area of the base.

The base can be any one of the six different faces of the prism.

*Note*: The formulas above are still valid even when the dimensions of the prism are not whole numbers.

### Right rectangular prism

- A right rectangular prism is a polyhedron in which:
  - the base is a rectangle
  - each cross-section parallel to the base is a rectangle that is exactly the same as the base
  - there are three right angles at each corner of the base.
- Volume of a right rectangular prism $=$ length $\times$ width $\times$ height
  $= \ell wh$

  OR
- Volume of a right rectangular prism $=$ Area of base $\times$ height
  $= Ah$

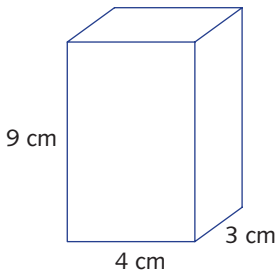
## Exercise 9H

Example 20

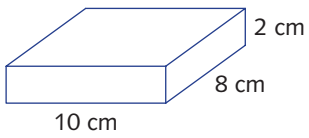
**1** Find the volume of a rectangular prism whose dimensions are 11 mm, 10 mm and 8 mm.

**2** Find the volume of each prism.

**a**




**b**

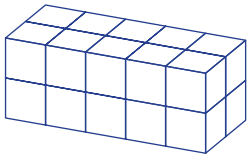




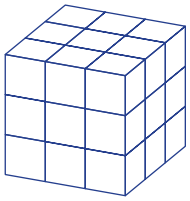
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**3** If ▱ is 1 unit cube, find the volume of each of these figures in unit cubes.

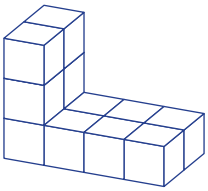
**a**



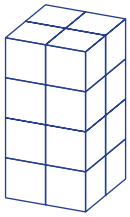
**b**



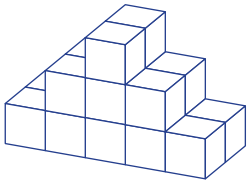
**c**



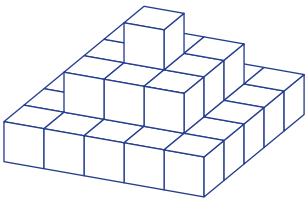
**d**



**e**



**f**



**4** The table below gives the lengths, widths, heights and volumes of various rectangular prisms. Copy the table and fill in the missing entries.

| | Length | Width | Height | Volume |
|---|---|---|---|---|
| **a** | 4 cm | 5 cm | 6 cm | |
| **b** | 7 cm | 11 cm | 10 cm | |
| **c** | | 5 cm | 2 cm | 60 $cm^3$ |
| **d** | 4 cm | 8 cm | | 128 $cm^3$ |

**5** A rectangular tank with a base that is 2 m by 1.5 m has a depth of 3 m. What is the volume of the tank?

**6** A tank in the form of a rectangular prism has base dimensions 2.5 m by 1.5 m and holds water to a depth of 3 m. If the depth is increased by 2 m, what is the increase in the volume of the water?

**7** A right rectangular prism has base area 45 $cm^2$ and height 15 cm. What is the volume of the prism?

**8** A right rectangular prism has dimensions 20 mm × 30 mm × 50 mm. What is the volume of the prism in:

**a** cubic millimetres? **b** cubic centimetres?

**9** A right rectangular prism has volume 450 $mm^3$. The height of the prism is 10 mm. Give three sets of whole number dimensions for such a prism.

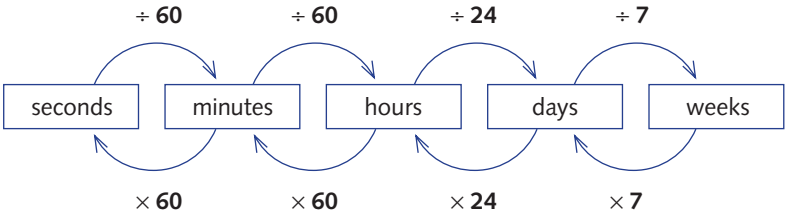
**10** A right rectangular prism has volume 240 $m^3$. The height of the prism is 10 m. Give three sets of whole number dimensions for such a prism.


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# 9I Time

Time is different from other measurements because it is not based on powers of 10.

- There are 60 seconds in 1 minute.
- There are 60 minutes in 1 hour.
- There are 24 hours in 1 day.
- There are 7 days in a week.




There are two ways of recording the time of day:

- Using **a.m.** and **p.m.** – For example, 5:30 p.m. means 5 hours 30 minutes after midday, 0:30 a.m. means 30 minutes after midnight. Remember that a.m. stands for *ante meridiem*, which is Latin for 'before noon', and p.m. stands for *post meridiem*, which is Latin for 'after noon'.
- Using the **24-hour system** – For example, 2235 means 22 hours and 35 minutes after midnight. This is the same as 10:35 p.m. 0530 means 5 hours and 30 minutes after midnight or 5:30 a.m. 1730 is the same as 5:30 p.m. Note that there is no need to indicate morning by using a.m., or afternoon by using p.m., when using 24-hour time. By convention, midnight is 0000, not 2400.

## Example 21

Calculate the number of:

**a** seconds in 19 minutes
**b** minutes in 16 hours
**c** hours in 6 days
**d** days and hours in 560 hours

## Solution

**a** $19 \times 60 = 1140$ seconds (60 seconds in a minute)

**b** $16 \times 60 = 960$ minutes (60 minutes in an hour)

**c** $6 \times 24 = 144$ hours (24 hours in a day)

**d** $24\overline{)56^{8}0}\ 2\ 3$ remainder 8

So 560 hours is 23 days and 8 hours.


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## Adding time

As time measurement is based on the numbers 7, 24 and 60, calculations involving time need to be handled carefully.

### Example 22

A truck driver drove for 5 hours and 45 minutes on Monday, 4 hours and 50 minutes on Tuesday and 6 hours and 30 minutes on Wednesday. What was the total time she spent driving?

#### Solution

$$\begin{array}{rrl} 5\text{ h} & 45\text{ m} & \text{(Add the minutes, add the hours.)} \\ +\ 4\text{ h} & 50\text{ m} & \\ +\ 6\text{ h} & +\ 30\text{ m} & \\ \hline 15\text{ h} & 125\text{ m} & \end{array}$$

Total time spent driving = 15 hours + 2 hours + 5 minutes
(Convert the minutes total to hours and minutes.)
= 17 hours 5 minutes

## Subtracting time and elapsed time

Calculating elapsed time is a skill that is made interesting by the fact that time is based on the numbers 24 and 60.

### Example 23

**a** Jane left home at 3:54 p.m. to travel to her aunt's house. She arrived at 5:40 p.m. How long did her journey take?

**b** Jane's father drove from their home to the aunt's house and it took him 48 minutes. How much longer did Jane take to arrive?

#### Solution

**a** At 3:54 p.m., there are 6 minutes to 4 p.m., and another 1 hour and 40 minutes to 5:40 p.m.

So Jane's total travel time = 6 minutes + 1 hour + 40 minutes
= 1 hour 46 minutes

**b** We need to work out the difference between Jane's travel time of 1 hour 46 minutes and her father's, 48 minutes.

12 minutes are needed to build up from 48 minutes to 1 hour, and then there are 46 minutes after that.
Total = 12 + 46
= 58 minutes
Jane took 58 minutes longer to arrive than her father.


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## Example 24

Marathon runners take about 3 hours to run 42 km. If a runner started at 11:23:00 a.m. (23 minutes past 11, no seconds) and finished at 2:40:49 p.m., what was the time taken to complete the marathon?

### Solution

The time taken to complete the marathon is calculated by building up to the next whole minute or hour.

| | Time | Hours | Minutes | Seconds |
|---|---|---|---|---|
| Start time | 11:23:00 a.m. | | | |
| Build up to | 12:00:00 p.m. | | 37 minutes | |
| Build up to | 2:00:00 p.m. | 2 hours | | |
| Build up to finish time | 2:40:49 p.m. | | 40 minutes | 49 seconds |
| Total | | 2 hours | 77 minutes | 49 seconds |
| Total time taken to complete the marathon (Convert minutes to hours and minutes) | | 3 hours | 17 minutes | 49 seconds |

## Example 25

A boat takes 8 days and 5 hours to complete its journey. So far, 3 days and 11 hours have elapsed. How much longer will the trip last?

### Solution

| | Time | Days | Hours |
|---|---|---|---|
| Start time | 3 days 11 hours | | |
| Build up to | 4 days | | 13 hours |
| Build up to finish time | 8 days 5 hours | 4 days | 5 hours |
| Total | | 4 days | 18 hours |

There are still 4 days and and 18 hours to go until the trip is complete.

## Time

- Measurement of time is based on the numbers 7, 12, 24 and 60.
- Building up to whole minutes, hours or days is helpful when calculating time differences and when adding times.


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## Exercise 9I

Example 21

**1** Calculate the number of:

**a** seconds in 23 minutes

**b** minutes in 24 hours

**c** hours in 17 days

**d** days and hours in 429 hours

**e** minutes in 3 days

**f** hours and minutes in 392 minutes

**g** hours, minutes and seconds in 5201 seconds

**2** Convert each of these times to 24-hour time.

**a** 2:30 p.m.

**b** 6 a.m.

**c** 11:49 p.m.

**d** Eight thirty-five in the evening

**e** Twenty-five to six in the morning

**f** Midday

**g** Midnight

**h** Five past three in the afternoon

**i** Two minutes to midnight

**3** Write each of these 24-hour times in 12-hour time using a.m. and p.m.

**a** 1300 **b** 0600 **c** 0820 **d** 2330

**4** Convert:

**a** $4\frac{1}{2}$ hours to minutes

**b** 289 minutes to hours and minutes

**c** $\frac{3}{4}$ of a year to months

**d** 1 week to hours

Example 22

**5** On one day, four students worked for 3 hours 42 minutes, 4 hours 19 minutes, 1 hour 26 minutes, and 3 hours 6 minutes, respectively. How many hours and minutes were spent working by all four students together?

**6** Mario worked for 17 days 18 hours in January, 28 days 21 hours in February and 25 days 3 hours in March. What was the total time he worked?

**7** Graham travelled for 3 days 23 hours by boat, then 26 hours by plane and finally 4 days 18 hours by train to be home for his mother's birthday. How long did his journey take?

Example 23

**8** What is the elapsed time, in days, hours and minutes, between:

**a** 12 noon and 3:45 p.m.?

**b** 6 p.m. and 8:30 p.m.

**c** 1200 and 1830?

**d** ten past eleven in the morning and three-thirty in the afternoon of the same day?

**e** 5:35 a.m. on Tuesday and 6:20 a.m. on the next Wednesday?

**f** 2:32 a.m. on Friday and 4:55 p.m. on the following Sunday?

**g** 1 p.m. on Thursday and 5:25 a.m. on the following Tuesday?

**9** When it is 3:03:52 p.m. (the last two digits indicate seconds) in Brisbane, the time is 7:03:52 a.m. on the same day in Madrid. We say that Brisbane is ahead of Madrid. What is the time difference between the two cities?


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**10** If I started a train trip at 1430 and finished it at 0625 the next day, how long did the journey take?

**11** Three people ran the Canberra marathon; their times were 2:43:52 (2 hours 43 minutes 52 seconds), 2:46:57 and 2:47:15. What were the time differences between the first and second competitors, and between the second and third?

**12** Suppose a child is born at 2349 on one day and his twin brother is born at 0008 the next day. How much older is the first child?

**13** The time zone difference from Melbourne to Los Angeles is 19 hours, Melbourne being ahead in time. What time would it be in Melbourne if it were 6:27 a.m. on Tuesday in Los Angeles?

**14** Sarah takes 4 hours and 45 minutes to read a 200-page novel, while Derek takes 5 hours and 2 minutes. Now they both read a 300-page novel. If they read at the same speeds as for the previous novel, how much faster than Derek will Sarah finish the 300-page novel?

# 9J Speed

Speed is a measure of how far something has travelled for a given unit of time.

## Constant speed

If the speed of an object does not change, we say that the object is moving with **constant speed**. A car travelling at a constant speed of 60 kilometres per hour would travel 60 kilometres in one hour. It would travel 120 km in 2 hours and 150 km in $2\frac{1}{2}$ hours. For example, if I travel at a constant speed for 100 kilometres and it takes me one hour to complete the journey, then my speed is 100 kilometres per hour. This can also be written as 100 km/h. If I travel 100 kilometres in 2 hours (at a constant speed), then my speed is 50 km/h.

The speed of a moving object is defined to be the distance travelled divided by the time the object takes to travel that distance. If we use the letter $d$ for the distance travelled and $t$ for the time it takes to travel that distance, then we write the formula for speed as:

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$
$$= \frac{d}{t}$$


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**Example 26**

Jung travels at a constant speed for 60 kilometres and it takes 4 hours to complete the journey. What is his speed?

**Solution**

$$\begin{aligned}\text{speed} &= \frac{\text{distance}}{\text{time}} \\ &= \frac{60}{4} \\ &= 15 \text{ km/h}\end{aligned}$$

**Example 27**

A man runs at 10 m/s for 11 seconds. How far does he run?

**Solution**

$$\begin{aligned}\text{distance} &= \text{speed} \times \text{time} \\ &= 10 \times 11 \\ &= 110 \text{ m}\end{aligned}$$

The man runs 110 m.

## Average speed

When driving a car or riding a bike, it is very rare for our speed to remain the same for a long period of time. Most of the time, especially in the city, we are slowing down or speeding up, so our speed is not constant. If we travel 20 kilometres in one hour, then we say that our **average speed** over that hour is 20 km/h, even though we must have travelled faster than this at some times and slower (perhaps even coming to a complete stop) at others. When we calculate speed, we often mean average speed.

**Example 28**

Paul rides 12 km on his bike in three quarters of an hour. What was his average speed?

**Solution**

$$\begin{aligned}\text{speed} &= \frac{\text{distance}}{\text{time}} \\ &= 12 \div \frac{3}{4} \\ &= 12 \times \frac{4}{3} \\ &= 16 \text{ km/h}\end{aligned}$$


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### Example 29

Kaelah travelled at an (average) speed of 50 km/h for 3 hours. How far did she travel?

#### Solution

An average speed of 50 km/h means that in 1 hour Kaelah travelled 50 km.

Hence, in 3 hours she travelled $3 \times 50 = 150$ km.

### Example 30

The speed of light is approximately 300 000 000 metres per second. How far, in kilometres, would a light ray travel in one minute?

#### Solution

In one second, light travels 300 000 000 metres.

Hence, in 60 seconds light travels

$$300\,000\,000 \times 60 \text{ metres} = 18\,000\,000\,000 \text{ metres}$$
$$= 18\,000\,000 \text{ kilometres}$$

## Exercise 9J

Example 26

**1** Copy and complete this table.

| | Speed | Distance | Time |
|---|---|---|---|
| **a** | | 100 km | 2 h |
| **b** | | 30 m | 6 min |
| **c** | | 15 km | $\frac{1}{2}$ h |
| **d** | 30 m/s | | 4 s |
| **e** | 55 km/h | | 11 h |
| **f** | 60 km/h | | $\frac{1}{3}$ h |

Example 27

**2** A child cycles for 90 minutes at 8 km/h. How far does he go?

Example 28

**3** A taxi drives 5.2 km to the airport in 13 minutes. What is its average speed in km per hour?

Example 29

**4** Maxine paddled her surf ski for 30 minutes at 2 km/h. How far did she go?

**5** A balloon travels at 24 km/h for 4 hours 40 minutes. How far does it travel?

**6** An aircraft travels 3650 km in 5 hours. What is its average speed?

**7** A train travels for 36 minutes at 80 km/h. How far does it go?

**8** A boat travels 120 nautical miles in 5 hours. What is its average speed?


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**9** A plane is flying at a speed of 840 km/h. How far will it travel between 10:30 a.m. and 11:15 a.m.?

Example 30

**10** A top athlete's average sprinting speed is recorded at 39.6 km/h.

**a** What is this speed in m/s?

**b** At this speed, how far could he travel in 18 seconds?

**11** Lucas took 25 minutes to walk 2.5 km and then 15 minutes to travel 17.5 km by train.

**a** What was the total distance Lucas travelled?

**b** What was his average speed when he was walking?

**c** What was his average speed on the train trip?

**d** What was his average speed for the entire trip?

**12** Henry travels 50 km by train in $\frac{3}{4}$ of an hour, and then cycles for 10 km in $\frac{4}{5}$ of an hour.

**a** How long, in hours and minutes, did his whole trip take?

**b** What is his average speed, in km/h, during the train trip?

**c** What is his average speed, in km/h, during the bike trip?

**d** What is his average speed over the whole trip?

**13** Linda rode 20 km uphill, which took her an hour. She then spent 40 minutes riding downhill the same distance. What was her average speed?

**14** Convert 60 km/h to metres per second. Convert $x$ km/h to metres per second.

**15** A standard triathlon race involves 1500 m of swimming, a 40 km bike ride, and a 10 km run. If a competitor took 20 minutes for the swim, 55 minutes for the ride and 35 minutes for the run, what was his average speed, in km/h, for the event?

# Review exercise

**1** Complete these conversions.

| | | |
|---|---|---|
| **a** 1607 mm = ____ m | **b** 1607 mm = ____ cm | **c** 197 mm = ____ cm |
| **d** 197 mm = ____ m | **e** 230 cm = ____ mm | **f** 230 cm = ____ m |
| **g** 55 cm = ____ mm | **h** 55 cm = ____ m | **i** 1.5 km = ____ cm |
| **j** 1.5 km = ____ m | **k** 8.3 m = ____ cm | **l** 8.3 m = ____ km |

**2** Complete these conversions.

| | | |
|---|---|---|
| **a** 3 kg = ____ mg | **b** 3 kg = ____ g | **c** 4.8 g = ____ kg |
| **d** 4.8 g = ____ mg | **e** 1029 mg = ____ g | **f** 1029 mg = ____ kg |
| **g** 329 kg = ____ g | **h** 329 kg = ____ mg | **i** 4 L = ____ mL |
| **j** 2.3 kL = ____ L | **k** 2950 mL = ____ L | **l** 250 km = ____ μm |


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**3** A 2.3 m length of timber is cut into 5 equal pieces. How long is each piece, in centimetres?

**4** 42.8 kilograms of fruit is shared between 8 families. How much fruit does each family receive?

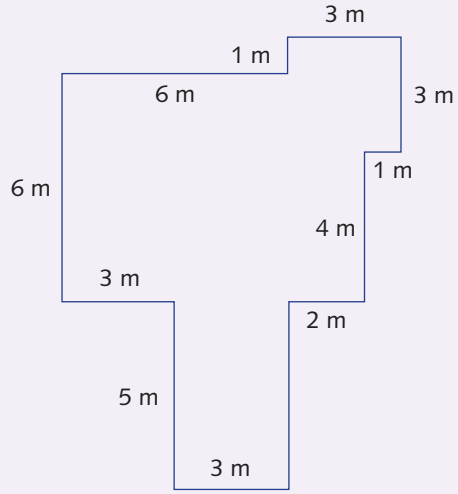
**5** A fence is required for a rectangular block of land that measures 51.6 m down each of the two sides and 16.85 m across the back, with no fence at the front. How long is the fence in total?

**6** If 35 lollies cost \$5.25, how much do 18 lollies cost?

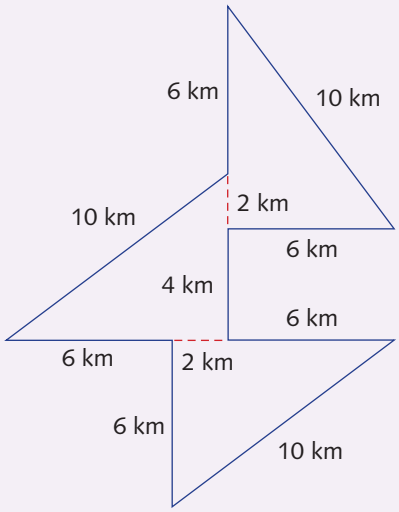
**7** I use 980 g of flour to make 14 dim sims. How much flour is needed for 17 dim sims? How much is needed for 3 dim sims?

**8** Calculate the perimeter and area of each of these figures, assuming that the non-slanting lines are either horizontal or vertical.

**a**




**b**




**9** Calculate the time that is 3 hours, 23 minutes and 59 seconds after each of these times. Express all answers as a.m. or p.m.

**a** 2:30 p.m.

**b** 11:43 p.m.

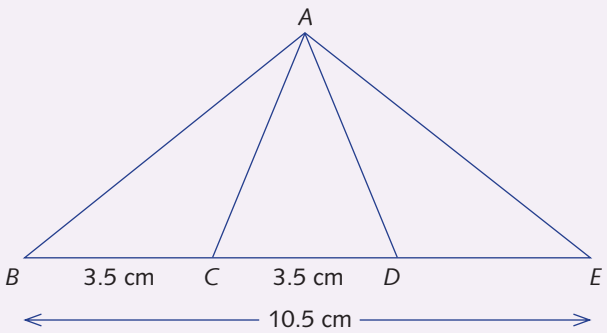
**c** 1:26:03 a.m.

**d** 1800

**e** Three forty-five in the afternoon

**10** At a supermarket, 250 g of chocolate costs \$4.50, while the price is \$12 for 1 kg 50 g of the same chocolate. Which is the more economical option?

**11** Find all 6 triangles in this figure and rank them by area.





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**12** A right-angled triangle has sides of 5, 12 and 13 cm. Find its area.

**13** If 200 $m^2$ of grass is required for the healthy grazing of 3 sheep, how much grass is needed to raise 15 sheep? How many healthy sheep can 2.5 hectares of land support?

**14** What is the perimeter, in metres, of a regular pentagon (all sides are equal), with side length 1.3 mm?

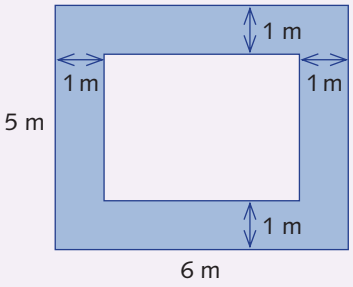
**15** Samantha runs at a speed of 8 m/s for 30 seconds. How far does she run?

**16** Yolanda runs 400 m in 52 seconds. What is her average speed in m/s?

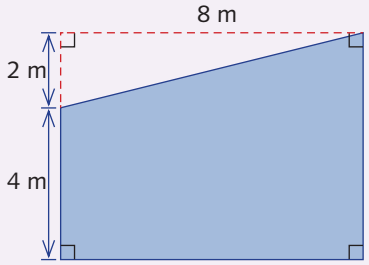
**17** Louise rides $5\frac{1}{2}$ km in $\frac{3}{4}$ of an hour. What is her speed in km/h?

**18** Find the area of each shaded region.

**a**




**b**



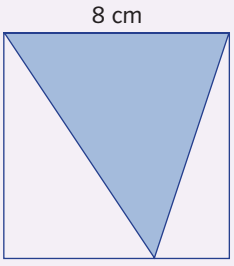

**19** What is the perimeter of a regular hexagon with side length 8.2 cm?

**20** Sole drives for 2 hours at 65 km/h and then at 80 km/h for 3 hours. What is Sole's average speed for the 5 hours?

**21** Jane's average walking speed is 8 km/h. If she walks for 3 hours and 20 minutes, how far has she walked? If she then hikes 10 km in 4 hours, what is her average speed for the entire journey?

**22** In bygone days, the imperial system of measurement was used. In this system, distance is measured in inches, feet, yards and miles. There are 12 inches in a foot, 3 feet in a yard, and 1760 yards in a mile. Given that 2.54 cm equals 1 inch, how many kilometres is a mile?

**23** A square of side length 8 cm has a triangle removed from it, as shown on the right. What is the remaining area?




**24** A yacht race started on Monday at 13:49:30 and finished when the last yacht arrived at its destination on Friday of the same week at 07:15:29. How long was the race in hours, minutes and seconds?


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**25** Elephants often weigh as much as 3 tonnes, while the heaviest mass a person can lift (usually in the Olympic games) is about 250 kg. What is the minimum number of people needed to lift an elephant?

**26** Triangle $ABC$ has side lengths $AC = 4$ cm, $AB = 3$ cm and $BC = 5$ cm.

$\angle BAC = 90°$. The point $X$ is on $BC$ and $AX$ is perpendicular to $BC$. Find:

**a** the area of triangle $ABC$

**b** the length $AX$

**27** **a** Find the volume of a right rectangular prism with dimensions 5.2 cm × 6 cm × 10 cm.

**b** A right rectangular prism has volume 625 $mm^3$. The height of the prism is 12.5 cm. Find the area of the base.

**c** The areas of the faces of a right prism are 35 $cm^2$, 50 $cm^2$ and 10 $cm^2$. Find the volume of the prism.

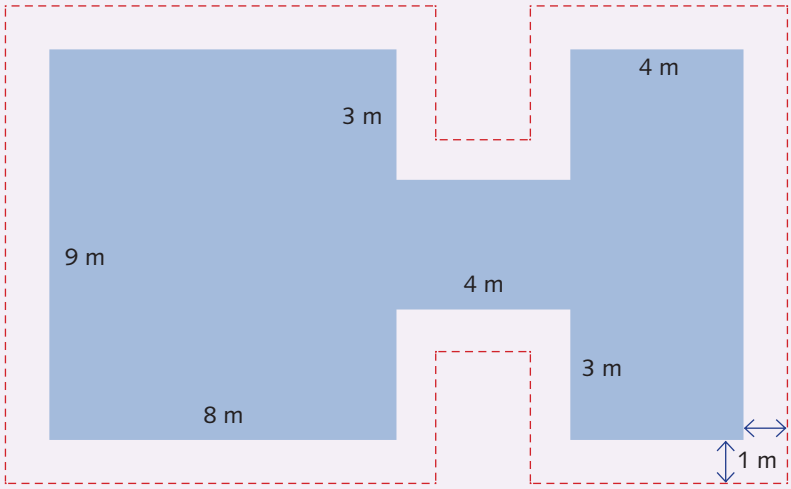
# Challenge exercise

**1** My dad told me that there are more microseconds in a minute than minutes in a century. Was he telling the truth?

**2** Draw three figures, based on rectangles and triangles, that have the same area but different perimeters.

**3** Draw three figures, based on rectangles and triangles, that have the same perimeter but different areas.

**4** A path 1 metre wide is being placed around the grassed area of a garden, as shown below. What is the area of the grassed part of the garden? What is its perimeter? What will the area of the path be?





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**5** If four days before tomorrow is Friday, what day is three days after yesterday?

**6** If birthday cards cost \$2.50 for a box of 12, \$1.25 for a packet of 3, or 50c each, what is the greatest number of cards that you can buy if you have \$14.90?

**7** If it takes 4 people 5 days to clean up the rubbish alongside a 180 km stretch of freeway, how many people will be needed to clean 250 km of freeway in 6 days? (Assume they all work at the same pace.)

**8** At how many different times are all the digits on a digital clock, which runs from 0:00 to 23:59, identical in a 24-hour period?

**9** I took my baby to the clinic to be weighed, but he would not keep still, so I held him and stood on the scales and the weight recorded was 75 kg. Then the nurse held the baby and stood on the scales, and the weight recorded was 69 kg. Finally, the nurse and I both stood on the scales, and the weight recorded was 137 kg. What did the baby weigh?

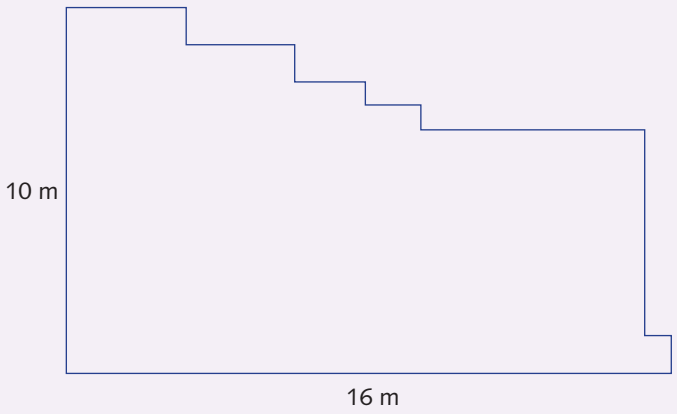
**10** A man plants 12 rows of beans in rows that are 10 m long. The plants in each row are spaced 50 cm apart. If the first and last plants in a row are 25 cm from the ends of the row, what is the total number of plants?

**11** Nine bus stops are equally spaced along a road. The distance from the first to the third stop is 600 m. How far is it from the first stop to the last?

**12** If 14 January 2006 was a Saturday, what day of the week will 14 January 2025 be?

**13** My average walking speed up a mountain was 3 km/h. I then ran down the same mountain at a speed of 12 km/h. What was my average speed for the entire trip up and down the mountain?

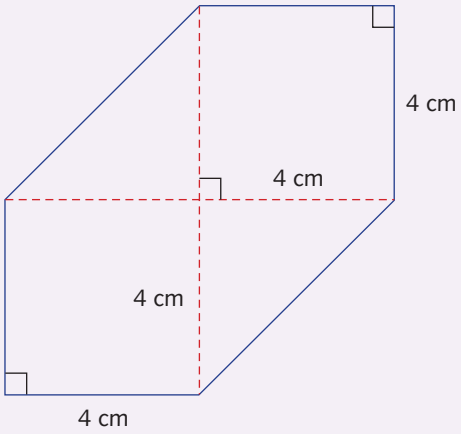
**14** Find the perimeter of the shape below. Once you figure out the trick, look at some of the previous perimeter exercises and see how quickly you can do them!





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**15** Find the area of the following figure, using two different methods.




**16** Imagine you have 16 tiles, each 1 cm square. Draw on grid paper some different rectangles that use all 16 tiles. You know that the area of each rectangle you have drawn is 16 $cm^2$. What is the perimeter of each rectangle, in centimetres?

**17** Imagine that you have a piece of string exactly 24 cm long. On grid paper, draw some different rectangular shapes that can be made using the entire 24 cm piece of string. You know that each of these rectangles has a perimeter of 24 cm. What is the area of each, in square centimetres? How small can you make the area of a rectangle with whole number side lengths? How large?

**18** Investigate the possible perimeters for a rectangle of area 64 $m^2$. Which of the alternatives has the smallest perimeter?

**19** Chase and Lana have 108 m of fencing with which to enclose an area of the schoolyard for a rectangular garden. Explore the possible areas of their garden, and state which alternative has the largest area.


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CHAPTER

# 10

# Review and problem-solving

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# 10A Review

## Chapter 1: Whole numbers

**1** Evaluate the following, using the mental strategies described in Chapter 1.

**a** 23 + 45 + 27 **b** 21 + 18 + 11 **c** 22 + 23 + 75

**d** 56 + 11 + 19 + 14 **e** 36 + 19 + 78 + 21 **f** 99 + 27 + 11 + 43

**2** Evaluate the following, using the addition algorithm, or try to do them mentally if you can.

**a** 692 + 149 **b** 3964 + 3829 **c** 1099 + 2058 + 4589

**d** 36 + 820 + 87 **e** 109 + 36 + 29 985 **f** 438 + 999 + 1053

**3** Evaluate the following, using the subtraction algorithm, or try to do them mentally if you can.

**a** 856 − 302 **b** 11 884 − 10 333 **c** 496 − 394

**d** 236 − 198 **e** 1403 − 1378 **f** 18 443 − 9876

**g** 1000 − 297 **h** 10 012 − 9999 **i** 10 101 − 8207

**4** Deon earned $126, $235, $154 and $165 over four Saturdays mowing lawns. How much did he earn in total?

**5** The lengths of material required for new curtains were 18 metres, 27 metres, 83 metres, 12 metres and 89 metres. What total length of material was required for all the curtains?

**6** The distance from Adelaide to Uluru is 1550 km, and the distance from Adelaide to Darwin via Uluru is 3050 km. How far is Uluru from Darwin?

**7** Jacqui is 183 cm tall, David is 152 cm tall and Adrian is 147 cm tall. How much taller than each of the others is Jacqui?

**8** Insert <, > or = to make these statements true.

**a** 43 + 7 □ 25 + 25 **b** 63 − 17 □ 26 + 19

**c** 18 + 18 + 18 □ 17 + 18 + 19 **d** 298 + 546 □ 1000 − 154

**9** Calculate:

**a** 27 × 5 + 27 × 95 **b** 2 × 43 × 5 **c** 25 × 82 × 4

**d** 125 × 6 **e** 138 × 7 **f** 143 × 12

**10** Use either the short or the long multiplication algorithm to calculate:

**a** 24 × 9 **b** 87 × 8 **c** 43 × 7

**d** 198 × 6 **e** 1035 × 8 **f** 22 486 × 3

**g** 333 × 21 **h** 6434 × 12 **i** 423 × 23

**j** 744 × 34 **k** 85 739 × 37 **l** 50 746 × 26

**m** 930 × 385 **n** 294 × 382 **o** 7584 × 4723


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**11** All of the 32 rows of seats are filled for the school assembly. If there are 17 seats in each row, how many children and teachers attend assembly?

**12** Each of 38 rabbit burrows has 19 rabbits in it. What is the total number of rabbits?

**13** Fizzoes come in packets of 11. There are 43 packets in a box, and 29 boxes in a carton. How many Fizzoes are there in each carton?

**14** A hotel has 17 rooms on each floor. If the hotel consists of three buildings, each with 27 floors, how many rooms are there in the hotel?

**15** Calculate each of the following, using the short division algorithm.

| | | |
|---|---|---|
| **a** $444 \div 2$ | **b** $844 \div 4$ | **c** $969 \div 3$ |
| **d** $497 \div 7$ | **e** $248 \div 8$ | **f** $729 \div 9$ |
| **g** $198 \div 3$ | **h** $176 \div 8$ | **i** $567 \div 9$ |
| **j** $2416 \div 8$ | **k** $1805 \div 5$ | **l** $4554 \div 6$ |
| **m** $432 \div 4$ | **n** $840 \div 8$ | **o** $924 \div 3$ |
| **p** $8948 \div 5$ | **q** $48\,567 \div 7$ | **r** $39\,573 \div 5$ |

**16** Evaluate:

**a** $5 \times 4 + 6 \div 2 - 1$ **b** $5 \times (4 + 6) \div 2 - 1$ **c** $5 \times 4 + (8 \div 2 - 1)$

## Chapter 2: Factors, multiples, primes and divisibility

**1** Find all the factors of:

**a** 39 **b** 48 **c** 144 **d** 420 **e** 1272 **f** 1250

**2** Which of the following numbers have 8 as a factor?

14, 24, 37, 38, 144, 196, 232, 333

**3** Which of the following numbers are multiples of 19?

47, 48, 76, 95, 109, 119, 127, 142, 152, 199

**4** Which of the following numbers have 186 as a multiple?

2, 3, 5, 6, 18, 62, 74, 86, 93

**5** List the prime numbers between 44 and 144.

**6** Find two prime numbers that sum to:

**a** 48 **b** 88 **c** 108 **d** 114 **e** 204 **f** 240

**7** Write in expanded form and evaluate:

**a** the powers of 4 up to $4^5$ **b** the powers of 10 up to $10^9$

**8** Express each number as a product of its prime factors.

**a** 24 **b** 96 **c** 150 **d** 162 **e** 324 **f** 392


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**9** Evaluate:

**a** $5^2$ **b** $9^2$ **c** $11^2$ **d** $28^2$ **e** $50^2$ **f** $100^2$

**10** Write the square root of:

**a** 36 **b** 400 **c** 196 **d** 441 **e** 729 **f** 1 000 000

**11** Write each of these numbers as the sum of 3 or 4 squares.

**a** 29 **b** 105 **c** 70 **d** 299

**12** Find three numbers for which the square of the number has none of the same digits as the cube of that number. One such number is 9, as $9^2 = 81$ and $9^3 = 729$, and the number 81 has none of the same digits as 729.

**13** Find the highest common factor of:

**a** 12 and 8 **b** 9 and 24 **c** 10 and 8

**d** 45 and 35 **e** 124 and 72 **f** 100 and 48

**14** Find the lowest common multiple of:

**a** 7 and 4 **b** 5 and 8 **c** 4 and 6

**d** 4 and 8 **e** 15 and 6 **f** 24 and 18

**15** Fill in the gaps in the 6-digit number 4__0__3__ to make a number that is divisible by:

**a** 2 **b** 10 **c** 3

**d** 2 and 5 **e** 3 and 8 **f** 3, 4 and 6

**g** 3 and 10 **h** 5 but not 10 **i** 2 but not 4

**16** Two factors of 403, namely 13 and 31, can each be written by reversing the digits of the other. Find three other numbers with this property.

## Chapter 3: An introduction to algebra

**1** Use algebra to write each of the following.

**a** The product of 6 and $z$

**b** The product of $x$ and $y$

**c** The difference of $x$ and 8 ($x$ is greater than 8)

**d** $m$ is multiplied by 5, and 3 is added to the result

**2** A packet of chocolates contains $n$ chocolates. How many chocolates are left in the packet if 5 are removed?

**3** There are 6 boxes, each containing $x$ mice. How many mice are there in all?

**4** Write each product without multiplication signs.

**a** $4 \times x \times 6 \times x$ **b** $5 \times x \times 7 \times x$ **c** $3 \times m \times m \times n \times n \times n$

**5** Rewrite each quotient, using fraction notation.

**a** $2x \div y$ **b** $3q \div 2p$ **c** $5w \div x$ **d** $x \div 3y$


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**6** Write each of the following in standard algebraic notation.

**a** The product of $z$ with itself

**b** The product of $5a$ and $b$

**c** The product of $4b$ and $3b$

**d** The quotient of $x$ divided by 7

**e** The quotient of $2p$ divided by $q$

**7** If $x = 5$, evaluate:

**a** $2x + 4$ **b** $5 - x$ **c** $2x^2 + 3$ **d** $20 - 2x$ **e** $7(x - 3)$

**8** The profit in selling $n$ crates of apples is $5n$ dollars. What is the profit if 100 crates of apples are sold?

**9** Simplify:

**a** $3x + 7x - x$

**b** $2a + 3a - a$

**c** $7xy - 3xy + 2xy$

**d** $11xy + 3xy - 2yx$

**10** Rewrite each statement, using algebra.

**a** A number, $x$, is multiplied by 6, and 2 is subtracted from the result.

**b** 3 is subtracted from a number, $x$, and the result is multiplied by 5.

**11** Write each statement, using algebra.

**a** A number, $a$, is squared, and the result is multiplied by 3.

**b** A number, $a$, is multiplied by 3, and the result is squared.

**12** Rewrite each expression without brackets.

**a** $(3x)^2$ **b** $(2z)^2$ **c** $(15x)^2$ **d** $(7c)^2$

**13** Assuming that $a$ is a multiple of 5, what are the next two multiples of 5?

## Chapter 4: Fractions – part 1

**1** Draw a number line marked with the whole numbers from 0 to 3, and indicate on it where the markers for $\frac{1}{2}$, $\frac{4}{2}$, $\frac{3}{4}$, $1\frac{3}{8}$ and $\frac{19}{8}$ are located.

**2** Draw a number line marked with the whole numbers from 0 to 3, and indicate on it where the markers for $\frac{2}{3}$, $\frac{1}{6}$, $\frac{8}{9}$, $2\frac{5}{6}$, $1\frac{1}{9}$ and $\frac{11}{6}$ are located.

**3** Write the fraction that represents each of these situations.

**a** 8 of 11 grandchildren visit their grandmother.

**b** Twenty minutes out of one hour are spent reading.

**c** One leg of a spider is broken.

**d** Four children out of a set of septuplets are girls.

**e** Only seven sailing boats of a fleet of 23 make it to the finish line before the storm.


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**4** Write two equivalent fractions for:

**a** $\frac{4}{8}$ **b** $\frac{3}{12}$ **c** $\frac{15}{20}$ **d** $\frac{2}{3}$ **e** $\frac{11}{12}$

**f** $\frac{5}{8}$ **g** $1\frac{1}{2}$ **h** $4\frac{1}{3}$ **i** $3\frac{5}{6}$ **j** $\frac{7}{24}$

**5** In each part, find the value of $n$ that makes the statement true.

**a** $\frac{n}{40} = \frac{15}{20}$ **b** $\frac{48}{40} = \frac{6}{n}$ **c** $\frac{10}{15} = \frac{n}{75}$

**d** $\frac{3}{8} = \frac{36}{n}$ **e** $\frac{18}{n} = \frac{90}{100}$ **f** $\frac{4}{n} = \frac{28}{49}$

**6** Reduce each fraction to its simplest form.

**a** $\frac{4}{12}$ **b** $\frac{9}{27}$ **c** $\frac{8}{92}$ **d** $\frac{12}{34}$ **e** $\frac{165}{200}$ **f** $\frac{18}{48}$

**g** $\frac{25}{125}$ **h** $\frac{26}{104}$ **i** $\frac{49}{105}$ **j** $\frac{500}{125}$ **k** $\frac{448}{64}$ **l** $\frac{196}{184}$

**7** Convert the following improper fractions to mixed numerals, and reduce them to simplest form.

**a** $\frac{8}{3}$ **b** $\frac{13}{6}$ **c** $\frac{17}{5}$ **d** $\frac{45}{40}$ **e** $\frac{120}{100}$ **f** $\frac{97}{32}$

**8** Find the next four numbers in each of the following sequences. (In each case, to go from one term to the next, you add the same fraction.)

**a** $\frac{4}{5}$, $1\frac{3}{5}$, $2\frac{2}{5}$, $3\frac{1}{5}$, ___, ___, ___, ___

**b** $\frac{5}{7}$, $1\frac{3}{7}$, $2\frac{1}{7}$, $2\frac{6}{7}$, ___, ___, ___, ___

**c** $\frac{1}{6}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, ___, ___, ___, ___

**9** Convert these mixed numerals to improper fractions.

**a** $3\frac{1}{6}$ **b** $4\frac{4}{5}$ **c** $5\frac{2}{3}$ **d** $1\frac{21}{22}$ **e** $6\frac{7}{9}$ **f** $2\frac{2}{19}$

**10** Write a fraction that:

**a** is closer to 1 than $\frac{7}{8}$

**b** is between $\frac{1}{3}$ and $\frac{2}{3}$

**c** is equivalent to $\frac{1}{2}$, but has a denominator greater than 50

**d** is between 4 and 5

**e** is larger than $\frac{4}{7}$ but smaller than $\frac{8}{9}$

**f** is larger than 2 but closer to 2 than $1\frac{11}{12}$


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**11** Order these fractions from smallest to largest.

**a** $\frac{1}{2}, \frac{1}{6}, \frac{1}{3}, \frac{1}{10}, \frac{1}{4}$ **b** $\frac{3}{4}, \frac{3}{8}, \frac{1}{8}, \frac{1}{4}, \frac{9}{8}$ **c** $\frac{1}{3}, \frac{4}{9}, \frac{5}{6}, \frac{3}{6}, \frac{2}{3}$

**d** $\frac{1}{18}, \frac{16}{17}, \frac{9}{18}, \frac{17}{19}, \frac{5}{16}$ **e** $\frac{7}{8}, \frac{4}{5}, \frac{5}{6}, \frac{3}{4}, \frac{3}{8}$ **f** $2\frac{1}{4}, \frac{3}{5}, 1\frac{3}{10}, 2\frac{2}{6}, 1\frac{1}{5}$

**12** Evaluate these sums, writing each answer as a fraction in simplest form.

**a** $\frac{1}{7}+\frac{4}{7}$ **b** $\frac{6}{8}+\frac{1}{8}$ **c** $\frac{4}{10}+\frac{4}{10}$ **d** $\frac{2}{3}+\frac{4}{3}$ **e** $\frac{5}{6}+\frac{3}{6}$ **f** $\frac{11}{12}+\frac{3}{12}$

**g** $\frac{1}{8}+\frac{1}{4}$ **h** $\frac{3}{12}+\frac{4}{6}$ **i** $\frac{8}{9}+\frac{1}{3}$ **j** $\frac{4}{5}+\frac{1}{4}$ **k** $\frac{3}{7}+\frac{2}{3}$ **l** $\frac{3}{10}+\frac{9}{11}$

**13** Evaluate:

**a** $\frac{2}{3}-\frac{1}{3}$ **b** $\frac{4}{7}-\frac{3}{7}$ **c** $\frac{27}{48}-\frac{19}{48}$ **d** $\frac{3}{4}-\frac{1}{8}$ **e** $\frac{1}{2}-\frac{1}{6}$ **f** $\frac{7}{8}-\frac{2}{4}$

**g** $\frac{3}{4}-\frac{1}{2}$ **h** $\frac{2}{3}-\frac{3}{5}$ **i** $\frac{2}{5}-\frac{1}{7}$ **j** $\frac{6}{7}-\frac{1}{3}$ **k** $\frac{9}{10}-\frac{3}{4}$ **l** $\frac{5}{6}-\frac{1}{9}$

**14** Write an algebraic expression for each of the following.

**a** Three different fractions that sum to exactly $\frac{5}{8}$

**b** Two fractions with a difference that is a little more than $\frac{1}{2}$

**c** Three different fractions that sum to exactly $\frac{1}{4}$

**d** Two fractions with a difference of exactly $\frac{3}{4}$

**15** Ella picks half a basket of grapes, Jonah picks $2\frac{1}{3}$ baskets and Barry picks $\frac{1}{4}$ of a basket. How many baskets were picked in total?

## Chapter 5: Fractions – part 2

**1** When Evelyn took $\frac{3}{4}$ of the chocolates, there were four chocolates left in the box. How many chocolates did Evelyn take?

**2** Evaluate:

**a** $\frac{3}{5}\times\frac{5}{6}$ **b** $\frac{2}{5}\times\frac{3}{4}$ **c** $\frac{7}{8}\times\frac{4}{7}$ **d** $\frac{3}{10}\times\frac{5}{6}$

**e** $\frac{1}{2}\times\frac{1}{3}\times\frac{1}{6}$ **f** $\frac{3}{4}\times\frac{5}{6}\times\frac{1}{2}$ **g** $\frac{4}{5}\times\frac{1}{2}\times\frac{2}{3}$ **h** $\frac{3}{4}\times\frac{5}{6}\times\frac{3}{8}$

**i** $\frac{14}{15}\times\frac{16}{21}\times\frac{5}{8}$ **j** $3\frac{1}{3}\times\frac{2}{5}$ **k** $5\frac{1}{4}\times1\frac{1}{14}$ **l** $7\frac{1}{3}\times2\frac{5}{11}$


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**3** Evaluate:

**a** $\frac{3}{4} \div \frac{3}{8}$ **b** $2\frac{1}{3} \div \frac{5}{8}$ **c** $1\frac{1}{5} \div 2\frac{1}{3}$ **d** $1\frac{1}{2} \div 3\frac{1}{3}$

**4** Evaluate:

**a** $4 \times \frac{1}{4}$ **b** $4 \div \frac{1}{4}$ **c** $\frac{2}{3} \times 3$ **d** $\frac{2}{3} \div 3$

**e** $3 \div \frac{1}{5}$ **f** $3 \times \frac{1}{5}$ **g** $\frac{1}{2} \div 2$ **h** $\frac{1}{2} \times 2$

**i** $\frac{1}{2} \div 17$ **j** $\frac{1}{2} \times 17$ **k** $\frac{2}{3} \times \frac{3}{4}$ **l** $\frac{2}{3} \div \frac{3}{4}$

**m** $\frac{2}{3} \times \frac{1}{3}$ **n** $\frac{2}{3} \div \frac{1}{3}$ **o** $\frac{4}{7} \times \frac{7}{5}$ **p** $\frac{4}{7} \div \frac{7}{5}$

**5** Evaluate:

**a** $\frac{3}{4} \times \frac{8}{9} + \frac{2}{3}$ **b** $\frac{2}{3} \times \frac{3}{4} + \frac{3}{4}$ **c** $\frac{5}{6} \times \frac{3}{10} + \frac{1}{2}$

**d** $\frac{3}{5} \div \frac{9}{10} + \frac{1}{3}$ **e** $\frac{3}{4} \div \frac{9}{16} - \frac{1}{6}$ **f** $\frac{2}{3}$ of $\frac{3}{4} \times \frac{8}{9}$

**g** $\frac{3}{5}$ of $\frac{5}{9} - \frac{2}{3}$ of $\frac{1}{6}$ **h** $\frac{7}{9} \div \frac{3}{4} \times \frac{14}{27}$

**6** Evaluate:

**a** $\dfrac{\frac{2}{3} + \frac{3}{4}}{\frac{5}{6} - \frac{2}{3}}$ **b** $\dfrac{\frac{5}{12} - \frac{1}{4}}{\frac{2}{3} + \frac{1}{6}}$ **c** $\dfrac{\frac{3}{4} \times \frac{8}{9}}{\frac{2}{3} - \frac{1}{6}}$

**7** A tank that is $\frac{3}{4}$ full contains 1341 L of water. How many litres does it hold when it is full?

**8** After cycling $\frac{5}{9}$ of a journey, a cyclist has 32 km further to go. What is the length of the journey?

**9** If I spend one-third of my money, then one-quarter of what is left, and finally one-fifth of the remainder, what fraction of my original money do I have left?

**10** Four bells start tolling together and ring every 1, $1\frac{1}{4}$, $1\frac{1}{5}$ and $1\frac{1}{6}$ seconds, respectively. After what period of time will they first ring all together again?

## Chapter 6: An introduction to geometry

**1** **a** Write down the complement of:

**i** 20° **ii** 45° **iii** 82° **iv** 18° **v** 76°

**b** Write down the supplement of:

**i** 140° **ii** 132° **iii** 85° **iv** 68° **v** 166°


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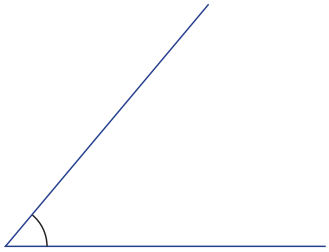
**2** Find the angle between the hour hand and the minute hand at:

**a** 3 p.m. **b** 9 a.m. **c** 4 p.m. **d** 3 a.m.

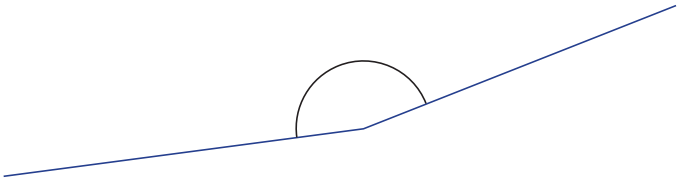
**e** 8 p.m. **f** 9 p.m. **g** 11 a.m. **h** 5 a.m.

**3** **a** Use your protractor to find the size of each angle.

**i**



**ii**



**b** Write down the size of the reflex angle for each angle in part **a**.

**4** Copy the diagram below, leaving about 6 cm of space above the diagram for angles to be drawn.

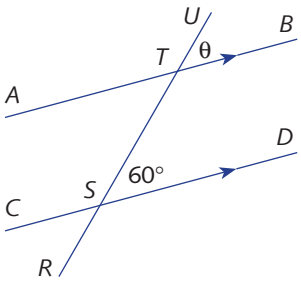



Use your protractor to construct rays, pointing upwards from $A$, that make angles with the ray $AP$ of:

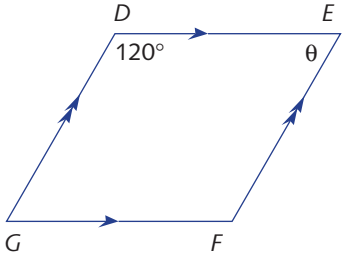
**a** 35° **b** 150° **c** 80°

**5** Find the values of the angles $\alpha$, $\beta$, $\gamma$ and $\theta$ in each diagram below. Give careful reasons for all your statements, and name the relevant parallel lines.

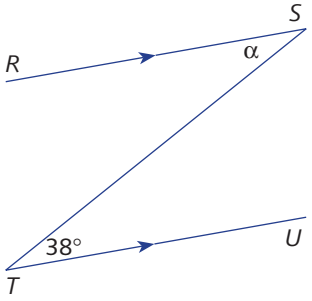
**a**



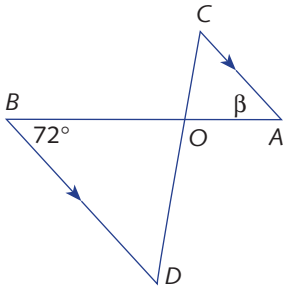

**b**



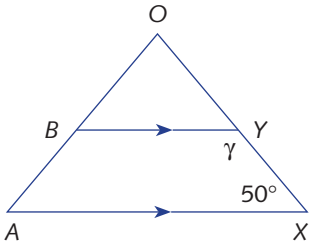

**c**




**d**




**e**

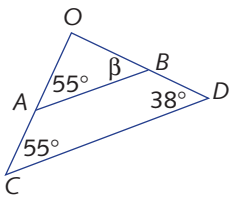




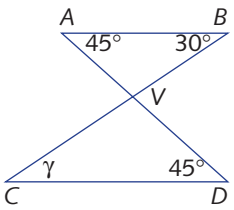
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**6** In each diagram below, give a reason why $AB \parallel CD$. Hence, find the values of $\alpha$, $\beta$, $\gamma$ and $\theta$.

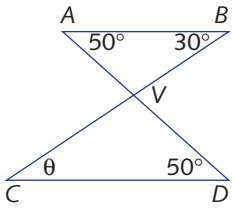
**a**



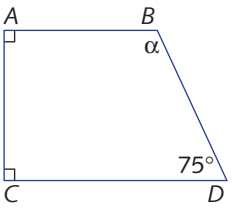

**b**




**c**




**d**




## Chapter 7: Algebra with fractions

**1** Write each of these expressions using algebraic notation. In each part, use $x$ as the pronumeral.

**a** A number is divided by 5, and 6 is added to the result.

**b** Five is added to a number, and the result is divided by 3.

**c** A number is divided by 8, and 5 is subtracted from the result.

**d** 4 is added to a number, and the result is divided by 11.

**e** 12 is subtracted from a number, and the result is divided by 12.

**f** A number is divided by 10, 4 is subtracted from the result, and the result of this is multiplied by 7.

**g** 7 is added to $\frac{5}{8}$ of a number.

**h** 12 is added to a number multiplied by $\frac{7}{8}$.

**i** A number is divided by 8, then 5 is subtracted from the result, then the result of this is multiplied by 9.

**2** For $x = 20$, find the value of:

**a** $\frac{x}{2}$ **b** $\frac{x}{5} + 3$ **c** $\frac{x+2}{3}$

**d** $\frac{x-2}{4}$ **e** $\frac{3(x+50)}{7}$ **f** $\frac{3x}{10} + 10$

**g** $3\left(\frac{x-20}{5}\right)$ **h** $60 - \frac{3x}{5}$ **i** $21 - \frac{360}{x}$

**3** A tank contains $n$ litres of water. 70 litres of water are added to the tank.

**a** How many litres of water are there now in the tank?

**b** The water is divided into 150 containers. How much water is there in each container?


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**4** Write each of the following expressions using algebraic notation.

**a** A number, $a$, is divided by 12, and 5 is subtracted from the result.

**b** 7 is added to a number, $a$, and the result is divided by $\frac{1}{2}$.

**c** 11 is subtracted from a number, $b$, and the result is divided by 5.

**d** A number, $b$, is subtracted from 10, and the result is divided by 15.

**5** Substitute $m = 16$ and $n = 20$ to evaluate:

**a** $\frac{m}{n}$ **b** $\frac{n}{m}$ **c** $\frac{m+n}{4}$ **d** $\frac{n-m}{5}$

**e** $\frac{m}{4} + 10$ **f** $\frac{20}{m} + 3$ **g** $\frac{m+n}{3}$ **h** $\frac{144}{m+n}$

**6** Write each of these expressions using algebraic notation.

**a** A number, $x$, is multiplied by 7 and divided by 2.

**b** A number, $x$, is multiplied by 11 and divided by 3.

**c** $\frac{1}{5}$ of $x$.

**d** A number, $y$, is multiplied by $\frac{3}{4}$.

**e** A number, $y$, is divided by 7, 11 is added to the result, and the result of this is multiplied by 13.

**7** Evaluate each of these expressions for $x = 35$.

**a** $\frac{2x}{3}$ **b** $\frac{3x}{7}$ **c** $\frac{2(x+7)}{5}$

**8** A piece of string is $x$ m in length. It is divided into 7 equal parts.

**a** Find the length of each part in terms of $x$.

**b** Find the length of each part for the following values of $x$.

**i** $x = 21$ **ii** $x = 42$ **iii** $x = 100$

## Chapter 8: Decimals

**1** Write the place value of the 6 in each of the following numbers.

**a** 26.308 **b** 610.495 **c** 1.6 **d** 2.9996

**e** 2.64 **f** 5.126 **g** 4.06 **h** 3.5396

**2** Write the larger number in each pair.

**a** 0.8 or 0.43 **b** 0.7 or 0.96 **c** 3.5 or 3.45

**d** 2.1 or 2.0987 **e** 9.8 or 9.215 23 **f** 10.3 or 10.45

**g** 1.000 03 or 1.03 **h** 4.56 or 4.92 **i** 0.39 or 0.21

**3** Write each set of numbers in order from smallest to largest.

**a** 3.45 3.061 3.059 3.1 3.009

**b** 4.32 4.203 4.222 22 4.0002 4.3


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**4** Express these decimals as fractions, simplifying where possible.

| | | | |
|---|---|---|---|
| **a** 0.3 | **b** 0.25 | **c** 0.6 | **d** 1.87 |
| **e** 4.3456 | **f** 2.05 | **g** 7.5 | **h** 10.306 |
| **i** 0.0004 | **j** 4.007 | **k** 9.9029 | **l** 10.375 |

**5** Express these fractions as decimals.

| | | | |
|---|---|---|---|
| **a** $\frac{3}{4}$ | **b** $\frac{1}{2}$ | **c** $\frac{1}{10}$ | **d** $\frac{45}{100}$ |
| **e** $\frac{6}{100}$ | **f** $\frac{234}{1000}$ | **g** $\frac{2}{5}$ | **h** $3\frac{4}{20}$ |
| **i** $\frac{21}{15}$ | **j** $\frac{2}{3}$ | **k** $\frac{5}{3}$ | **l** $\frac{5}{7}$ |

**6** Complete these calculations.

| | | |
|---|---|---|
| **a** 4.938 + 3.85 | **b** 34.692 − 28.85 | **c** 2.9032 + 83.184 |
| **d** 28.5 × 12 | **e** 234.008 − 198.34 | **f** 78.45 ÷ 0.5 |
| **g** 5.205 × 100 | **h** 67.0296 + 344.3222 | **i** 27.55 ÷ 0.3 |
| **j** 12.0034 + 1.295 773 | **k** 26.093 × 6 | **l** 289.5914 ÷ 0.04 |

**7** Copy and complete the table with the equivalent fraction or decimal.

| | Fraction | Decimal |
|---|---|---|
| **a** | | 0.375 |
| **b** | $\frac{2}{5}$ | |
| **c** | | 0.75 |
| **d** | $2\frac{3}{4}$ | |
| **e** | | 3.125 |

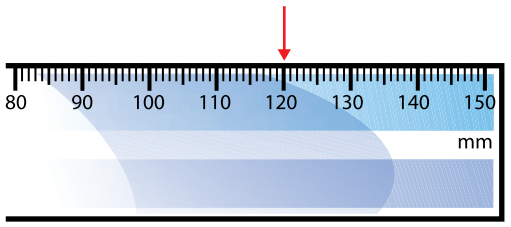
**8** Write each fraction as a recurring decimal.

| | | | | |
|---|---|---|---|---|
| **a** $\frac{1}{6}$ | **b** $\frac{2}{7}$ | **c** $\frac{3}{11}$ | **d** $\frac{5}{6}$ | **e** $\frac{6}{7}$ |

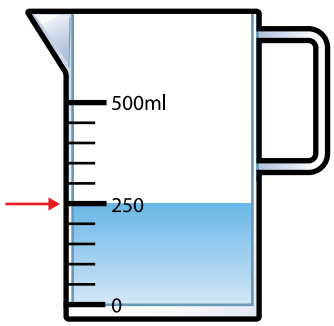
## Chapter 9: Measurement

**1** The diagrams below show the readings for measurements taken using different measuring tools. The arrow indicates what the measurement was in each case. Read each scale and write down the reading. Include the unit of measurement.

**a**



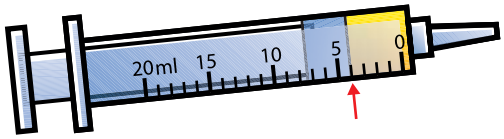

**b**



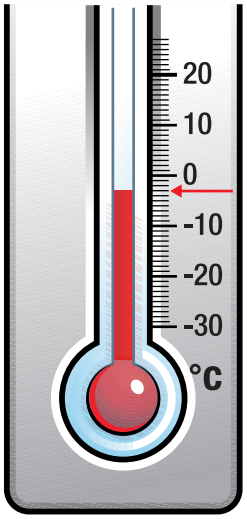



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**c**




**d**




**2** Complete these conversions.

**a** 12 mm = ____ cm

**b** 143 mm = ____ cm or ____ m

**c** 2.3 m = ____ cm or ____ mm

**d** 0.8 km = ____ m

**e** $\frac{2}{5}$ cm = ____ mm

**f** 3 kg = ____ g

**g** 324 g = ____ mg or ____ kg

**h** 25 L = ____ mL

**i** 2305 mL = ____ L

**j** 3.2 L = ____ mL

**k** 0.4 m = ____ cm

**3** Three different packets of muesli were purchased. Packet A contained 785 g, packet B contained 1.2 kg and packet C contained 1050 g. If these were mixed together in one large container, what would be the total mass of the muesli, in both kilograms and grams?

**4** Aled knows that 1 mL of water has a mass of 1 g. If he has the following items in his school bag, which itself weighs 1.2 kg, what is the mass of Aled's bag and its contents?

- Laptop computer (3.4 kg)
- Sandwich (230 g)
- Water bottle (bottle 15 g, 375 mL of water)
- Apple (55 g)
- School books (945 g)
- Pencil case (387 g)

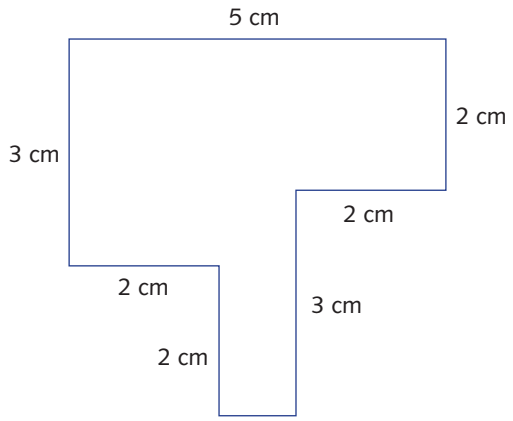
**5** If 4 chocolate blocks cost $9.48, how much do 3 chocolate blocks cost?

**6** If a generator needs 32 L of fuel to run for 5 days, for how long can it run on 112 L?

**7** Which is the best value, 8 apples for $2.72 or 53 apples for $19.61?


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**8** Find the perimeter and area of this figure.




**9** What are the perimeter and area of a rectangle with length and width:

**a** 112 and 18 centimetres?

**b** 5 and 8 metres?

**c** 82 and 34 mm?

**d** $5x$ and $3y$?

**10** Calculate the time elapsed, in hours, minutes and seconds, between:

**a** 6 p.m. on Sunday and 6:25 a.m. on Tuesday

**b** 8:25 a.m. and 3:08 p.m. on the same day

**c** two finish times for a race: 03:34:21 and 04:25:56

**11** What is the time:

**a** 3 hours, 24 minutes and 30 seconds after 2:30 p.m.?

**b** two days, 17 hours and 3 minutes before midnight on Tuesday?

**c** 3 hours, 18 minutes after 11:28 a.m.?

# 10B Problem-solving

## Evens and odds

Two whole numbers are added together. If their sum is odd, which statements below are always true? Which are always false? Which are sometimes true and sometimes false?

**1** Their quotient is not a whole number.

**2** Their product is even.

**3** Their difference is even.

**4** Their product is more than their sum.

**5** If 1 is added to one of the numbers and the product is found, it will be even.


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## The Collatz conjecture

Choose any whole number to start with.

If it is odd, multiply it by 3 and then add 1.

If it is even, divide it by 2.

Then repeat this process on the number just obtained. Keep repeating the procedure.

For example, if you start with 58, the resulting chain of numbers is:

58, 29, 88, 44, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, …

The Collatz conjecture, made by Lothar Collatz in 1937, claims that, if you repeat this process over and over, starting with any whole number greater than zero, eventually you will finish up with the sequence … 4, 2, 1, 4, 2, 1.

A **conjecture** is a statement that is thought to be true but has not been proved mathematically to be true for all cases. Although the Collatz conjecture has been shown to work — often very quickly — for many whole numbers, there are some quite small numbers that take a very long time to come down to … 4, 2, 1, 4, 2, 1.

Apply this process to all the whole numbers greater than zero and less than or equal to 30. For each one, find:

- how many steps it takes to reach 1 the first time
- the largest number in the sequence. (For the sequence above, 58 takes 19 steps and reaches a maximum of 88.)

Look for shortcuts and work with a partner if you like.

## Totient numbers

**1** A **totient number** is the number of fractions between 0 and 1 (not including 0 or 1) for a given denominator that cannot be reduced to a simpler equivalent fraction. The totient number of 2 is 1, since we have $\frac{1}{2}$; of 3 it is 2, since we have $\frac{1}{3}$ and $\frac{2}{3}$; and of 4 it is also 2, since we have $\frac{1}{4}$ and $\frac{3}{4}$ $\left(\frac{2}{4}\right.$ can be reduced to $\left.\frac{1}{2}\right)$. The totient number of 5 is 4, since we have $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$ and $\frac{4}{5}$; and of 6 it is 2, since we have $\frac{1}{6}$ and $\frac{5}{6}$. Find the totient numbers for all denominators up to 12.

**2** For any denominator $n$, there are $n$ fractions between 0 and 1 (including 0 but not 1). Of these fractions, some will be counted towards the totient number of $n$, but others will cancel down and count towards the totient number of one of the factors of $n$. Using this information and the totient numbers from the previous question, calculate the totient numbers for 15, 18, 20 and 24.

**3** The totient number is related to the prime factors of the original number, since these will determine which fractions can be cancelled. Using this information, calculate the totient numbers of 72, 81, 98 and 100.

## Last digits of powers

### Square numbers

Without using a calculator, can you say which of this set of numbers could not be square numbers?

8 116 801, 251 301 659, 3 186 842, 20 720 704


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Yes, you can, just by checking the last digit of each number.

Square numbers can only end in six different digits.

Do a bit of experimentation with a calculator and find the four different digits that square numbers never end in. (This eliminates the third number in this set.)

Now check out the pairs of digits that your odd square numbers end with. What digits are possible in the tens column of an odd square number? (This should eliminate the second number in this set.)

Complete these sentences, using what you have discovered.

- In a square number, the last digit can only be one of ___, ___, ___, ___, ___ or ___.
- The second-last digit of an odd square number is always ___.

### Cube numbers

Cube numbers behave rather differently.

A bit more experimentation will show that a cube number can end in *any* digit. This digit depends on the last digit of the number being cubed.

Complete this table:

| If a number ends in | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| Its cube will end in | | | | | | | | | | |

### Fourth powers

Fourth powers are in fact just square numbers that have been squared.

For example, $7^4 = 7 \times 7 \times 7 \times 7 = 7^2 \times 7^2 = 49 \times 49 = 2401$.

Since $4^2 = 16$ and $9^2 = 81$, the last digit of a fourth power can only be 0, 1, 5 or 6.

### Fifth powers

Fifth powers have a magic of their own. Do a bit of experimentation to find out what it is.

## Obstinate numbers

An odd number can usually be written as the sum of a prime number and a power of two. This is true for all odd numbers greater than 1 and less than 100.

For example, if we choose 23, we can say that it is equal to $19 + 2^2$ (19 is prime and $2^2 = 4$, and $19 + 4 = 23$).

We could just as easily have said that it is $7 + 2^4$, as 7 is prime and $7 + 16 = 23$.

But $21 + 2^1$ and $15 + 2^3$ do not work, as 21 and 15 are not prime numbers.

Some odd numbers can be expressed like this in many ways.

For example, $61 = 59 + 2^1 = 57 + 2^2 = 53 + 2^3 = 29 + 2^5$.

But others are more difficult to express in this way.

Try to find as many pairs as you can for these numbers:

**a** 45 **b** 29 **c** 59 **d** 95


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There are some odd numbers that cannot be expressed as the sum of a prime and a power of two. These have been called **obstinate numbers**.

An example of an obstinate number is 251, as the working out below shows.

$251 - 2^1 = 249 = 3 \times 83$

$251 - 2^2 = 247 = 13 \times 19$

$251 - 2^3 = 243 = 3 \times 81$

$251 - 2^4 = 235 = 5 \times 47$

$251 - 2^5 = 219 = 3 \times 73$

$251 - 2^6 = 187 = 11 \times 17$

$251 - 2^7 = 123 = 3 \times 41$

The next power of 2 is $2^8 = 256$, which is too large to subtract from 251, so 251 is obstinate.

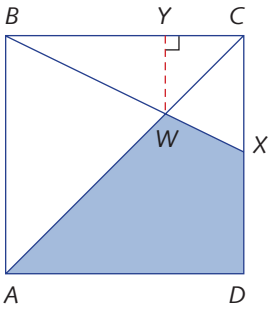
In fact, 251 is the third obstinate number. The first two are between 100 and 150.

Find these two numbers, keeping track of how you eliminated the other 23 odd numbers between 100 and 150.

Remember to be systematic. Making a list of the powers of two up to $2^7 = 128$ might be a good place to start. Look for shortcuts and patterns as you go.

## Areas

**1** $ABCD$ is a square with side length 3 cm. $X$ is the midpoint of side $CD$ and $Y$ is a point on $BC$ such that $BY = 2$ cm.

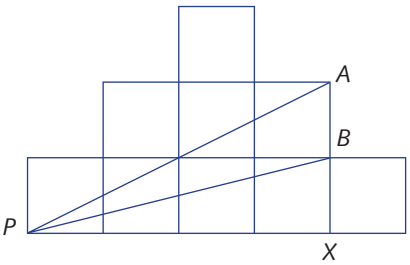



**a** Find the area of triangles $BWA$ and $CWX$.

**b** Find the area of triangle $BAC$ and hence the area of triangle $BWC$.

**c** Find the area of the shaded region.

**2** Each square in the diagram below has side length 4 cm.

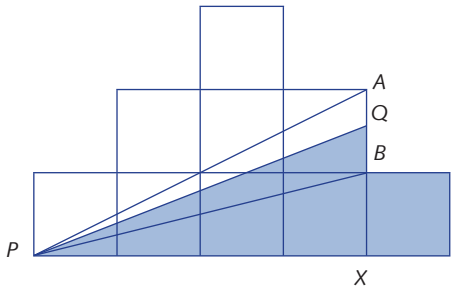



**a** Find the total area of the nine squares.

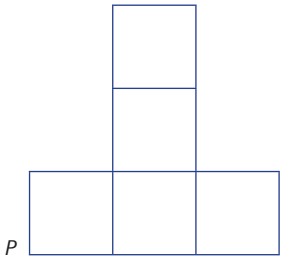
**b** Find the area of triangle $PAX$ and the area of triangle $PBX$.


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**c** In the figure below, find the position of point $Q$ on line interval $AB$ such that the shaded region is half the area of the nine squares.




**3** Draw a line through $P$ that splits the 5 squares below into equal areas.




Prove that your answer is correct.

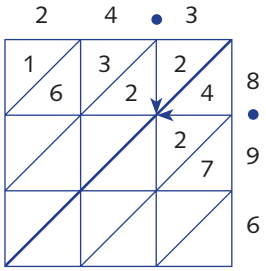
## The grid method of multiplying decimals

Many different ways of multiplying have been used over the centuries.
The **grid** (or **lattice**) **method** was used extensively. Here we adapt it to multiply decimals.

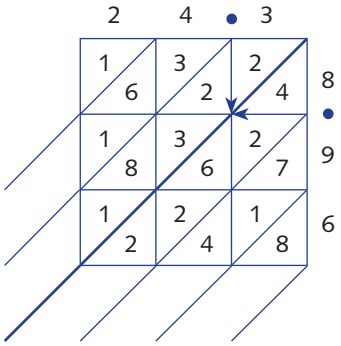
Consider the product $24.3 \times 8.96$. Follow the procedure outlined to fill in the grid. The first few numbers have been entered.

$3 \times 8 = 24$: The 2 is placed in the top position.

$4 \times 8 = 32$: The 3 is placed in the top position.




Notice the arrows in the grid extending inwards from the decimal points. The diagonal on which these arrows meet (shown as a bold line) separates numbers greater than or equal to 1 from numbers less than 1. Now fill in the grid and extend the diagonals.

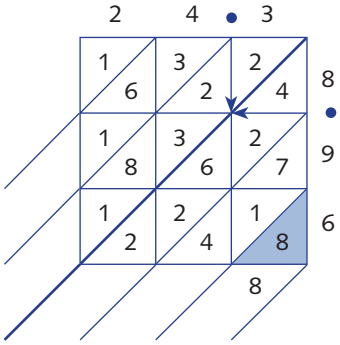




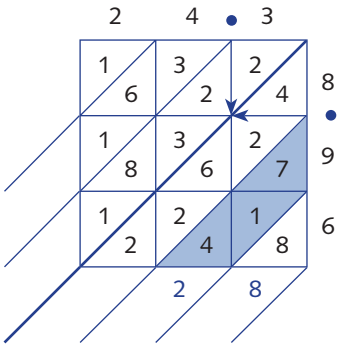
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Next, add along the diagonals, starting at the bottom right. The sum for the first diagonal is 8. This 8 represents $\frac{8}{1000}$.

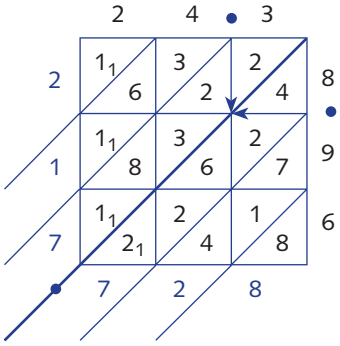
Check your answers using another method for long multiplication.




The sum for the second diagonal is $7 + 1 + 4 = 12$. This 12 represents $\frac{12}{100} = \frac{1}{10} + \frac{2}{100}$. The 1 is carried to the next diagonal.




The grid is now complete. The answer is 217.728. Note that the decimal point in the answer appears at the end of the bold diagonal.




Try these problems using this method.

**1** $0.243 \times 0.896$ **2** $4.567 \times 8.32$ **3** $6.892 \times 5.789$

# 10C Number bases

The number system we use is the **decimal system**. The word *decimal* comes from the Latin word *decem*, which means 'ten'. As we saw in Chapter 1, we can write any number in expanded form using powers of 10. For example:

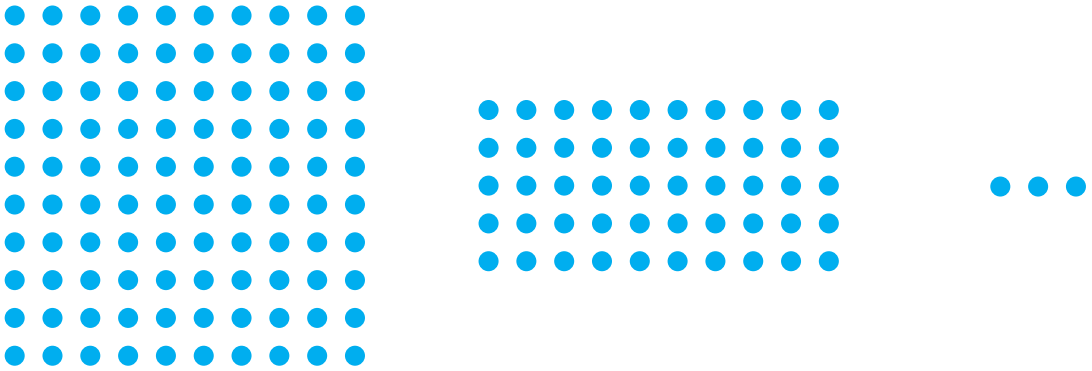
$45 = 4 \times 10 + 5 \times 1$

$3412 = 3 \times 10^3 + 4 \times 10^2 + 1 \times 10 + 2$

Another way to describe this is to say that it is a **place-value system** with **base 10**.


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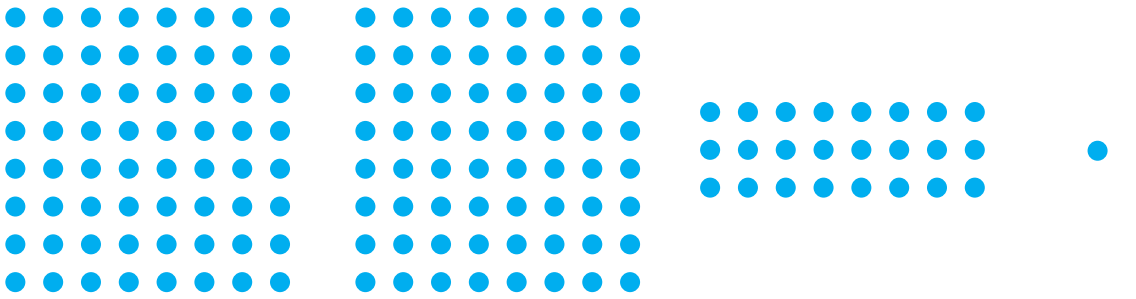
If we have a number of buttons, they can be arranged in groups to show the place values base 10.



$1 \times 10^2$ $\quad$ $5 \times 10$ $\quad$ $3 \times 1$

The number is $1 \times 10^2 + 5 \times 10 + 3 \times 1 = 153$.

Suppose we now use powers of 8 instead of powers of 10. Since $8^2 = 64$ and $8^3 = 512$, we cannot form any groups of $8^3$ out of 153 buttons. Two groups of $8^2$ gives 128. This leaves 25 buttons, which is 3 groups of 8 and 1 single button.



$1 \times 8^2$ $\quad$ $1 \times 8^2$ $\quad$ $3 \times 8$ $\quad$ $1 \times 1$

This can be written as $2 \times 8^2 + 3 \times 8 + 1$.

The number is 231 in base 8. We write this as $231_8$. We read this number as 'two three one, base eight'.

We have shown that $153_{10} = 231_8$.

The largest digit to be used in base $n$ is $n - 1$. For example, in base 10 we use the digits 0 to 9, in base 8 we use the digits 0 to 7, and so on.

**Example 1**

Consider 17 base 10. Rearrange the dots to show the base 8 groupings of dots.



**Solution**

There are 2 lots of 8 and one lot of 1. We can write the number as $21_8$.

$21_8 = 2 \times 8^1 + 1 \times 1$




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### Example 2

Write 39 base 10 in base 4.

### Solution

First note that $4^2 = 16$ and $4^3 = 64$.

There are two lots of 16 in 39, as $2 \times 16 = 32$. This leaves 7, which is one lot of 4 and three lots of 1.

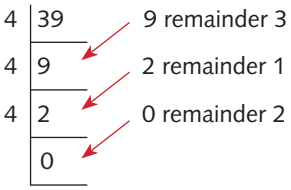
$39 = 2 \times 4^2 + 1 \times 4 + 3$

We can write 39 (base 10) = 213 (base 4) or $39_{10} = 213_4$.

This is read as 'three nine base 10 is equal to two one three, base 4'.

An algorithm for changing base is shown below.

The number in the previous example is used to illustrate this.




The remainders, in reverse order, give the base 4 number ($39_{10} = 213_4$). Can you explain why this works?

### Example 3

Write the base 7 number for:

**a** 27 **b** 52

### Solution

**a** $27 = 3 \times 7 + 6 \times 1$

Therefore, the number is 36 base 7. That is, $27_{10} = 36_7$.

**b** $52 = 1 \times 7^2 + 0 \times 7 + 3 \times 1$

Therefore, the number is 103 base 7. That is, $52_{10} = 103_7$.


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## Exercise 10C

**1** Write the base 4 number for each of the following (which are in base 10).

**a** 34 **b** 56 **c** 78 **d** 100 **e** 130

**2** Write the base 8 number for each of the following (which are in base 10).

**a** 20 **b** 34 **c** 70 **d** 100 **e** 512 **f** 600

**3** Write each of the following as a base 10 number.

**a** $546_7$ **b** $323_4$ **c** $201_3$ **d** $121_{16}$

# 10D Binary numbers

Base 2 is a very useful base. Computers use base 2 for their calculations because a logic, digital or computer switch (like a light switch) usually has only two positions: 'on' and 'off'. 'On' can be represented by 1, and 'off' by 0.

Base 2 has a special name: the **binary system**. It has only 2 digits: 0 and 1.

For example:

$101\text{ (base 2)} = 1 \times 2^2 + 0 \times 2 + 1 = 5\text{ (base 10) or } 101_2 = 5_{10}$

$100111_2 = 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2 + 1 = 39\text{ (base 10) or } 10111_2 = 39_{10}$

### Example 4

Write 24 in binary form.

**Solution**

$24 = 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 0 = 11000\text{ (base 2)}$

## Converting from binary to decimal

Consider the binary number $10101_2$.

| $2^4$ | $2^3$ | $2^2$ | 2 | units |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 1 |

$10101_2 = 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2 + 1 = 21_{10}$


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## Addition and subtraction

The setting out for binary addition and subtraction is the same as for base 10 addition and subtraction.

The following are a few simple additions:

$1_2 + 1_2 = 10_2$

$1_2 + 1_2 + 1_2 = 11_2$

$10_2 + 10_2 = 100_2$

$10_2 + 10_2 + 1_2 = 101_2$

$10_2 + 10_2 + 10_2 = 110_2$

### Example 5

Find:

**a** $11_2 + 101_2$

**b** $1011_2 + 11111_2$

#### Solution

**a**
$$\begin{array}{r} {\scriptstyle 1\ 1\ \ \ } \\ 1\ 1\ 0\ 1 \\ +\ \ 1\ 0\ 1 \\ \hline 1\ 0\ 0\ 0 \end{array}$$

**b**
$$\begin{array}{r} {\scriptstyle 1\ 1\ 1\ 1\ 1\ } \\ 1\ 0\ 1\ 1\ 1\ 1 \\ +\ \ \ \ 1\ 0\ 1\ 1\ 1 \\ \hline 1\ 0\ 1\ 0\ 1\ 0 \end{array}$$

The 'carry' process follows the same principle as is used in base 10.

So $11_2 + 101_2 = 1000_2$ and $11111_2 + 1011_2 = 101010_2$.

### Example 6

Find $1101_2 - 11_2$.

#### Solution

$$\begin{array}{r} 1\ 1^{1}\ 0\ 1 \\ -\ \ 1\ 1 \\ \hline 1\ 0\ 1\ 0 \end{array} \quad \text{or} \quad \begin{array}{r} 1\ {}^{0}\!\not{1}^{1}\ 0\ 1 \\ -\ \ 1\ 1 \\ \hline 1\ 0\ 1\ 0 \end{array}$$

That is, $1101_2 - 11_2 = 1010_2$.



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**Example 7**

Calculate $1101_2 + 11101_2 + 10011_2$.

**Solution**

$$\begin{array}{r} \phantom{+}\ 1\ 1\ 0\ 1 \\ \phantom{+}\ 1\ 1\ 1\ 0\ 1 \\ +\ 1_1 0_1 0_1 1_1 1 \\ \hline 1\ 1\ 1\ 1\ 0\ 1 \end{array}$$

That is, $1101_2 + 11101_2 + 10011_2 = 111101_2$.

## Exercise 10D

**1** Write the binary number for each of the following numbers, which are in base 10.

**a** 5 **b** 10 **c** 12 **d** 16 **e** 45

**2** Write the base 10 number for each of the following numbers, which are in base 2.

**a** 1001 **b** 111 **c** 1011 **d** 1100 **e** 1001001

**3** Find the result of each of the following additions in the binary system.

**a** 11 + 101 **b** 1111 + 101 **c** 1010 + 11 **d** 1010 + 1010

**4** Find the result of each of the following subtractions in the binary system.

**a** 11101 − 101 **b** 11011 − 1100 **c** 111 − 11

**d** 1001 − 111 **e** 110011 − 1111

**5** Find the result of each of the following additions in the binary system.

**a**
$$\begin{array}{r} 1\ 1\ 1 \\ +\ 1\ 1\ 1 \\ \hline 1\ 1\ 1 \end{array}$$

**b**
$$\begin{array}{r} 1\ 0\ 1\ 0 \\ +\ \ 1\ 0\ 1 \\ \hline 1\ 1\ 1\ 1 \\ \hline \end{array}$$

**c**
$$\begin{array}{r} 1\ 1\ 1\ 1 \\ +\ \ 1\ 1\ 1 \\ \hline 1\ 0\ 0\ 0 \\ \hline \end{array}$$

**6** A binary number is added to 10100 to give 111111. Find this binary number.

**7** A binary number is added to 111 to give 1000. Find this binary number.


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CHAPTER

# 11

Number and Algebra

# Integers

You have probably come across examples of negative numbers already. These are the numbers that are less than zero. They are necessary in all sorts of situations. For example, they are used in the measurement of temperature.

The temperature 0°C is the temperature at which water freezes, known as the *freezing point*. The temperature that is 5 degrees colder than freezing point is written as –5°C.

In some Australian cities, the temperature drops below zero. Canberra's lowest recorded temperature is –10°C. The lowest temperature ever recorded in Australia is –23°C, at Charlotte's Pass in NSW. Here are the lowest recorded temperatures at some other places:

| | |
|---|---|
| Alice Springs | –7°C |
| Paris | –24°C |
| London | –21°C |

Negative numbers are also used to record heights below sea level.

For example, the surface of the Dead Sea in Israel is 424 metres below sea level. This can be written as –424 metres. This is the lowest point on land anywhere on Earth. The lowest point on land in Australia is at Lake Eyre, which is 15 metres below sea level. This is written as –15 metres.

*continued over page*



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Brahmagupta, an Indian mathematician, wrote important works on mathematics and astronomy, including a work called *Brahmasphutasiddhanta* (*The Opening of the Universe*), which he wrote in the year 628 CE. This book is believed to mark the first appearance of negative numbers. Brahmagupta gives the following rules for positive and negative numbers in terms of fortunes (positive numbers) and debts (negative numbers). By the end of this chapter, you will be able to understand his words.

A debt subtracted from zero is a fortune.
A fortune subtracted from zero is a debt.
The product of zero multiplied by a debt or fortune is zero.
The product of zero multiplied by zero is zero.
The product or quotient of two fortunes is a fortune.
The product or quotient of two debts is a fortune.
The product or quotient of a debt and a fortune is a debt.
The product or quotient of a fortune and a debt is a debt.

# 11A Negative integers

The whole numbers, together with the negative whole numbers, are called the **integers**. These are:

…, –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, …

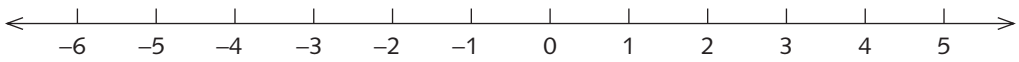
The numbers 1, 2, 3, 4, 5, … are called the **positive integers**.

The numbers …, –5, –4, –3, –2, –1 are called the **negative integers**.

The number 0 is neither positive nor negative.

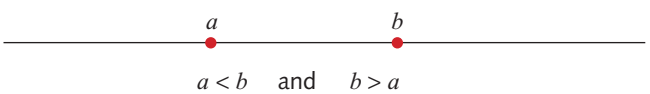
## The number line

The integers can be represented by points on a **number line**. The line is infinite in both directions, with the positive integers to the right of zero and the negative integers to the left of zero. The integers are equally spaced.




An integer $a$ is **less than** another integer $b$ if $a$ lies to the left of $b$ on the number line. The symbol $<$ is used for **less than**. For example, $-3$ is to the left of $-1$, so $-3 < -1$.

An integer $b$ is **greater than** another integer $a$ if $b$ lies to the right of $a$ on the number line. The symbol $>$ is used for **greater than**. For example, 1 is to the right of $-5$, so $1 > -5$.




$a < b$ and $b > a$

A practical illustration of this is that a temperature of $-8°$C is colder than a temperature of $-3°$C, and $-8 < -3$. Also, $0°$ is warmer than $-5°$C, and $0 > -5$.


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### Example 1

List all the integers less than 5 and greater than –3.

#### Solution

–2, –1, 0, 1, 2, 3, 4

### Example 2

**a** Arrange the following integers in increasing order.
–6, 6, 0, 100, –1000, –5, –100, 8

**b** Arrange the following integers in decreasing order.
–25, 1000, –500, –26, 53, 100, 56

#### Solution

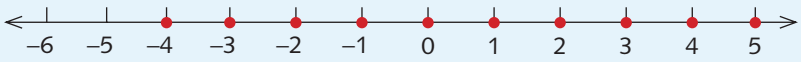
**a** –1000, –100, –6, –5, 0, 6, 8, 100

**b** 1000, 100, 56, 53, –25, –26, –500

### Example 3

Draw a number line and mark on it with dots all the integers less than 6 and greater than –5.

#### Solution



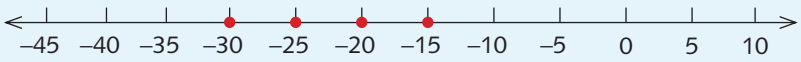

### Example 4

**a** The sequence 10, 5, 0, –5, –10, … is 'going down by fives'. Write down the next four numbers, and mark them on the number line.

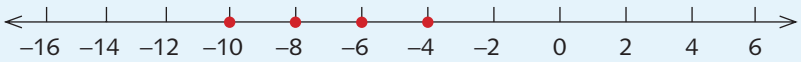
**b** The sequence –16, –14, –12, … is 'going up by twos'. Write down the next four numbers, and mark them on the number line.

#### Solution

**a** The next four numbers are –15, –20, –25, –30.




**b** The next four numbers are –10, –8, –6, –4.



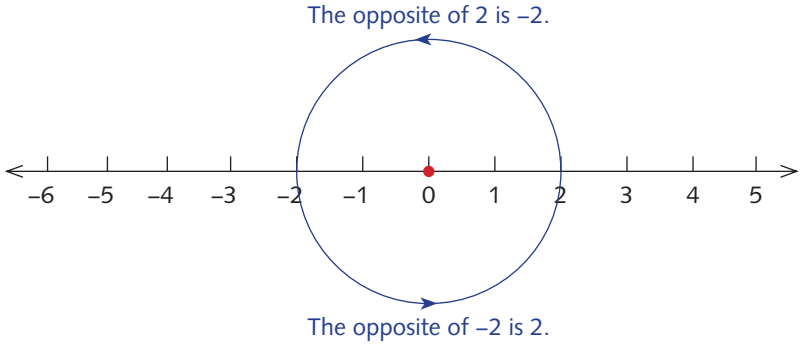


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## The opposite of an integer

The number –2 is the same distance from 0 as 2, but lies on the opposite side of zero. We call –2 the **opposite** of 2. Similarly, the opposite of –2 is 2.

The operation of forming opposites can be visualised by putting a pin in the number line at 0 and rotating the number line by 180°.




Notice that the opposite of the opposite is the number we started with. For example, $-(-2) = 2$.

*Note*: The number 0 is the opposite of itself. That is, $-0 = 0$. No other number has this property.

## Exercise 11A

Example 1

**1** **a** List the integers less than 3 and greater than –5.

**b** List the integers greater than –8 and less than –1.

**c** List the integers less than –4 and greater than –10.

**d** List the integers greater than –132 and less than –123.

Example 2

**2** **a** Arrange the following integers in increasing order.
–10, 10, 0, 100, –100, –6, –1000, 5

**b** Arrange the following integers in decreasing order.
–30, 45, –45, –550, –31, 26, –26, 55

Example 3

**3** **a** Draw a number line and mark on it the numbers –2, –4, –6 and –8.

**b** Draw a number line and mark on it the numbers –1, –3, –5 and –7.

**c** Draw a number line and mark on it the integers less than 0 and greater than –8.

**d** Draw a number line and mark on it the integers less than 3 and greater than –3.

Example 4

**4** The sequence –15, –13, –11, … is 'going up by twos'. Write down the next three terms. (Draw a number line to help you.)

**5** The sequence 3, 1, –1, … is 'going down by twos'. Write down the next three terms. (Draw a number line to help you.)

**6** The sequence –50, –45, –40, … is 'going up by fives'. Write down the next three terms. (Draw a number line to help you.)


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**7** Give the opposite of each integer.

**a** 5 **b** –4 **c** –10 **d** –12

**e** 7 **f** –8 **g** –4 **h** –3

**8** Simplify:

**a** –(–2) **b** –(–7) **c** –(–20)

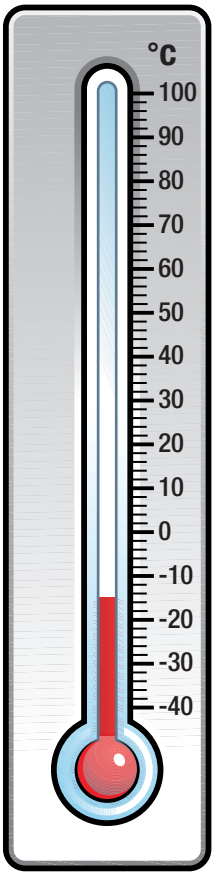
**d** –(–(–10)) **e** –(–(–30)) **f** –(–(–(–40)))

**9** Insert the symbol > or < in each box to make a true statement.

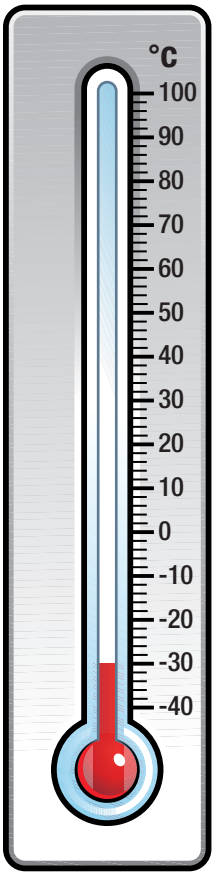
**a** 3 ☐ 5 **b** 3 ☐ –5 **c** –7 ☐ –4 **d** 2 ☐ – (–3)

**10** Write down the reading for each thermometer shown below.

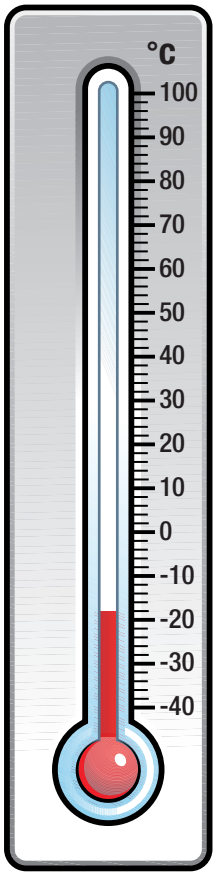
**a**



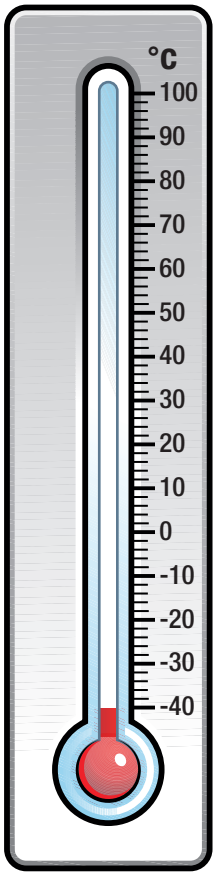

**b**




**c**




**d**






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# 11B Addition and subtraction of a positive integer

If a submarine is at a depth of −250 m and then rises by 20 m, its final position is −230 m. This can be written −250 + 20 = −230.

Joseph has $3000 and he spends $5000. He now has a debt of $2000, so it is natural to interpret this as 3000 − 5000 = −2000.

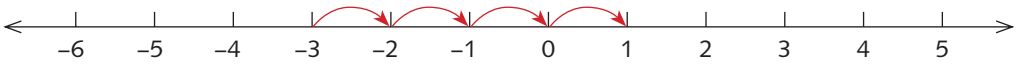
These are examples of adding and subtracting a positive integer.

## The number line and addition

The number line provides a useful picture for addition and subtraction of integers.

## Addition of a positive integer

When you add a positive integer, move to the *right* along the number line.




For example, to calculate −3 + 4, start at −3 and move 4 steps to the right. We see that −3 + 4 = 1.

We can interpret −3 + 4 in terms of money:

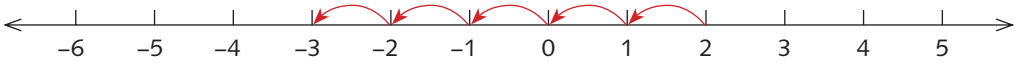
> I start with a debt of $3 but I then earned $4. I now have $1.

## Subtraction of a positive integer

We will start by thinking of subtraction as **taking away**.

When you subtract a positive integer, move to the *left* along the number line.

For example, to calculate 2 − 5, start at 2 and move to the left 5 steps. We see that 2 − 5 = −3.




We can interpret 2 − 5 in terms of money:

> I had $2 and I spent $5. I now have a debt of $3.


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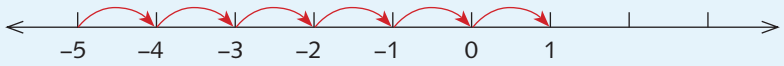
### Example 5

Write the answers to these additions.

**a** $-5 + 6$ **b** $-7 + 12$ **c** $-11 + 20$

### Solution

**a**




$-5 + 6 = 1$ (Start at $-5$ on the number line and move 6 steps to the right.)

**b** $-7 + 12 = 5$ (Start at $-7$ on the number line and move 12 steps to the right.)

**c** $-11 + 20 = 9$ (Start at $-11$ on the number line and move 20 steps to the right.)

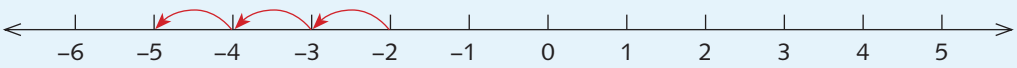
### Example 6

Find the value of:

**a** $-2 - 3$ **b** $6 - 9$ **c** $-4 - 11$ **d** $3 - 12 - 8$

### Solution

**a**




Start at $-2$ and move 3 steps to the left. We see that $-2 - 3 = -5$.

**b** $6 - 9 = -3$ (Start at 6 and move 9 steps to the left.)

**c** $-4 - 11 = -15$ (Start at $-4$ and move 11 steps to the left.)

**d** $3 - 12 - 8 = -17$ (Start at 3 and move 12 steps to the left and then 8 steps to the left.)

## Exercise 11B

You may wish to visualise these calculations on a number line.

Example 5

**1** Calculate these additions.

**a** $-5 + 7$ **b** $-2 + 3$ **c** $-5 + 10$ **d** $-1 + 4$

**e** $-12 + 16$ **f** $-5 + 2$ **g** $-6 + 12$ **h** $-5 + 10$

**i** $-11 + 4$ **j** $-12 + 4$ **k** $-8 + 10$ **l** $-1 + 12$

Example 6

**2** Calculate these subtractions.

**a** $5 - 6$ **b** $6 - 12$ **c** $-5 - 10$ **d** $-11 - 10$

**e** $-7 - 16$ **f** $-5 - 2$ **g** $-6 - 2$ **h** $5 - 10$

**i** $-11 - 4$ **j** $-12 - 5$ **k** $-10 - 9$ **l** $-5 - 8$



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Example 6d

**3** Work from left to right to calculate the answer.

| | | | |
|---|---|---|---|
| **a** $15 - 6 - 8$ | **b** $6 - 12 - 5$ | **c** $-8 - 10 - 11$ | **d** $-11 + 10 - 20$ |
| **e** $-7 - 16 - 20$ | **f** $5 - 2 - 10$ | **g** $-6 - 2 - 20$ | **h** $5 - 10 + 20$ |
| **i** $-11 - 4 - 30$ | **j** $-12 - 5 + 20$ | **k** $-20 - 30 - 10$ | **l** $-5 + 6 - 7$ |

**4** Work from left to right to calculate the answer.

| | | |
|---|---|---|
| **a** $11 - 10 - 20 - 15$ | **b** $-2 - 3 - 4 - 5$ | **c** $20 - 9 - 7 - 4$ |
| **d** $-11 + 1 + 2 + 8 + 1$ | **e** $-20 - 2 - 4 + 6$ | |

**5** **a** Johanne has a total amount of \$3400 and spends \$5000. What is Johanne's debt?

**b** Francis has a debt of \$4670, but earns \$3456 and pays off a portion of the debt. How much does Francis owe now?

**c** David has a debt of \$3760, but earns \$4000 and pays off the debt. How much does David have now?

**6** **a** A submarine is at a depth of $-320$ m and then rises by 40 m. What is the new depth of the submarine?

**b** The temperature in a freezer is $-17°C$. The freezer is turned off and in 10 minutes the temperature has risen by $8°C$. What is the temperature of the freezer now?

**7** Place either a plus (+) or a minus (−) sign in each box to make these statements true.

**a** $1 \square 2 \square 3 \square 4 = -2$

**b** $3 \square 10 \square 9 \square 5 = 7$

**c** $1 \square 2 \square 3 \square 4 \square 5 = -1$

# 11C Addition and subtraction of a negative integer

In the previous section, we considered addition and subtraction of a positive integer. In this section, we will add and subtract negative integers.

## Addition of a negative integer

Adding a negative integer to another integer means that you take a certain number of steps to the *left* on a number line.

The result of the addition $4 + (-6)$ is the number you get by moving 6 steps to the left, starting at 4.



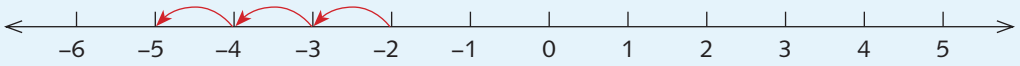

$4 + (-6) = -2$


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**Example 7**

Find $-2 + (-3)$.

**Solution**




$-2 + (-3)$ is the number you get by moving 3 steps to the left, starting at $-2$. That is, $-5$.

Notice that $-2 - 3$ is also equal to $-5$.

All additions of this form can be completed in a similar way. For example:

$4 + (-7) = -3$ and note that $4 - 7 = -3$

$-11 + (-3) = -14$ and note that $-11 - 3 = -14$

This suggests the following rule.

**To add a negative integer, subtract its opposite**

For example:

$4 + (-10) = 4 - 10$
$= -6$

$-7 + (-12) = -7 - 12$
$= -19$

## Subtracting a negative integer

We have already seen that adding $-2$ means taking 2 steps to the left. For example:




$7 + (-2) = 5$

We want subtracting $-2$ to be the reverse of the process of adding $-2$. So to subtract $-2$, we take 2 steps to the *right*. For example:




$7 - (-2) = 9$

There is a very simple way to state this rule:

**To subtract a negative number, add its opposite**

For example:

$7 - (-2) = 7 + 2$
$= 9$


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## Example 8

Evaluate:

**a** $12 + (-3)$ **b** $-3 + (-7)$ **c** $6 - (-18)$

**d** $-12 - (-6)$ **e** $4 - (-15)$ **f** $-25 - (-3)$

### Solution

**a** $12 + (-3) = 12 - 3$
$= 9$

**b** $-3 + (-7) = -3 - 7$
$= -10$

**c** $6 - (-18) = 6 + 18$
$= 24$

**d** $-12 - (-6) = -12 + 6$
$= -6$

**e** $4 - (-15) = 4 + 15$
$= 19$

**f** $-25 - (-3) = -25 + 3$
$= -22$

## Example 9

Calculate:

**a** $6 - (-3) + (-8)$ **b** $-14 + (-7) - (-15)$

### Solution

**a** $6 - (-3) + (-8) = 6 + 3 - 8$
$= 9 - 8$
$= 1$

**b** $-14 + (-7) - (-15) = -14 - 7 + 15$
$= -21 + 15$
$= -6$

## Example 10

The temperature on Saturday at 1 a.m. was $-13°C$ and the temperature at 2 p.m. was $-2°C$. Calculate the rise in temperature.

### Solution

Rise in temperature $= -2 - (-13)$
$= -2 + 13$
$= 11°C$


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## Exercise 11C

Example 7, 8a, b

**1** Find:

**a** $5 + (-2)$ **b** $-6 + 2$ **c** $-5 + 10$ **d** $-11 + (-4)$

**e** $-12 + 16$ **f** $-5 + (-2)$ **g** $-6 + (-2)$ **h** $-5 + (-10)$

**i** $11 + (-4)$ **j** $-12 + 4$ **k** $-20 + (-30)$ **l** $-110 + 100$

Example 8c, d, e, f

**2** Find:

**a** $5 - (-6)$ **b** $6 - (-12)$ **c** $-5 - (-10)$

**d** $11 - (-4)$ **e** $-12 - (-16)$ **f** $-5 - (-2)$

**g** $-6 - (-2)$ **h** $5 - (-10)$ **i** $-11 - 4$

**j** $-12 - (-4)$ **k** $-15 - (-20)$ **l** $-30 - (-100)$

Example 9

**3** Evaluate:

**a** $15 - 26 + (-25)$ **b** $-10 - 12 + 8$ **c** $-39 + 54 - 1$

**d** $31 - 41 - (-9)$ **e** $6 + 12 - 16$ **f** $-28 - (-35) - (-2)$

**g** $-36 - 17 + 26$ **h** $5 - (-21) + 45$ **i** $16 + (-4) - (-4)$

**j** $-92 + 54 - (-82)$ **k** $-900 + 1000 - (-100)$ **l** $-500 + 2000 - (-50)$

**4** Write the answers to these subtractions.

**a** $23 - (-20)$ **b** $-78 - (-56)$ **c** $-65 - (-78)$

**5** Evaluate:

**a** $45 - 50$ **b** $30 - (-5)$ **c** $60 - (-5)$

**d** $4 - 11 - 21 + 40$ **e** $12 - 20 + 30$ **f** $7 - 10 - 20$

**g** $7 - (-15) + 20$ **h** $-11 - 10 - (-4)$ **i** $-30 + 50 - 45 - (-6)$

**j** $-34 + 60 - (-5) + 10$ **k** $43 + 50 - (-23)$ **l** $-10 - 45 + 30$

Example 10

**6** The temperature in Moscow on a winter's day went from a minimum of $-19°C$ to a maximum of $-2°C$. By how much did the temperature rise?

**7** The temperature in Ballarat on a very cold winter's day went from $-3°C$ to $7°C$. What was the rise in temperature?

**8** The table below shows temperatures at 5 a.m. and 2 p.m. for a number of cities. Complete the table.

| Temperature at 5 a.m. | Temperature at 2 p.m. | Rise in temperature (°C) |
|---|---|---|
| −10 | 5 | |
| −15 | 5 | |
| −25 | −3 | |
| −20 | −15 | |
| −7 | −5 | |
| −11 | 2 | |
| −13 | 5 | |


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**9** A meat pie in the microwave rises in temperature by about 9°C for each minute of heating. If you take a frozen meat pie out of the freezer, where it has been stored at −14°C, how long does it have to be in the microwave before it reaches 40°C?

**10** The temperature in Canberra on a very cold day went from 11°C to −3°C. What was the change in temperature?

**11** The table below shows the temperatures inside and outside a building on different days.

| Day | Temperature inside (°C) | Temperature outside (°C) |
|---|---|---|
| M | 20 | 25 |
| T | 13 | 18 |
| W | 24 | 20 |
| T | 10 | −5 |
| F | −5 | −10 |
| S | 3 | −6 |

For each day, calculate (Temperature inside − Temperature outside). What does it mean if the result of this calculation is negative?

**12** Jane has just received her first credit card, and has already used it to buy some clothes. The balance is −$140. She spends another $70 at the grocery store the next day. At the end of the week, she will be paid $280. If she uses this to pay off her credit card, how much will Jane have left?

**13** Find $1 - 2 + 3 - 4 + 5 - 6 + \ldots\ldots + 99 - 100$.

# 11D Multiplication involving negative integers

## Multiplication with negative integers

$5 \times (-3)$ means 5 lots of −3 added together. That is:

$$\begin{aligned} 5 \times (-3) &= (-3) + (-3) + (-3) + (-3) + (-3) \\ &= -15 \end{aligned}$$

Just as $8 \times 6 = 6 \times 8$, we will take $-3 \times 5$ to be the same as $5 \times (-3)$.

All products such as $5 \times (-3)$ and $-3 \times 5$ are treated in the same way.

For example:

$$\begin{aligned} -6 \times 3 &= 3 \times (-6) \\ &= -18 \end{aligned} \qquad \begin{aligned} -15 \times 4 &= 4 \times (-15) \\ &= -60 \end{aligned}$$

The question remains as to what we might mean by multiplying two negative integers together. We first investigate this by looking at a multiplication table.

In the left-hand column below, we are taking multiples of 5. The products go down by 5 each time.


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In the right-hand column, we are taking multiples of –5. The products go up by 5 each time.

| | |
|---|---|
| $2 \times 5 = 10$ | $2 \times (-5) = -10$ |
| $1 \times 5 = 5$ | $1 \times (-5) = -5$ |
| $0 \times 5 = 0$ | $0 \times (-5) = 0$ |
| $-1 \times 5 = -5$ | $-1 \times (-5) = ?$ |
| $-2 \times 5 = -10$ | $-2 \times (-5) = ?$ |

The pattern suggests that it would be natural to take $-1 \times (-5)$ to equal 5 and $-2 \times (-5)$ to equal 10 so that the pattern continues in a natural way.

All products such as $-5 \times (-2)$ and $-5 \times (-1)$ are treated in the same way. For example:

$-6 \times (-2) = 12$ $\qquad$ $-3 \times (-8) = 24$

We have the following rules.

### The sign of the product of two integers

- The product of two positive numbers is a positive number.
- The product of a negative number and a positive number is a negative number. For example:

$$\begin{aligned} -4 \times 7 &= -(4 \times 7) \\ &= -28 \end{aligned}$$

- The product of two negative numbers is a positive number. For example:

$$\begin{aligned} -4 \times (-7) &= 4 \times 7 \\ &= 28 \end{aligned}$$

### Example 11

Evaluate each of these products.

**a** $3 \times (-20)$ **b** $-6 \times 10$ **c** $-25 \times (-30)$

**d** $15 \times (-40)$ **e** $-12 \times 8 \times 2$ **f** $-4 \times (-8) \times (-3)$

### Solution

**a** $3 \times (-20) = -60$

**b** $-6 \times 10 = -60$

**c** $\begin{aligned} -25 \times (-30) &= 25 \times 30 \\ &= 750 \end{aligned}$

**d** $15 \times (-40) = -600$

**e** $\begin{aligned} -12 \times 8 \times 2 &= (-96) \times 2 \\ &= -192 \end{aligned}$

**f** $\begin{aligned} -4 \times (-8) \times (-3) &= 32 \times (3) \\ &= -96 \end{aligned}$


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## Exercise 11D

**1** Calculate each multiplication.

**a** $-1 \times 5$ **b** $6 \times (-1)$ **c** $-6 \times (-1)$ **d** $16 \times (-1)$

**e** $0 \times (-3)$ **f** $-3 \times (-2)$ **g** $6 \times (-3)$ **h** $-3 \times 8$

**i** $-7 \times 8$ **j** $-9 \times 0$ **k** $-2 \times (-5)$ **l** $-4 \times 6$

Example 11

**2** Calculate each multiplication.

**a** $5 \times (-2)$ **b** $6 \times (-2)$ **c** $5 \times (-1)$ **d** $11 \times (-4)$

**e** $12 \times (-16)$ **f** $-5 \times 2$ **g** $-6 \times 2$ **h** $-5 \times 10$

**i** $-11 \times 4$ **j** $-12 \times 4$ **k** $-20 \times (-6)$ **l** $16 \times (-3)$

**m** $-7 \times (-18)$ **n** $-13 \times (-13)$ **o** $-19 \times 8$ **p** $15 \times (-4)$

**q** $-17 \times (-9)$ **r** $-6 \times (-17)$ **s** $-14 \times 20$ **t** $-12 \times (-15)$

**3** Evaluate:

**a** $3 \times (-2) \times (-6)$ **b** $-4 \times (-7) \times (-6)$ **c** $60 \times (-4) \times (-10)$

**d** $-45 \times (-7) \times 20$ **e** $45 \times (-3) \times (-20)$ **f** $-34 \times (-3) \times (-2)$

**g** $-3 \times (-1) \times (-4)$ **h** $6 \times (-3) \times (-1)$ **i** $-1 \times (-1) \times (-6)$

**4** Copy and complete these multiplications.

**a** $2 \times \ldots = -30$ **b** $5 \times \ldots = -65$ **c** $-7 \times \ldots = 42$ **d** $\ldots \times (-8) = 56$

**e** $5 \times \ldots = -30$ **f** $\ldots \times (-6) = 30$ **g** $-7 \times \ldots = -84$ **h** $(-1) \times \ldots = -8$

**i** $\ldots \times (-9) = 0$ **j** $-8 \times \ldots = -96$ **k** $\ldots \times 3 = -81$ **l** $-5 \times \ldots = 80$

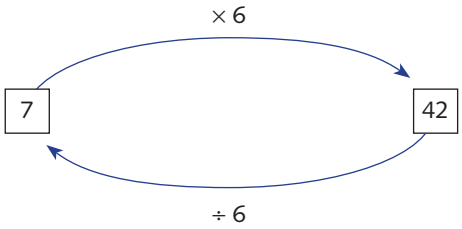
# 11E Division involving negative integers

### Division with negative integers

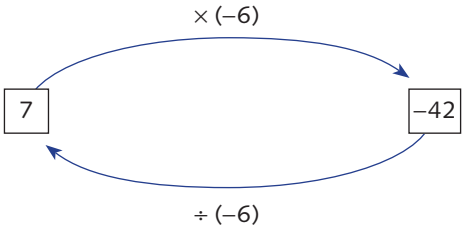
Every multiplication statement, for non-zero numbers, has equivalent division statements. For example, $7 \times 3 = 21$ is equivalent to $21 \div 3 = 7$ or $21 \div 7 = 3$. We will use this fact to establish the rules for division involving integers.

Here are some more examples:

$7 \times 6 = 42$ is equivalent to $42 \div 6 = 7$



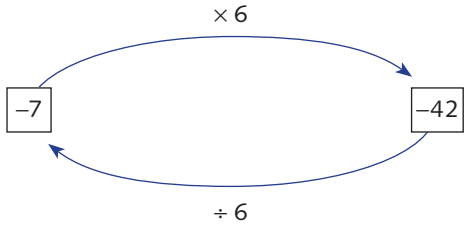

$7 \times (-6) = -42$ is equivalent to $-42 \div (-6) = 7$



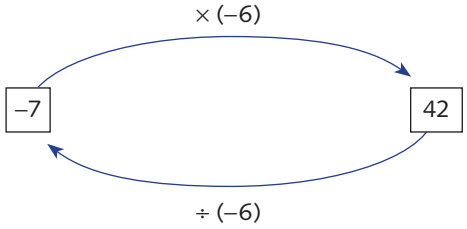


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$-7 \times 6 = -42$ is equivalent to $-42 \div 6 = -7$




$-7 \times (-6) = 42$ is equivalent to $42 \div (-6) = -7$




## The sign of the quotient of two integers

- The quotient of a positive number and a negative number is a negative number. For example: $28 \div (-7) = -4$
- The quotient of a negative number and a positive number is a negative number. For example: $-28 \div 7 = -4$
- The quotient of two negative numbers is a positive number. For example: $-28 \div (-7) = 4$

Notice that the rules for the sign of a quotient are the same as the rules for the sign of a product.

### Example 12

Evaluate each of these divisions.

**a** $-45 \div 9$ **b** $-20 \div (-4)$ **c** $63 \div (-9)$

#### Solution

**a** $-45 \div 9 = -5$ **b** $-20 \div (-4) = 5$ **c** $63 \div (-9) = -7$

As before, we use another way of writing division. For example, $-16 \div 2$ can be written as $\frac{-16}{2}$.

### Example 13

Evaluate:

**a** $\frac{-45}{9}$ **b** $\frac{-36}{-4}$ **c** $\frac{60}{-12}$

#### Solution

**a** $\frac{-45}{9} = -5$ **b** $\frac{-36}{-4} = 9$ **c** $\frac{60}{-12} = -5$


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# Exercise 11E

Example 12

**1** Calculate each division.

**a** $-15 \div 3$ **b** $-26 \div 2$ **c** $-35 \div 7$ **d** $-21 \div 3$

**e** $-120 \div 3$ **f** $15 \div (-3)$ **g** $36 \div (-2)$ **h** $45 \div (-5)$

**i** $21 \div (-7)$ **j** $456 \div (-1)$ **k** $-51 \div (-3)$ **l** $-72 \div (-12)$

**m** $-100 \div (-50)$ **n** $-121 \div 11$ **o** $-64 \div (-4)$ **p** $-144 \div (-6)$

**q** $-39 \div (-13)$ **r** $-500 \div (-10)$ **s** $-162 \div 6$ **t** $-396 \div 11$

Example 13

**2** Evaluate:

**a** $\frac{5}{-1}$ **b** $\frac{-5}{-1}$ **c** $\frac{6}{-2}$ **d** $\frac{8}{-4}$

**e** $\frac{-1}{-1}$ **f** $\frac{1}{-1}$ **g** $\frac{-50}{-1}$ **h** $\frac{-2}{1}$

**i** $\frac{-10}{2}$ **j** $\frac{12}{-3}$ **k** $\frac{-9}{-3}$ **l** $\frac{-6}{6}$

**3** Calculate each division.

**a** $\frac{-48}{-12}$ **b** $\frac{-52}{13}$ **c** $\frac{-60}{12}$ **d** $\frac{-112}{-8}$

**e** $\frac{132}{-4}$ **f** $\frac{-600}{5}$ **g** $\frac{-225}{15}$ **h** $\frac{292}{-4}$

**i** $\frac{-80}{10}$ **j** $\frac{696}{-24}$ **k** $\frac{-196}{14}$ **l** $\frac{-1000}{-100}$

**m** $\frac{-144}{6}$ **n** $\frac{256}{-4}$ **o** $\frac{-98}{-7}$ **p** $\frac{-288}{-16}$

**4** Copy and complete these divisions.

**a** $50 \div \ldots = -10$ **b** $-45 \div \ldots = 9$ **c** $-312 \div \ldots = -3$

**d** $\ldots. \div (-13) = -13$ **e** $\ldots. \div (-20) = 20$ **f** $\ldots. \div (-25) = 0$

**g** $-60 \div \ldots. = -5$ **h** $\ldots. \div 15 = -30$ **i** $121 \div \ldots = -11$

**5** Evaluate:

**a** $45 \times (-10) \div 3$ **b** $-34 \times (-3) \times (-2)$ **c** $6 \times (-10) \div 5$

**d** $-10 \times 20 \div (-5)$ **e** $-5 \times 12 \div (-6)$ **f** $16 \div (-8) \times (-25)$

**6** Put multiplication ($\times$) or division ($\div$) signs in each box to make each statement true.

**a** $8 \square 4 \square (-2) \square 3 = -12$ **b** $3 \square 4 \square (-5) \square 6 = -10$


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# 11F Indices and order of operations

You need to be particularly careful with the order of operations when working with negative integers.

For example, $-4^2 = -16$ and $(-4)^2 = 16$. In the first case, 4 is first squared and then the opposite is taken. In the second case, $-4$ is squared.

Notice how different the two answers are.

Remember that multiplication is done before addition unless there are brackets. For example:

$$-12 + 2 \times 5 = -12 + 10 \quad = -2 \qquad \text{and} \qquad (-12 + 2) \times 5) = -10 \times 5 \quad = -50$$

The same general conventions that we have previously stated for whole numbers also apply when dealing with negative integers.

## Order of operations

- Evaluate expressions inside brackets first.
- In the absence of brackets, carry out operations in the following order:
  - powers
  - multiplication and division from left to right
  - addition and subtraction from left to right.

### Example 14

Evaluate:

**a** $(-6)^2$ **b** $-6^2$ **c** $-6 - 5 + 4$ **d** $6 \times (-2) + 8$

**e** $-20 \div 2 \times 10$ **f** $2 \times (-4) \div 8$ **g** $-6 + 3 \times (-2)$ **h** $11 - (-3) \times (-5)$

### Solution

**a** $(-6)^2 = -6 \times (-6)$
$= 36$

**b** $-6^2 = -(6 \times 6)$
$= -36$

**c** $-6 - 5 + 4 = -11 + 4$
$= -7$

**d** $6 \times (-2) + 8 = -12 + 8$
$= -4$

**e** $-20 \div 2 \times 10 = -10 \times 10$
$= -100$

**f** $2 \times (-4) \div 8 = -8 \div 8$
$= -1$

**g** $-6 + 3 \times (-2) = -6 + (-6)$
$= -12$

**h** $11 - (-3) \times (-5) = 11 - 15$
$= -4$


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### Example 15

Evaluate:

**a** $3 \times (-6 + 8)$ **b** $6 - (5 + 4)$

**c** $-3 + 6 \times (7 - 12)^2$ **d** $3 \times (-6) + 3 \times 8$

### Solution

**a** $3 \times (-6 + 8) = 3 \times 2$
$= 6$

**b** $6 - (5 + 4) = 6 - 9$
$= -3$

**c** $-3 + 6 \times (7 - 12)^2 = -3 + 6 \times (-5)^2$
$= -3 + 6 \times 25$
$= -3 + 150$
$= 147$

**d** $3 \times (-6) + 3 \times 8 = -18 + 24$
$= 6$

### Example 16

Evaluate:

**a** $4 \times (-6) \div 2 + 3$ **b** $-7 + 36 \div (-2)^2 + 4$

### Solution

**a** $4 \times (-6) \div 2 + 3 = -24 \div 2 + 3$
$= -12 + 3$
$= -9$

**b** $-7 + 36 \div (-2)^2 + 4 = -7 + (36 \div 4) + 4$
$= -7 + 9 + 4$
$= 6$

## Exercise 11F

Example 14

**1** Evaluate:

**a** $-6 + 20 - 15$ **b** $-4 - (-10) + 20$ **c** $-6 + 12 - 15$

**d** $-4 + 11 - (-15)$ **e** $-15 + 7 - 8$ **f** $65 - (-34) + 50$

**g** $-12 + 20 - 50$ **h** $-50 - 23 - 47$ **i** $-20 - (-25) + 60$

**2** Evaluate:

**a** $-3 \times (-16) + 8$ **b** $-4 + 6 \times 11 - 14$

**c** $-6 - 18 \times 4$ **d** $-6 \times (-3) + 12$

**e** $-2 \times (-6 + 16) - 25$ **f** $-15 + 5 \times (-3) + 12$

**g** $-11 + 5 \times 12 + (-15)$ **h** $-18 - 4 \times 26 - (-12)$


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Example 15

**3** Evaluate:

**a** $-(3 - 17)$ **b** $-(27 - 54)$ **c** $12 + (4 - 16)$

**d** $-43 + (6 - 11)$ **e** $15 - 21 + 4 \times (-3)$ **f** $-3 \times (56 - 87)$

**g** $-14 \times (2 - 11)$ **h** $5 \times (13 - 41)$ **i** $-7 \times (11 - 18)$

**j** $(34 + 34) - (-5) \times (-120)$ **k** $(50 + 70) \times (-3) - 5 \times (-2)$

**4** Evaluate:

**a** $(-8)^2$ **b** $-(11)^2$ **c** $2 \times (-4)^2$ **d** $-9 \times (-3)^2$

**e** $(-10)^2 \times -(3)^2$ **f** $(-12)^2$ **g** $(-5)^3$ **h** $(-2)^4$

**i** $(-2)^5$ **j** $(-2)^6$ **k** $(-1)^3$ **l** $(-1)^4$

**5** Evaluate:

**a** $2 \times (-2)^6$ **b** $3 \times (-2)^5$ **c** $4 \times (-4)^3$ **d** $5 \times (-2)^2$

**e** $3 \times (-4)^2$ **f** $2 \times (-1)^5$ **g** $4 \times (-3)^3$ **h** $7 \times (-1)^{23}$

**6** Evaluate:

**a** $-3 \times (-16 + 8)$ **b** $-4 + 6 \times (11 - 12)$

**c** $-6 - (15 + 4)$ **d** $-6 \times (-2 + 12)$

**e** $-2 \times (-6 + 16) - 20$ **f** $-89 + 5 \times (-32 + 12)$

**g** $-71 + 5 \times (51 + (-35))$ **h** $-18 - 4 \times (26 - (-12))$

**7** Evaluate:

**a** $40 \div (-5) \div 8$ **b** $80 \times (-3) \div 10$

**c** $50 \div 10 \times 2$ **d** $60 \times (-5) \div 25$

**8** Evaluate:

**a** $(-10)^2 + 2 \times (-10)$ **b** $(-10)^2 \times (-10)^3$

**c** $2 \times (-10)^3 + 10^2$ **d** $-2 \times (-10)^2 \times (-10)$

Example 16

**9** Evaluate:

**a** $3 \times (-12) \div 4 + 1$ **b** $-5 + 49 \div (-7)^2 + 2$ **c** $-4 \times 6 \div 8 - 5$

**d** $3 - 50 \div (3 - 8)^2 - 2$ **e** $14 - 3 \times 6 \div (-2)$ **f** $7 - 3^2 \times (1 - 3)^2$

**g** $5 \times (-14) \div (-7) - 3$ **h** $16 + 12 \div (-2)^2 - 4$

**10** A shop manager buys 200 shirts at \$16 each and sells them for a total of \$3000. Calculate the total purchase price, and subtract this from the total amount gained from the sale. What does this number represent?

**11** A man puts \$1000 into a bank account every month for 12 months. Initially, he had \$3000 in the bank.

**a** How much does he have in the account at the end of 12 months, given that he has not withdrawn any money?

**b** At the end of the 12 months, he writes a cheque for \$20 000. How much does he now have left?


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**12** A pizza shop runs a delivery van at a cost of $200 a day to deliver pizzas from the shop to its customers. Each pizza costs $3 to make and sells for $9.

**a** If the pizza shop delivers 90 pizzas in a day, how much money does it make?

**b** The price of a pizza increases to $10 and the cost of making a pizza is unchanged. How much money does the pizza shop make if 90 pizzas are delivered?

**c** If the price of a pizza decreases to $8 and the cost of making it increases to $4, how much does the pizza shop make or lose if it delivers 45 pizzas in a day?

**13** The local charity is planning to run a fair, and is trying to decide how much to charge for entry. The hall where it plans to hold it will cost $500 to rent for the day. It plans to charge $5 per person for entry, and to give each person a show bag that costs $2 to produce.

**a** If 120 people come to the fair, how much money will the charity make?

**b** If the charity decides instead to charge $8 per person, and 120 people attend, how much will it make or lose?

**c** If it charges $5 per child and $8 per adult, and 60 children and 60 adults attend, how much will it make or lose?

# Review exercise

**1** Complete each addition.

| | | |
|---|---|---|
| **a** $25 + (-2)$ | **b** $-36 + 22$ | **c** $-35 + 50$ |
| **d** $-51 + (-44)$ | **e** $-32 + 16$ | **f** $-45 + (-23)$ |
| **g** $-160 + (-20)$ | **h** $-50 + (-10)$ | **i** $110 + (-40)$ |
| **j** $-120 + 40$ | **k** $35 + (-3)$ | **l** $-72 + 22$ |
| **m** $-75 + 50$ | **n** $-91 + (-44)$ | **o** $-65 + 59$ |
| **p** $-60 + (-25)$ | **q** $-165 + (-25)$ | **r** $-55 + (-10)$ |
| **s** $115 + (-45)$ | **t** $-125 + 43$ | **u** $-332 + (-215)$ |

**2** In an indoor cricket match, a team has made 25 runs and lost 7 wickets. What is the team's score? (A run adds 1 and a wicket subtracts 5.)

**3** The temperature in June at a base in Antarctica varied from a minimum of $-60°C$ to a maximum of $-35°C$. What was the value of:

**a** maximum temperature − minimum temperature?

**b** minimum temperature − maximum temperature?


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**4** The temperature in Canberra had gone down to −3°C. The temperature in a heated house was a cosy 22°C. What was the value of:

**a** (inside temperature) − (outside temperature)?

**b** (outside temperature) − (inside temperature)?

**5** Complete each multiplication.

**a** $125 \times (-2)$ **b** $-36 \times 11$ **c** $-35 \times 50$

**d** $-51 \times (-40)$ **e** $-3 \times 16$ **f** $-50 \times (-23)$

**g** $-160 \times (-20)$ **h** $-50 \times (-10)$ **i** $11 \times (-40)$

**j** $-120 \times 20$ **k** $-20 \times (-5)$ **l** $-25 \times (-4)$

**6** Complete each division.

**a** $125 \div (-5)$ **b** $-36 \div 9$ **c** $-35 \div 5$

**d** $-51 \div (-3)$ **e** $-16 \div (-4)$ **f** $-50 \div (-10)$

**g** $-160 \div (-20)$ **h** $-1500 \div (-10)$ **i** $110 \div (-40)$

**j** $-120 \div 20$ **k** $-196 \div (-14)$ **l** $625 \div (-25)$

**7** Evaluate each expression.

**a** $-4 \times (6 - 7)$ **b** $7 \times (11 - 20)$ **c** $-3 \times (5 + 15)$

**d** $-6 \times (-4 - 6)$ **e** $-12 \times (-6 + 20)$ **f** $-(-4)^2$

**g** $(3 - 7) \times (11 - 15)$ **h** $(10 - 3) \times (-3 + 10)$ **i** $(-5 - 10) \times (10 - 4)$

**8** Start with the number −5, add 11 and then subtract 20. Multiply the result by 4. What is the final result?

**9** Start with −100, subtract 200 and then add −300. Divide the result by 100. What is the final result?

**10** Evaluate:

**a** $(-8)^2$ **b** $-8^2$ **c** $-11 - 15 + 14$

**d** $16 \times (-2) + 10$ **e** $-3 \times (-8) + 100 - 150$ **f** $-200 \div 2 \times 10$

**g** $2 \times (-6) \div 8$ **h** $4 \times (-6) \div (-3)$

**11** Evaluate:

**a** $5 \times (-7 + 18)$ **b** $-13 + 16 \times (7 - 12)$

**c** $16 - (15 + 14)$ **d** $16 \times (-12 + 8)$

**e** $-3 \times (-16 + 20) - 25$ **f** $3 \times (-8) + 3 \times 18$

**g** $5 \times (-7^2) + 3 \times 42$ **h** $7 \times (-3)^2 + 3 \times (-4)$


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# Challenge exercise

**1** What is the least product you could obtain by multiplying any two of the following numbers:

$-8, -6, -1, 1$ and $4$?

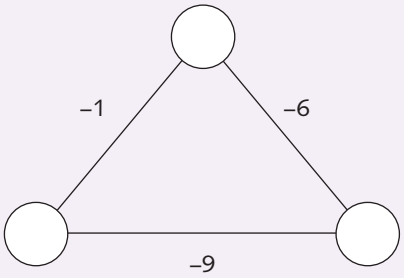
**2** Evaluate:

**a** $(-1)^{1000}$

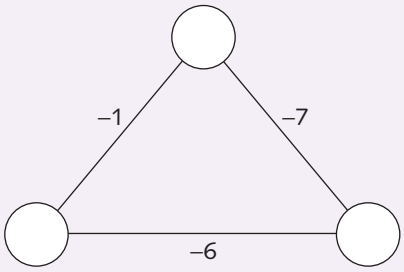
**b** $(-1)^{1001}$

**3** The integers on the edges of each triangle below are given by the sum of integers which are to be placed in the circles. Find the numbers in the circles.

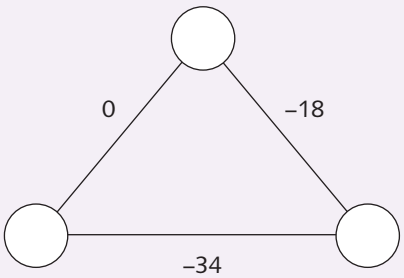
**a**



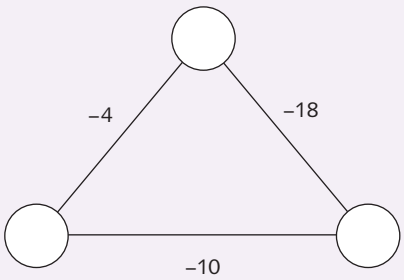

**b**




**c**




**d**




**4** Put the three numbers $4, -2$ and $-7$ into the boxes below

$\square + \square - \square = \square$

so that the answer is:

**a** $-1$

**b** $9$

**c** $-13$


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**5** Put the three numbers 5, –5 and –4 into the boxes below

$\square + \square - \square = \square$

so that the answer is:

**a** 6

**b** –14

**c** 4

**6** Find the number that must be placed in the box to make the following statement true.

$3 - \square + (-5) = 0$

**7** Place brackets in each statement below to make the statement true.

**a** $5 + (-3) \times 3 + 4 = 14$

**b** $5 + (-3) \times 3 + 4 \times 2 = 4$

**c** $5 - 5 \times 6 + 7 \times 6 - 5 = 37$

**8** This is a magic square. All rows, columns and diagonals have the same sum. Complete the magic square.

| | | |
|---|---|---|
| 2 | –5 | 0 |
| | | |
| | | –4 |

**9** **a** Find the value of $2 - 4 + 6 - 8 + 10 - 12$ by:

**i** working from left to right

**ii** pairing the numbers $((2 - 4) + (6 - 8) + (10 - 12))$

**b** Evaluate $2 - 4 + 6 - 8 + 10 - 12 + 14 - 16 + \ldots + 98 - 100$.

**10** Evaluate $100 + 99 - 98 - 97 + \ldots + 4 + 3 - 2 - 1$.

**11** The average of five numbers was 2. If the smallest number is deleted, the average is 4. What is the smallest number?

**12** Find the value of:

**a** $(1 - 3) + (5 - 7) + (9 - 11) + (13 - 15) + (17 - 19)$

**b** $1 - 3 + 5 - 7 + 9 - 11 + \ldots + 101 - 103$


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CHAPTER

# 12

Number and Algebra

# Algebra and the Cartesian plane

This chapter deals with the substitution of negative numbers into algebraic expressions. The following example illustrates why this is important.

In the United States, temperature is measured on the Fahrenheit scale, while in Australia we use the Celsius scale. It is useful to be able to convert from one scale to the other. For example, if the temperature in a town in the US is $-5$°C, what is the temperature in degrees Fahrenheit (°F)? The rule for converting to °F is to multiply the Celsius temperature value by $\frac{9}{5}$ and add 32 to the result.

We can write this in algebraic notation as $F = \frac{9}{5}C + 32$, where $F$ and $C$ are the temperature values in the Fahrenheit and Celsius scales, respectively.

In the formula $F = \frac{9}{5}C + 32$ substituting $C = -5$ gives:

$$\begin{aligned} F &= \frac{9}{5} \times (-5) + 32 \\ &= -9 + 32 \\ &= 23 \end{aligned}$$

In this chapter, we will also see how such situations can be illustrated on the Cartesian plane.

The Cartesian plane is used to combine algebra and geometry. The Cartesian plane is very important in mathematics. The idea is simple but extremely useful.



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# 12A Substitution with integers

In Chapter 3, we substituted positive whole number values for pronumerals. In Chapter 7, we substituted positive fractions. We will now look at how to substitute negative integer values, as illustrated in the following examples.

## Example 1

Evaluate each expression for $x = -5$.

**a** $4x + 3$ **b** $-4x + 3$ **c** $4(x + 3)$

**d** $-4(x + 3)$ **e** $-4x^2$ **f** $(-4x)^2$

### Solution

**a** $4x + 3 = -20 + 3$
$= -17$

**b** $-4x + 3 = 20 + 3$
$= 23$

**c** $4(x + 3) = 4 \times (-2)$
$= -8$

**d** $-4(x + 3) = -4 \times (-2)$
$= 8$

**e** $-4x^2 = -4 \times 25$
$= -100$

**f** $(-4x)^2 = (20)^2$
$= 400$

## Example 2

Evaluate each expression for $m = -5$, $n = 6$ and $p = -10$.

**a** $m + n$ **b** $m + p$ **c** $m - p$

**d** $mp$ **e** $np$ **f** $\frac{p}{m}$

### Solution

**a** $m + n = -5 + 6$
$= 1$

**b** $m + p = -5 + (-10)$
$= -15$

**c** $m - p = -5 - (-10)$
$= -5 + 10$
$= 5$

**d** $mp = -5 \times (-10)$
$= 50$

**e** $np = 6 \times (-10)$
$= -60$

**f** $\frac{p}{m} = \frac{-10}{-5}$
$= 2$

Problems in words can often be translated into algebra.

## Example 3

The temperature is now 12°C.

**a** What is the new temperature if the temperature drops by 15°C?

**b** What is the new temperature if the temperature drops by $x$°C?

**c** Find the new temperature if:

**i** $x = 10$ **ii** $x = 20$


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### Solution

**a** New temperature $= 12°C - 15°C$
$= -3°C$

**b** New temperature $= (12 - x)°C$

**c** **i** If $x = 10$, new temperature $= (12 - 10)°C$
$= 2°C$

**ii** If $x = 20$, new temperature $= (12 - 10)°C$
$= -8°C$

### Example 4

Christina has \$100 in a bank account. She takes \$$x$ from the bank account every day.

**a** How much money does she have in the account after 4 days?

**b** How much does she have left in the account after 4 days if:

**i** $x = 10$? **ii** $x = 20$? **iii** $x = 25$?

**c** Interpret the outcome in words if $x = 30$.

### Solution

**a** Amount left $= \$(100 - 4x)$

**b** **i** Amount left $= 100 - 40$
$= \$60$

**ii** Amount left $= 100 - 80$
$= \$20$

**iii** Amount left $= 100 - 100$
$= \$0$

**c** Amount left $= 100 - 120$
$= -20$ dollars

Christina has overdrawn her account by \$20.

## Exercise 12A

Example 1

**1** Evaluate each expression for $x = -2$.

**a** $2x$ **b** $-x$ **c** $x + 2$ **d** $x - 3$

**e** $2x + 3$ **f** $x^3$ **g** $-x^3$ **h** $(-x)^2$

**i** $3 - x$ **j** $3 - 2x$ **k** $5 + 2x$ **l** $2 - 5x$

**2** Evaluate each expression for $x = -30$.

**a** $2x + 3$ **b** $-x + 6$ **c** $2x - 4$ **d** $5 - x$

**e** $6 - 2x$ **f** $5 - 4x$ **g** $-\frac{x}{2}$ **h** $\frac{x - 4}{2}$


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Example 2

**3** Substitute $m = -4$, $n = 3$ and $p = -24$ to evaluate:

**a** $m + n$ **b** $m + p$ **c** $m - p$ **d** $mp$

**e** $np$ **f** $\frac{p}{m}$ **g** $mnp$ **h** $\frac{p}{n}$

**4** Evaluate each expression for $x = -2$.

**a** $5x + 4$ **b** $-5x + 4$ **c** $5(x + 4)$

**d** $-5(x + 4)$ **e** $-5x^2$ **f** $(-5x)^2$

**5** Evaluate each expression for $a = -3$.

**a** $5 + 2a$ **b** $6 - 3a$ **c** $2a + 3$

**d** $4 - a$ **e** $(-a)^3$ **f** $-a^3$

**g** $(-2a)^2$ **h** $(-2a)^3$ **i** $a^3 + 2$

**6** Evaluate each expression for $z = -3$.

**a** $-3z$ **b** $z^4$ **c** $5 - 2z$

**d** $(2z)^3$ **e** $(-z)^3$ **f** $-2z^2$

**7** Substitute $m = -20$, $n = -10$ and $p = 50$ to evaluate:

**a** $m + n$ **b** $p + m$ **c** $n - p$ **d** $mn$

**e** $mp$ **f** $\frac{m}{p}$ **g** $\frac{p}{m}$ **h** $mnp$

**8** Evaluate each expression for $w = 10$.

**a** $-40w$ **b** $-w^4$ **c** $10 - 2w$

**d** $(-2w)^3$ **e** $(-w)^3 + w^2$ **f** $w^3 - 10w^2$

**9** Evaluate each expression for $w = -2$.

**a** $-20w$ **b** $-w^4$ **c** $10 - 2w$

**d** $(-2w)^3$ **e** $(-w)^3 + w^2$ **f** $w^3 - 10w^2$

Examples 4

**10** Buffy has \$1000 in a bank account. She takes \$$x$ from the bank account every day.

**a** How much money does she have in the account after:

**i** 1 day? **ii** 5 days?

**b** Find the value of her bank account after 5 days if:

**i** $x = 100$ **ii** $x = 200$ **iii** $x = 250$

Example 3

**11** The temperature in a room drops by $x$°C every hour. The temperature in the room at 12 p.m. is 25°C.

**a** What will the temperature be at:

**i** 1 p.m.? **ii** 6 p.m.?

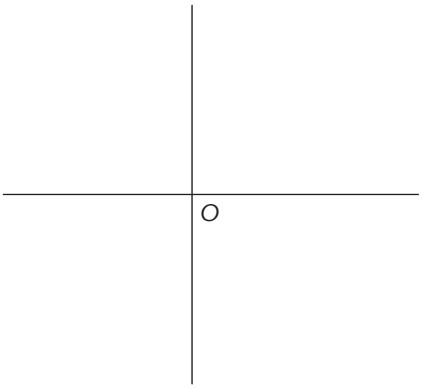
**b** If $x = 6$, what will the temperature be at 5 p.m.?


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# 12B The Cartesian plane

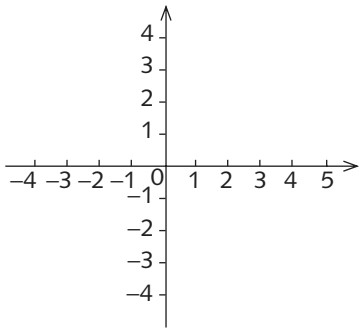
We have previously represented numbers as points on the number line. This idea can be extended to represent points in the plane by pairs of numbers.

We start with two perpendicular straight lines. They intersect at a point $O$ called the **origin**. We leave the right-angle sign out for clarity.

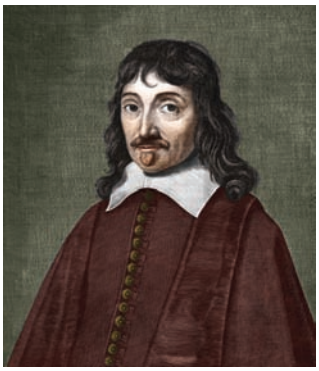



Each of the lines is called an **axis**. The plural of *axis* is *axes*.

Next we mark off internals of unit length along each axis, and mark each axis as a number line with 0 at the point $O$. The arrows are drawn to show that the axes extend infinitely, in both directions, and to indicate which is the positive direction.

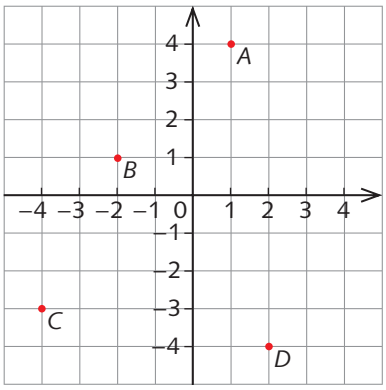



The axes are called the **coordinate axes** or sometimes the **Cartesian coordinate axes**. They are named after the French mathematician and philosopher René Descartes (1596–1650). He introduced coordinate axes to show how algebra could be used to solve geometric problems. Although the idea is simple, it revolutionised mathematics.



Now we can imagine adding vertical and horizontal lines to the diagram through the integer points on the axes. We can describe each point where the lines meet by a pair of integers. This pair of integers is called the **coordinates** of the point. The first number is the **horizontal coordinate** and the second number is the **vertical coordinate**.

For example, the coordinates of the point labelled $A$ are $(1, 4)$. This is where the line through the point 1 on the horizontal axis and the line through the point 4 on the vertical axis meet. We move 1 unit to the right of the origin and 4 units up to reach $A$.




The point $D$ has coordinates $(2, -4)$. We move 2 units to the right of the origin and 4 units down to get $D$.

The point $B$ has coordinates $(-2, 1)$. We move 2 units to the left of the origin and 1 unit up to get $B$.

The point $C$ has coordinates $(-4, -3)$. We move 4 units to the left of the origin and 3 units down.


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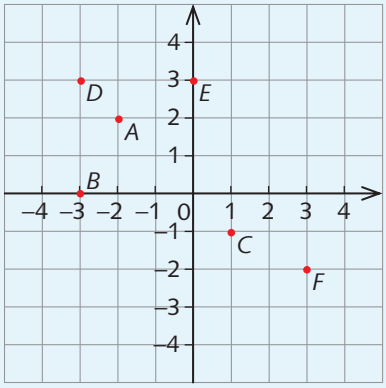
## Example 5

On a number plane, plot the points with the given coordinates.

**a** $A(-2, 2)$ **b** $B(-3, 0)$ **c** $C(1, -1)$

**d** $D(-3, 3)$ **e** $E(0, 3)$ **f** $F(3, -2)$

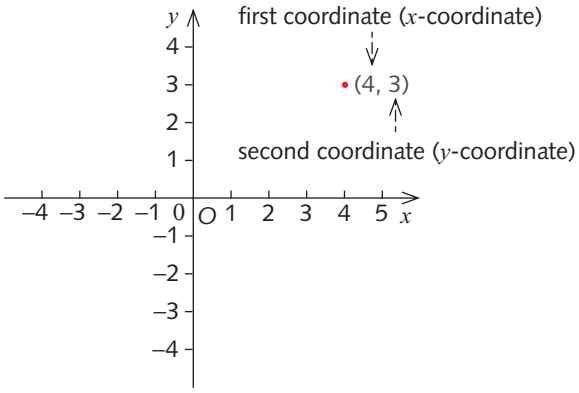
### Solution




Remember, the first coordinate tells us where to go from the origin in the horizontal direction. If it is negative, we go to the left of the origin; if it is positive, we go to the right of the origin.

The second coordinate tells us where to go from the origin in the vertical direction. If it is negative, we go below the origin; if it is positive, we go above the origin.

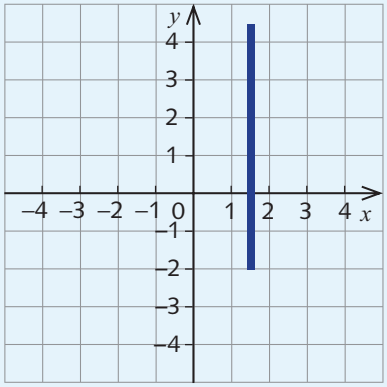
We have labelled the horizontal axis the $x$-axis, and the vertical axis the $y$-axis.




The first coordinate is usually called the ***x*-coordinate** and the second coordinate is usually called the ***y*-coordinate**.

## Example 6

Plot the following points on the grid and join them with straight-line intervals in the order they are given to complete the picture.



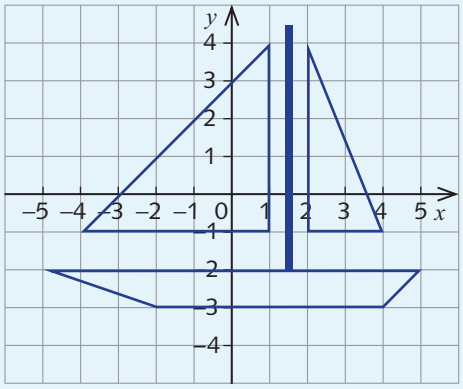

Shape 1: Join $(2, 4)$ to $(4, -1)$ to $(2, -1)$ to $(2, 4)$.

Shape 2: Join $(1, 4)$ to $(1, -1)$ to $(-4, -1)$ to $(1, 4)$.

Shape 3: Join $(5, -2)$ to $(4, -3)$ to $(-2, -3)$ to $(-5, -2)$ to $(5, -2)$.

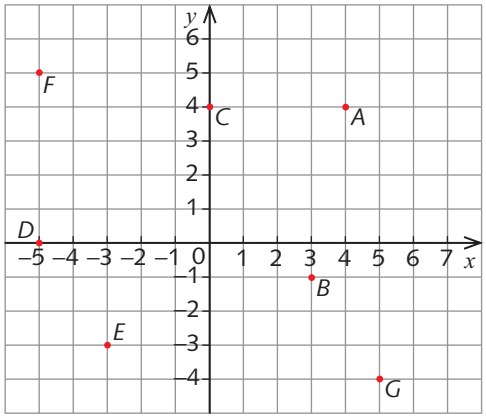

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**Solution**




## Exercise 12B

**1** Give the coordinates of points $A$ to $G$.




Example 5

**2** On a number plane, plot the points with the given coordinates.

**a** $A(5, 1)$ **b** $B(-2, 4)$

**c** $C(-3, -3)$ **d** $D(3, -1)$

**e** $E(2, 4)$ **f** $F(0, -4)$

**g** $G(-5, 1)$ **h** $H(-5, -2)$

**3** On a number plane, plot the points with the given coordinates.

**a** $A(-4, -1)$ **b** $B(-2, -3)$ **c** $C(2, -2)$ **d** $D(4, -1)$

**e** $E(-1, 4)$ **f** $F(-3, -2)$ **g** $G(-3, 2)$ **h** $H(0, -4)$

Example 6

**4** Plot these points on a grid and join them in the given order to draw three pictures.

**a** $(2, 3), (2, 2), (-1, 2), (-1, 0), (1, 0), (1, -1), (-1, -1), (-1, -3), (2, -3), (2, -4), (-2, -4), (-2, 3), (2, 3)$.

**b** $(5, 0), (2, 1), (3, 3), (1, 2), (0, 5), (-1, 2), (-3, 3), (-2, 1), (-5, 0), (-2, -1), (-3, -3), (-1, -2), (0, -5), (1, -2), (3, -3), (2, -1), (5, 0)$.

**c** Shape 1: Join $(2, 4)$ to $(2, 2)$ to $(0, 2)$ to $(0, 4)$ to $(2, 4)$.

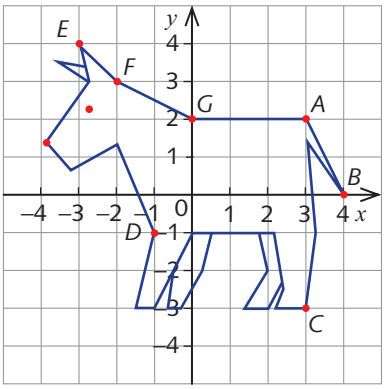
Shape 2: Join $(3, 0)$ to $(2, 1)$ to $(2, -1)$ to $(0, -1)$ to $(0, 1)$ to $(-1, 0)$ to $(-2, 1)$ to $(1, 2)$ to $(4, 1)$ to $(3, 0)$.

Shape 3: Join $(2, -1)$ to $(3, -4)$ to $(2, -4)$ to $(1, -2)$ to $(0, -4)$ to $(-1, -4)$ to $(0, -1)$ to $(2, -1)$.

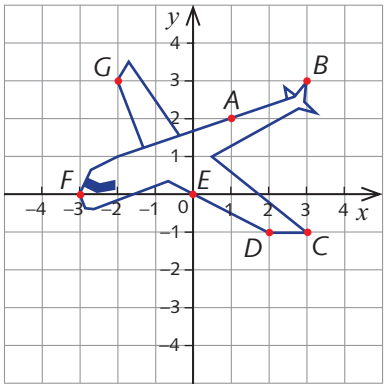

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**5** Write down the coordinates of the points labelled $A$ to $G$ in each diagram.

**a**




**b**




**6** Draw coordinate axes and mark on them the integer points 1 cm apart.

**a** Plot the points $A(0, 1)$, $B(3, 1)$, $C(3, 4)$ and $D(0, 4)$, and join them to form $AB$, $BC$, $CD$ and $DA$. Describe the shape formed and evaluate its area.

**b** Plot the points $A(-2, 0)$, $B(4, 0)$ and $C(1, 4)$, and join them to form $AB$, $BC$ and $CA$. Describe the shape formed and evaluate its area.

**c** Plot the points $A(-4, -4)$, $B(7, -4)$, $C(7, 1)$ and $D(-4, 1)$, and join them to form $AB$, $BC$, $CD$ and $DA$. Describe the shape formed and evaluate its area.

**d** Plot the points $A(-6, 4)$, $B(-1, 4)$ and $C(-6, 1)$, and join them to form $AC$, $CB$ and $BA$. Describe the shape formed and evaluate its area.

**e** Plot the points $A(0, 2)$ and $B(1, 4)$, and draw the line passing through them. Now plot the points $C(-2, 5)$ and $D(0, 4)$, and draw the line passing through these points. Describe the relationship between the lines.

**f** Plot the points $A(0, 1)$, $B(4, 3)$, $C(10, 3)$ and $D(6, 1)$, and join them to form $AB$, $BC$, $CD$ and $DA$. Describe the shape formed and evaluate its area.

**7** **a** On a grid, join $(0, 0)$ to $(3, 1)$ to $(4, 2)$ to $(4, 4)$ to $(2, 4)$ to $(1, 3)$ to $(0, 0)$ to draw one petal of a flower.

**b** Complete, and then plot, the following list of points to form a second petal the same shape as the first.

Join $(0, 0)$ to $(3, -1)$ to $(\_\_, -2)$ to $(4, \_\_)$ to $(\_\_, \_\_)$ to $(\_\_, \_\_)$ to $(0, 0)$.

**c** Draw the remaining two petals of the same shape to complete the flower.

**d** Write down the ordered list of points required to draw each of the petals in part **c**.


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# 12C Completing tables and plotting points

The following examples show how an understanding of the number plane can help us with algebra, and vice versa.

Two students play a simple game to improve their multiplication of integers. Liam calls out a number and Andrea multiplies it by 2. Liam starts at –2 and calls out to Andrea each integer up to 2. Their results are recorded in a table.

We can write the rule as:

Andrea's number = 2 × Liam's number

and the table is:

| **Liam's number** | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **Andrea's number** | –4 | –2 | 0 | 2 | 4 |

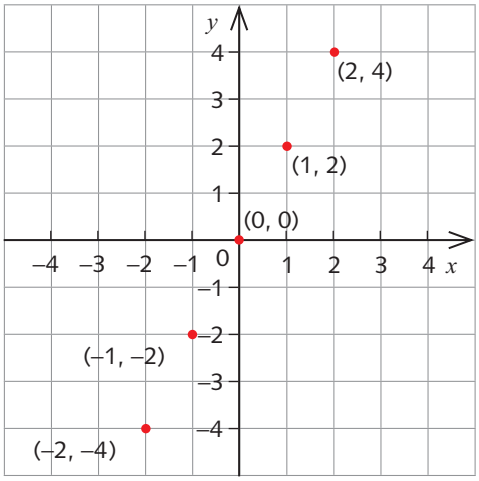
If we denote Liam's number by $x$ and Andrea's number by $y$, then we can write the rule as:

$y = 2x$

and the table can now be written as:

| $x$ | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | –4 | –2 | 0 | 2 | 4 |

We can also plot the points in the table on the number plane, as shown below.




The points (–2, –4), (–1, –2), (0, 0), (1, 2) and (2, 4) are plotted. What do you notice about these points?

A line can be drawn through all of the points. Try it!

We can follow the same kind of procedure for any similar rule – a table can be formed and the corresponding points plotted.


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## Example 7

For each given rule, complete the table, list the coordinates of the points, and plot the points on a number plane.

**a** $y = -x$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

**b** $y = x + 1$

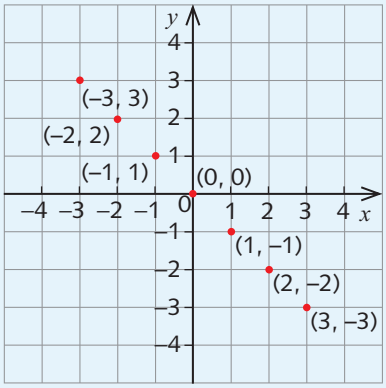
| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

## Solution

**a** $y = -x$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | 3 | 2 | 1 | 0 | −1 | −2 | −3 |

The points are (−3, 3), (−2, 2), (−1, 1), (0, 0), (1, −1), (2, −2) and (3, −3).

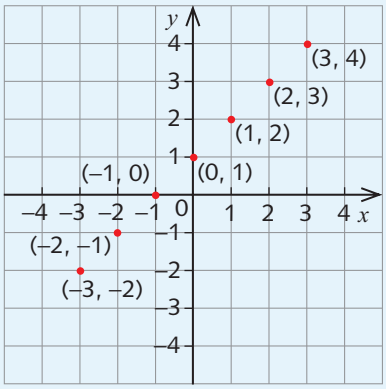



**b** $y = x + 1$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | −2 | −1 | 0 | 1 | 2 | 3 | 4 |

The points are (−3, −2), (−2, −1), (−1, 0), (0, 1), (1, 2), (2, 3) and (3, 4).

Check that a straight line can be drawn through all the points.





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# Exercise 12C

Example 7

**1** For each given rule, complete the table, list the coordinates, and plot the corresponding set of points on a number plane. Check that each set of points lies on a line.

**a** $y = 3x$

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

**b** $y = -2x$

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

**c** $y = x - 2$

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

**d** $y = x + 2$

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

**e** $y = 2x + 1$

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

**f** $y = 1 - x$

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

**g** $y = 3 - 2x$

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

**2** For each given rule, complete the table, list the coordinates, and plot the corresponding set of points on a number plane.

**a** $y = x + \frac{1}{2}$

| $x$ | –2 | –1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

**b** $y = x - \frac{1}{2}$

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |


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**c** $y = 2x + \frac{1}{2}$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

**d** $y = -x + \frac{1}{2}$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

**3** Complete the table for each given rule.

**a** $y = 5x - 7$

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

**b** $y = 9 - 4x$

| $x$ | −4 | −3 | −2 | −1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | | | |

**4** Complete the table for the rule, list the coordinates, and plot the corresponding set of points on a number plane. Note that they do not lie on a straight line.

**a** $y = x^2$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

**b** $y = (1 - x)^2$

| $x$ | −2 | −1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

# 12D Finding rules

In the previous section we looked at completing tables and plotting the corresponding points. In this section we will find a rule that fits a table or a plot of points.

## Example 8

Find a rule for each of the following tables and then express the rule in algebra.

**a**

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 5 | 10 | 15 | 20 | 25 |

$y = \ldots$

**b**

| $t$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $d$ | −4 | −1 | 2 | 5 | 8 |

$d = \ldots$


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## Solution

**a** Each $x$ value is multiplied by 5 to obtain the corresponding $y$ value.
A rule for the table is $y = 5x$.

**b** Each $t$ value is multiplied by 3 and 2 is added to it to obtain the corresponding $d$ value.
A rule for the table is $d = 3t + 2$.

## Example 9

Tiles are formed into the letter 'X' as shown below.

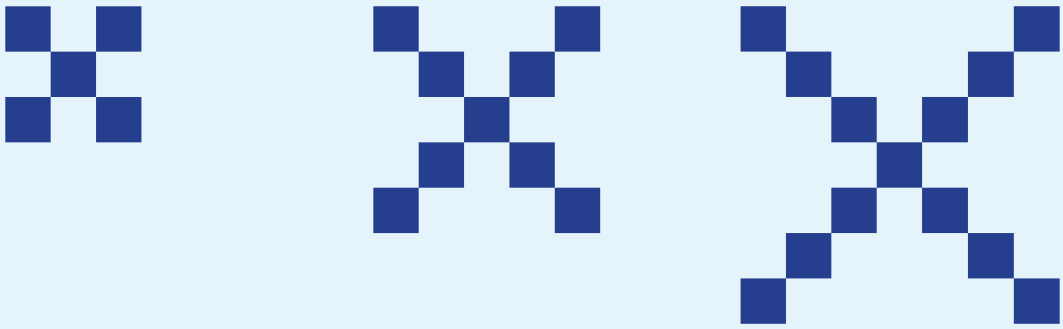


**Diagram 1** **Diagram 2** **Diagram 3**

**a** Copy and complete the table below, where $n$ is the number of the diagram.

| Diagram number ($n$) | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Number of tiles ($t$) | 5 | 9 | 13 | | | |

**b** How does the number of tiles increase as we move from one diagram to the next?

**c** Plot the points $(n, t)$ for values of $n$ from 1 to 6, using your table of values.

**d** Write a rule that tells us the number of tiles we need for the $n$th diagram.

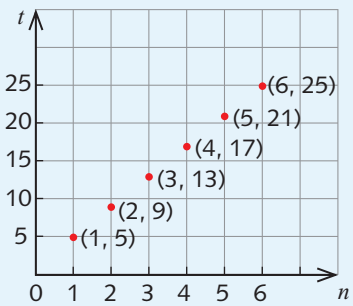
## Solution

**a**

| Diagram number ($n$) | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Number of tiles ($t$) | 5 | 9 | 13 | 17 | 21 | 25 |

**b** We need an extra four tiles each time we make a bigger 'X'.

**c**




**d** Multiply the $n$ value by 4 and add 1.
A rule for the number of tiles is $t = 4n + 1$.

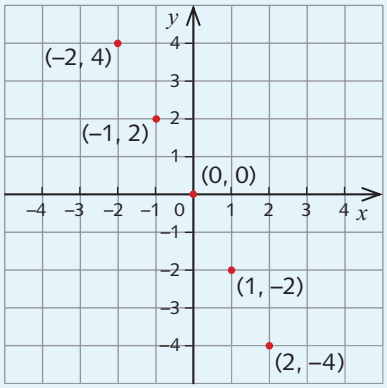

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Sometimes the patterns involve negative numbers.

### Example 10

Plot the points $(-2, 4)$, $(-1, 2)$, $(0, 0)$, $(1, -2)$ and $(2, -4)$ on a number plane and give a rule connecting the $y$-coordinate to the $x$-coordinate.

### Solution




The rule is $y = -2x$.

## Exercise 12D

Example 8

**1** Say in words how to find the $y$ value given the corresponding $x$ value and then write the rule in algebra. Each of these tables gives a set of coordinates that lie on a straight line.

**a**

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 2 | 3 | 4 | 5 | 6 |

**b**

| $t$ | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $d$ | 0 | 3 | 6 | 9 | 12 |

**c**

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 4 | 6 | 8 | 10 | 12 |

**d**

| $t$ | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $d$ | 12 | 9 | 6 | 3 | 0 |

**e**

| $m$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $n$ | 0 | 3 | 6 | 9 | 12 |

**f**

| $x$ | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | 3 | 1 | –1 | –3 | –5 |

Example 9

**2** A pile of matchsticks is used to make the following pattern of shapes. The first diagram uses three matches to form one triangle. The second diagram uses five matches to form two triangles.

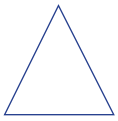


Diagram 1

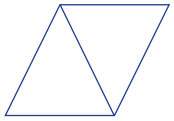


Diagram 2

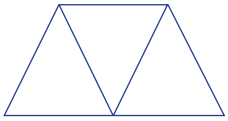


Diagram 3

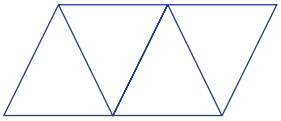


Diagram 4


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**a** Count the number of matches used to make each diagram, and complete the table below.

| Number of triangles ($t$) | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Number of matches ($m$) | 3 | 5 | 7 | | | |

**b** How many matches do we add each time to create an extra triangle?

**c** Plot the points $(t, m)$ for values of $t$ from 1 to 6, using your table of values.

**d** Write a rule that tells us the number, $m$, of matches we need to make any number of triangles.

**3** The first diagram shows four chairs placed around one square table. The second diagram shows six chairs placed around two square tables. The third diagram shows eight chairs placed around three square tables. Consider the number of chairs needed each time an extra table is added to the row.

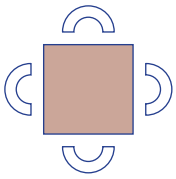


Diagram 1

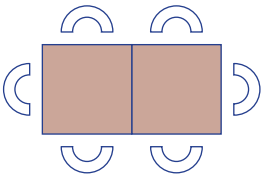


Diagram 2

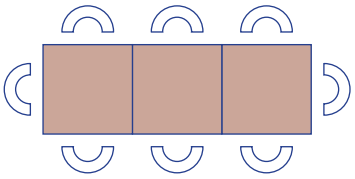


Diagram 3

**a** Count the number of chairs used to make each diagram, and complete the table below.

| Number of tables ($t$) | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Number of chairs ($c$) | 4 | 6 | 8 | | | |

**b** How many extra chairs are needed each time a table is added?

**c** Plot the points $(t, c)$ for values of $t$ from 1 to 6, using your table of values.

**d** Write a rule that tells us the number, $c$, of chairs we need to place around any number, $t$, of tables.

**4** Tommy the terrible 2-year-old emptied the kitchen cupboards and used all the cans of food to make a tower as in diagram 1. When his mother discovered what he had done, she noticed the tower was in the shape of an 'L'. Not wanting to miss an opportunity to teach Tommy the alphabet, she proceeded to pack the cans away four at a time as shown in the following diagrams.

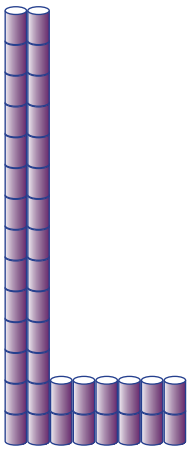


Diagram 1

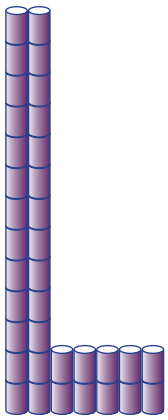


Diagram 2

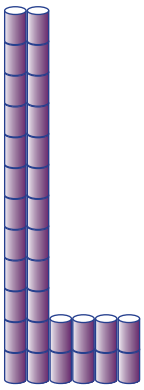


Diagram 3


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**a** How many cans of food did Tommy use to build his first tower?

**b** Count the number of cans used to make each tower, and complete the table below.

| Diagram number ($n$) | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Number of cans ($c$) | 40 | 36 | 32 | | | |

**c** Plot the points $(n, c)$ for values of $n$ from 1 to 6, using your table of values.

**d** Write a rule that tells us the number of cans needed to create each 'L'.

**e** How many different 'L's can Tommy's mother make before the tower loses its 'L' shape?

Example 10

**5** For each of the following, plot the points on a number plane and give a rule connecting the $y$-coordinate to the $x$-coordinate.

**a** $(-2, -2)$, $(-1, -1)$, $(0, 0)$, $(1, 1)$, $(2, 2)$

**b** $(-2, -1)$, $(-1, 0)$, $(0, 1)$, $(1, 2)$, $(2, 3)$

**c** $(-2, -4)$, $(-1, -2)$, $(0, 0)$, $(1, 2)$, $(2, 4)$

**d** $(-2, -4)$, $(-1, -3)$, $(0, -2)$, $(1, -1)$, $(2, 0)$

**6** Sarah is given \$1000 for her 18th birthday. She decides to use it to sponsor a child in Africa at a cost of \$20 each month.

**a** Complete the table below to show how much money Sarah has left at the end of each month.

| Month ($m$) | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Dollars ($d$) | 1000 | 980 | | | | | |

**b** Write a rule to show how many dollars, $d$, Sarah has left after $m$ months.

**c** How much money will Sarah have after 10 months?

**d** For how many months can Sarah sponsor the child?

**7** Frank recently turned 16 and got his learner's permit. His mother supervises him driving the family car to and from school each day, a trip which takes him 30 minutes each way. Frank keeps a log of the total hours he has driven. The table shows the total number, $h$, of hours Frank has driven after $w$ weeks.

| Week number ($w$) | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Number of driving hours ($h$) | 5 | 10 | 15 | | | |

**a** Complete the table.

**b** Write a rule that tells us the number of driving hours, $h$, after $w$ weeks.

**c** How many hours of driving will Frank have done after 12 weeks?

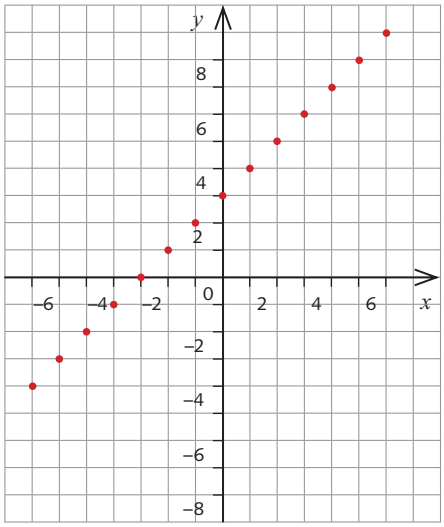
**d** How many weeks driving will Frank need to do to complete 45 hours of driving?


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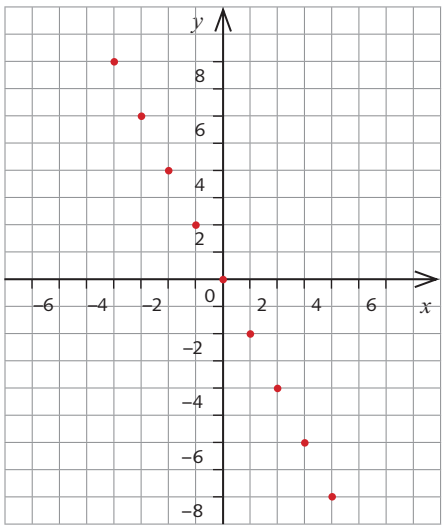
**8** Match each diagram with the correct rule from the list below.

$y = 3x$ $\quad y = -2x$ $\quad y = x + 3$
$y = -2x - 1$ $\quad y = -2x + 1$ $\quad y = x - 4$

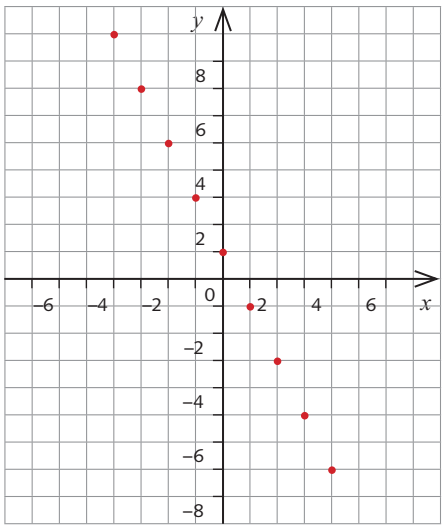
**a**



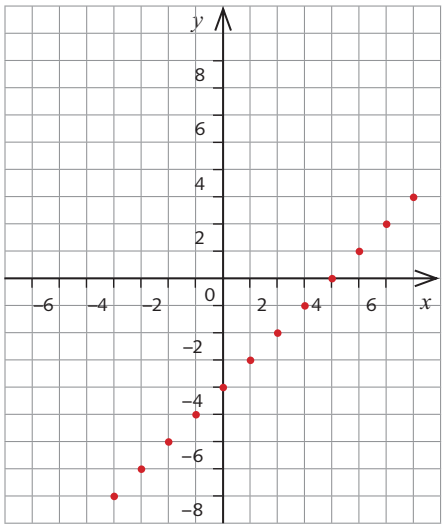

**b**



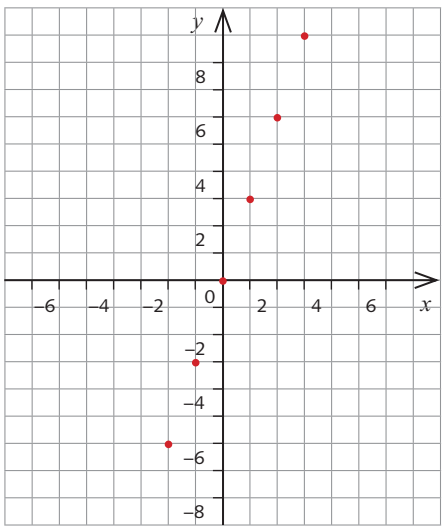

**c**



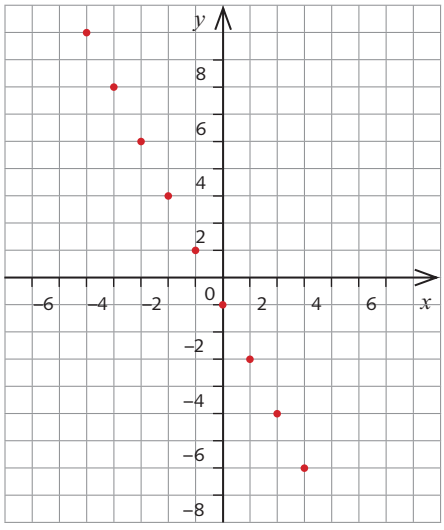

**d**




**e**




**f**





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# Review exercise

**1** Given that $m = -1$, $n = 2$ and $p = -6$, evaluate:

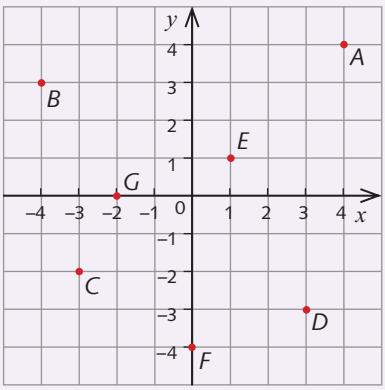
**a** $m + n$ **b** $m + p$ **c** $m - p$ **d** $mp$ **e** $np$ **f** $\frac{p}{m}$

**2** On a number plane, plot each of the points whose coordinates are given below.

**a** $A(1, 1)$ **b** $B(2, -3)$ **c** $C(0, 6)$

**d** $D(-4, 0)$ **e** $E(-4, -2)$ **f** $F(-4, 5)$

**3** Write down the coordinates of each of the points $A$ to $G$.




**4** Make up a table with integer $x$ values from $-3$ to 3 for each of the rules given below. List the corresponding coordinates and plot the points. Check that a line can be drawn through the points.

**a** $y = 3 - x$ **b** $y = 2x - 3$

**5** Evaluate each expression for $x = -3$.

**a** $10 - x$ **b** $10 + x$ **c** $x^2$ **d** $x^3$

**e** $-x^2$ **f** $(-5x)^2$ **g** $-25x^2$ **h** $5 - 5x$

**6** David has \$600 in a bank account. He takes \$$x$ from the account every week.

**a** How much money does he have in the account after:

**i** 1 week? **ii** 5 weeks?

**b** Find the value of his bank account after 5 weeks if:

**i** $x = 100$ **ii** $x = 200$

**7** The temperature in a freezer drops by $2x$°C every hour after 6 p.m. until it reaches $-5$°C. The temperature in the freezer at 6 p.m. is 20°C.

**a** What will the temperature be at:

**i** 7 p.m.? **ii** 11 p.m.?

**b** If $x = \frac{1}{2}$, what will the temperature be in 8 hours?

**c** If $x = 2$, what will the temperature be in 5 hours?

**d** If $x = 2\frac{1}{2}$, when does the temperature reach $-5$°C?


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**8** $ABCD$ is a square. The coordinates of $A$, $B$ and $C$ are $(0, 0)$, $(0, 6)$ and $(6, 6)$, respectively. What are the coodinates of $D$?

**9** $ABCD$ is a rectangle. The coordinates of $A$, $B$ and $C$ are $(1, -6)$, $(1, 2)$ and $(7, 2)$. What are the coordinates of $D$?

**10** Fill in the boxes to give a rule for each of the following tables.

**a**

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 3 | 4 | 5 | 6 | 7 |

$y = x + \square$

**b**

| $t$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $d$ | 4 | 6 | 8 | 10 | 12 |

$d = \square \times t + \square$

**c**

| $m$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $n$ | 0 | 4 | 8 | 12 | 16 |

$n = \square \times m - \square$

**d**

| $p$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $q$ | −5 | −3 | −1 | 1 | 3 |

$q = \square \times p - \square$

**e**

| $t$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $d$ | 12 | 9 | 6 | 3 | 0 |

$d = \square \times t + \square$

**f**

| $x$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | −1 | −4 | −7 | −10 | −13 |

$y = \square \times x - \square$

# Challenge exercise

**1** David and Angela have 10 CDs to divide between them.

**a** Copy and complete the following table showing how the CDs can be divided.

| CDs for David | 0 | 1 | | | | | | | | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| CDs for Angela | 10 | 9 | | | | | | | | 0 |

Let $x$ be the number of CDs that David has and $y$ be the number of CDs that Angela has.

**b** Write coordinates corresponding to each column of the table.

**c** Plot these points on a number plane with the $x$- and $y$-axes labelled from 0 to 10.

**d** Write a rule for $y$ in terms of $x$.


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**2** David and his twin brother Andrew are to share 10 CDs with Angela in such a way that the twins receive CDs in pairs, and have at least one pair of CDs.

**a** Copy and complete the table below, showing how the CDs can be divided.

| Pairs of CDs for David and Andrew | 5 | 4 | | | |
|---|---|---|---|---|---|
| Single CDs for Angela | 0 | 2 | | | |

Let $x$ be the number of pairs of CDs that the twins receive, and $y$ be the number of CDs that Angela receives.

**b** Write coordinates corresponding to each column of the table.

**c** Plot these points on a number plane, with the $x$- and $y$-axes labelled from 0 to 10.

**d** Write a rule for $y$ in terms of $x$.

**3** **a** $ABCD$ is a square. $A$ has coordinates (4, 5), $D$ has coordinates (8, 5) and $B$ has coordinates (4, 9). Find the coordinates of $C$.

**b** $OXYZ$ is a square. $O$ is the origin and $X$ is the point with coordinates (0, 5). Give the possible coordinates for the points $Y$ and $Z$.

**c** $ABCD$ is a square. $A$ has coordinates (0, 0) and $B$ has coordinates (4, 4). Find the possible coordinates of $C$ and $D$.

*Note*: In questions 4, 5 and 6, the number plane axes have markers at 1 cm intervals; that is, the point (1, 0) is 1 cm from the origin etc.

**4** $AB$ is an interval on the number plane. $A$ has coordinates (5, 0) and $B$ has coordinates (10, 0). Describe the points $C$ such that triangle $ABC$ has area 20 cm$^2$.

**5** $AB$ is an interval on the number plane. $A$ has coordinates (0, 4) and $B$ has coordinates (0, 10). Points $C$ and $D$ are such that $ABCD$ is a square of area 36 cm$^2$. Find the possible coordinates of $C$ and $D$.

**6** $AB$ is an interval on the number plane. $A$ has coordinates (0, 4) and $B$ has coordinates (0, 10). Points $C$ and $D$ are such that $ABCD$ is a rectangle of area 42 cm$^2$. Find the possible coordinates of $C$ and $D$.

**7** A very large garden grows pineapples and mangoes. The manager of the garden insists that the fruit is stacked as follows.

- Mangoes are placed in stacks of 10.
- Pineapples are placed in stacks of 5.

**a** List all the different ways you can choose 30 pieces of fruit.

| Stacks of mangoes | Stacks of pineapples |
|---|---|
| 3 | 0 |
| ⋮ | ⋮ |
| 0 | 6 |

Let $x$ be the number of mango stacks and $y$ be the number of pineapple stacks.

**b** List the coordinates (stacks of mangoes, stacks of pineapples).

**c** Plot these points on a number plane.

**d** Write a rule for $y$ in terms of $x$.


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**8** The admission prices to an agriculture show are:

Adults: \$9 each  Children: \$2 each

A group of people arrives at the ticket counter and pays a total of \$90.

Let $x$ be the number of adults and $y$ be the number of children in the group.

**a** List the integer coordinates $(x, y)$ that satisfy the rule $9x + 2y = 90$.

**b** Plot these points on a number plane.

**c** Find the number of children and the number of adults if the total number of people in the group is:

**i** 38  **ii** 31

**9** A square has vertices with coordinates $O(0, 0)$, $A(a, 0)$, $B(a, a)$, $C(0, a)$.

**a** State the area $s$ of the square in terms of $a$.

**b** Complete the table of values.

| $a$ | $\frac{1}{2}$ | 1 | $1\frac{1}{2}$ | 2 | $2\frac{1}{2}$ |
|---|---|---|---|---|---|
| $s$ | | | | | |

**c** Plot these points on a number plane. Note that they do not lie on a straight line.


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CHAPTER

13

Measurement and Geometry

# Triangles and constructions

Geometrical constructions are an enjoyable and practical part of geometry. They have been used for centuries by builders and others professions. Most constructions involve triangles, so we begin with the geometry of triangles.

The study of triangles was undertaken by the Babylonians as early as 3000 BCE. They knew some methods for working out the areas of some triangles. The ancient Egyptians also worked on the measurement of side lengths and areas of triangles.

The ancient Greeks introduced systematic methods for investigating the properties of triangles.

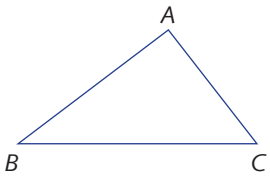
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# 13A Angles in triangles

In this section, we will prove two useful results about the angles of any triangle. You may have seen these two results already, but proving them may be new to you.

## Triangles

A **triangle** is formed by taking any three non-collinear points $A$, $B$ and $C$ and joining the three intervals $AB$, $BC$ and $CA$. These intervals are called the **sides** of the triangle, and the three points are called its **vertices** (the singular is **vertex**.)



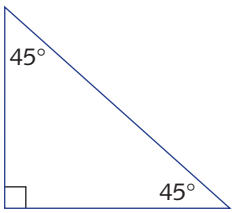

The triangle to the right is called the 'triangle $ABC$'.

This is written in symbols as $\Delta ABC$.

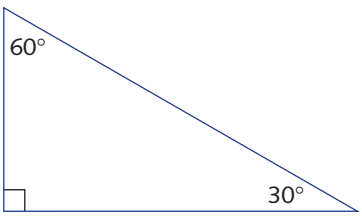
## Investigating the interior angles of a triangle

The first important result about triangles is that the sum of the three interior angles of a triangle is always 180°, whatever the triangle may look like. Here are three ways of demonstrating this result.

**1** Draw a number of different-looking triangles, measure their three angles and check that their sum is 180°. If you have set squares, they provide excellent examples of triangles.

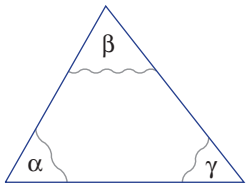


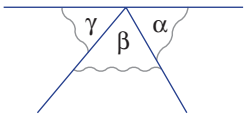

$90° + 45° + 45° = 180°$



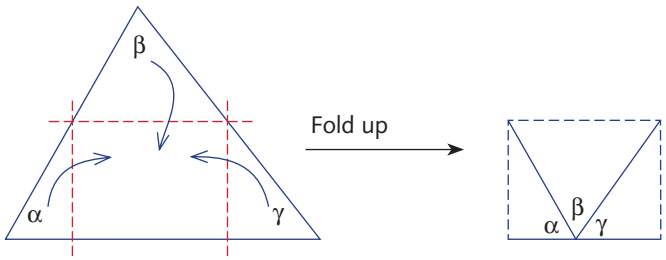

$90° + 60° + 30° = 180°$

**2** Cut out a triangle. Tear the corners off and place them together so that they form a straight angle.




**3** Cut out a triangle. Fold it without any tearing to demonstrate that the three interior angles form a straight angle.




The third demonstration is close to a proof.


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## Proving that the sum of the interior angles is 180°

Doing measurements and experiments on any number of different triangles does not prove a general result – however many triangles you check, there are always more. Here is an argument that establishes the result for any triangle.

The statement of the result is called a **theorem**. This comes from a Greek word meaning 'a thing to be gazed upon' or 'a thing contemplated by the mind' – our word 'theatre' comes from the same root.

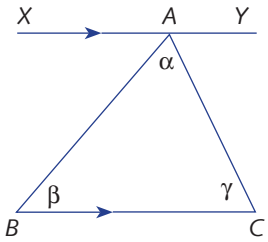
*Theorem*: The sum of the interior angles of a triangle is 180°.

*Proof*: Let $\Delta ABC$ be a triangle.

Let $\angle BAC = \alpha$, $\angle B = \beta$ and $\angle C = \gamma$.

We must prove that $\alpha + \beta + \gamma = 180°$.

Draw the line $XAY$ parallel to $BC$ through the vertex $A$.




Then $\angle XAB = \beta$ (alternate angles, $XY \parallel BC$),

and $\angle YAC = \gamma$ (alternate angles, $XY \parallel BC$).

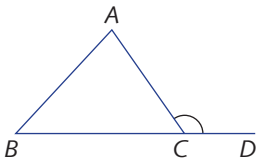
Hence, $\alpha + \beta + \gamma = 180°$ (straight angle at $A$).

## A shorter notation for angles

In the above proof, we referred to $\angle B$ and $\angle C$, rather than to $\angle ABC$ and $\angle ACB$. We can use this shorter notation because there is only one non-reflex angle at each of the vertices $B$ and $C$. However, there are several angles at the vertex $A$, so we have to use a longer form, such as $\angle BAC$ and $\angle XAB$, to show precisely which one we mean.

## The exterior angles of a triangle

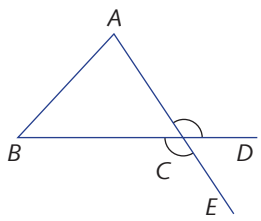
Let $\Delta ABC$ be a triangle, with the side $BC$ produced to $D$. (The word 'produced' means 'extended'.) Then the marked angle $\angle ACD$ formed by the side $AC$ and the extension $CD$ is called an **exterior angle** of the triangle.




The angles $\angle A$ and $\angle B$ are called the **opposite interior angles**, because they are opposite the exterior angle at $C$.

An exterior angle and the interior angle adjacent to it are adjacent angles on a straight line, so they are supplementary:

$\angle ACD + \angle ACB = 180°$ (straight angle at $C$)




There are two exterior angles at each vertex, as shown in the diagram to the right. Because the two angles are vertically opposite, they are equal in size:

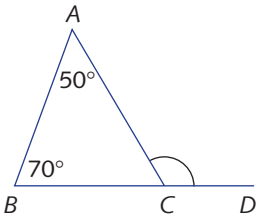
$\angle ACD = \angle BCE$ (vertically opposite angles at $C$)


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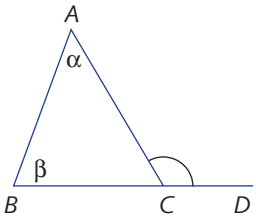
## The exterior angle theorem

The vital fact about exterior angles is that an exterior angle of a triangle is equal to the sum of the two opposite interior angles.

This theorem can be proven using the angle sum of the triangle. The argument on the left below gives a particular case, while the argument on the right gives the general case.




In the diagram above,
$\angle ACB + 50° + 70° = 180°$
(angle sum of $\Delta ABC$)
$\angle ACB = 180° - (50° + 70°)$
so $\angle ACD = 50° + 70°$
(straight angle at $C$)
$= 120°$




In the diagram above,
$\angle ACB + \alpha + \beta = 180°$
(angle sum of $\Delta ABC$)
$\angle ACB = 180° - (\alpha + \beta)$
so $\angle ACD = \alpha + \beta$
(straight angle at $C$)

The theorem can also be proven without using the angle sum of a triangle result by drawing a parallel line. A formal proof of this is given below.

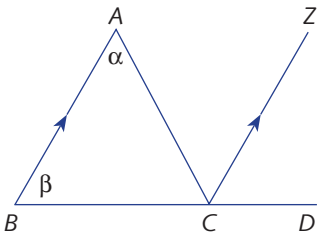
*Theorem*: An exterior angle of a triangle equals the sum of the opposite interior angles.

*Proof*: Let $\Delta ABC$ be a triangle, with the side $BC$ produced to $D$.

Let $\angle A = \alpha$ and $\angle B = \beta$.

We need to prove that $\angle ACD = \alpha + \beta$.

Draw the ray $CZ$ through $C$ parallel to $BA$ as shown in the diagram.




Then $\angle ZCD = \beta$ (corresponding angles, $BA \parallel CZ$),

and $\angle ACZ = \alpha$ (alternate angles, $BA \parallel CZ$).

Hence, $\angle ACD = \alpha + \beta$ (adjacent angles at $C$).

### Two theorems about the angles of a triangle

- The sum of the interior angles of a triangle is 180°.
- An exterior angle of a triangle is equal to the sum of the opposite interior angles.

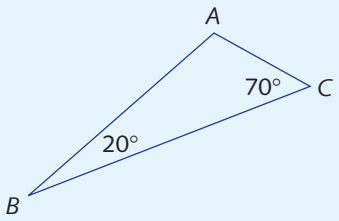
These two theorems can now be used in geometrical problems.

Always be specific and name the triangles and angles involved.


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### Example 1

Find $\angle A$ in the triangle opposite.



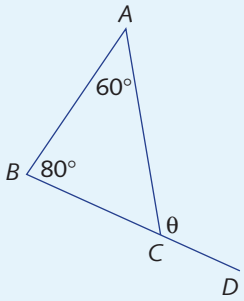

### Solution

$\angle A + 20° + 70° = 180°$ (angle sum of $\Delta ABC$)
so $\angle A = 90$

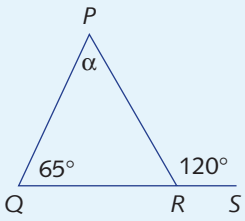
### Example 2

Find $\theta$ and $\alpha$ in the diagrams.

**a**




**b**




### Solution

**a** $\theta = 60° + 80°$ (exterior angle of $\Delta ABC$)
so $\theta = 140°$

**b** $\alpha + 65° = 120°$ (exterior angle of $\Delta PQR$)
so $\alpha = 55°$

## Exercise 13A

**1** **a** Draw a large triangle $ABC$. Then produce (extend) the side $AB$ to $D$.

**b** Measure the three interior angles of the triangle and confirm that their sum is 180°.

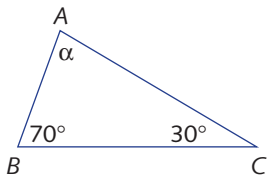
**c** Measure the exterior angle $\angle CBD$ and confirm that it is the sum of $\angle A$ and $\angle C$.


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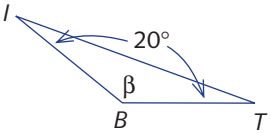
Example 1

**2** Use the interior angle sum of a triangle to find $\alpha$, $\beta$, $\gamma$ or $\theta$ in the diagrams below.

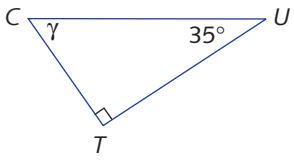
**a**



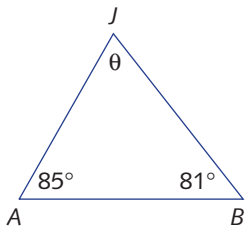

**b**



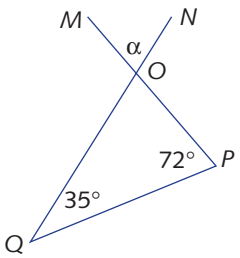

**c**



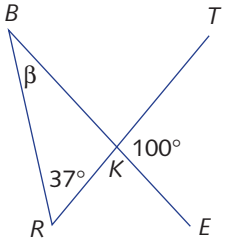

**d**



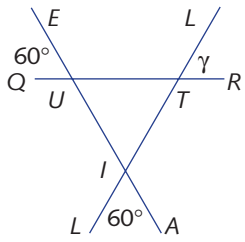

**e**



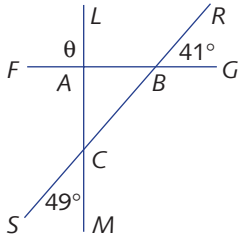

**f**



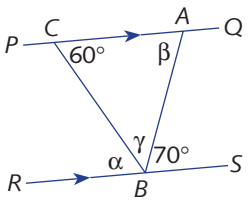

**g**




**h**



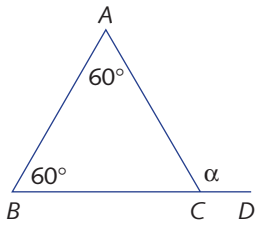

**i**



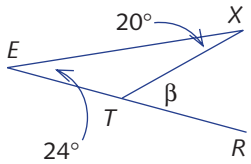

Example 2

**3** Use the exterior angle theorem to find $\alpha$, $\beta$, $\gamma$ or $\theta$ in the diagrams below.

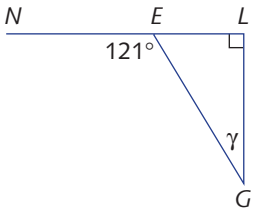
**a**



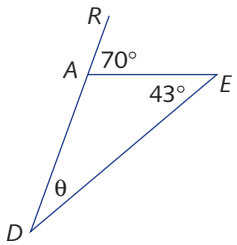

**b**



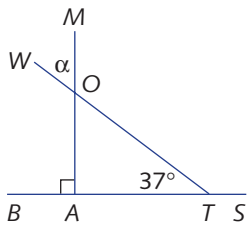

**c**



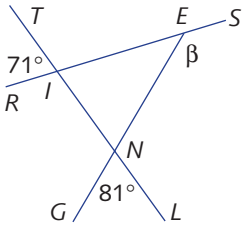

**d**



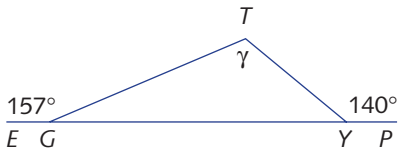

**e**



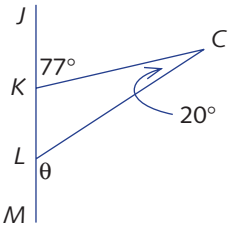

**f**




**g**




**h**




**4** Explain your answers to these questions.

**a** Can a triangle have two obtuse angles?

**b** Can a triangle have two right angles?

**c** What is the minimum number of acute angles a triangle can have?

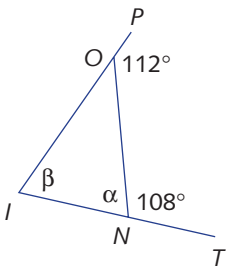

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**d** Can a triangle have an acute exterior angle?

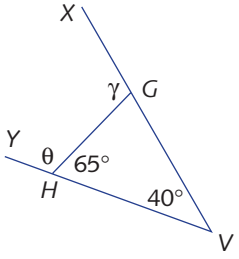
**e** Can a triangle have two acute exterior angles?

**5** Use the exterior angle theorem to find $\alpha$, $\beta$, $\gamma$ and $\theta$ in each diagram. Give reasons.

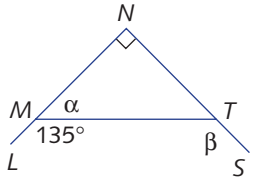
**a**



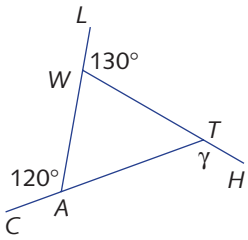

**b**



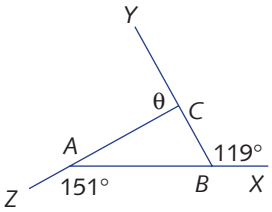

**c**



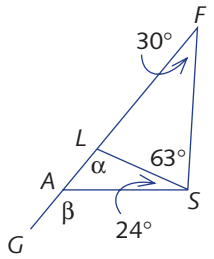

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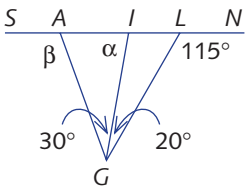

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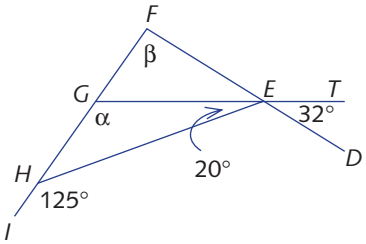

**f**




**g**

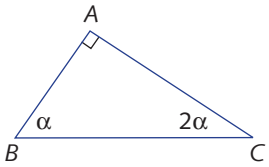



**h**

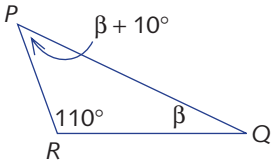



**6** Find the values of the pronumerals in the diagrams below.

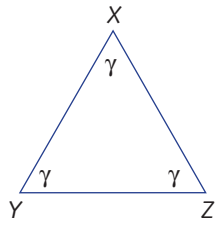
**a**



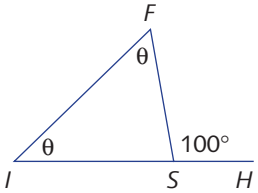

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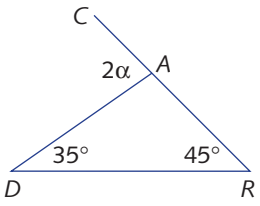

**c**



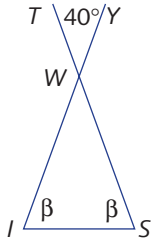

**d**



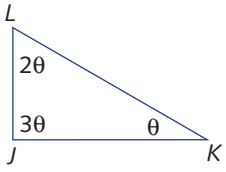

**e**



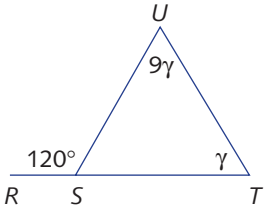

**f**




**g**




**h**

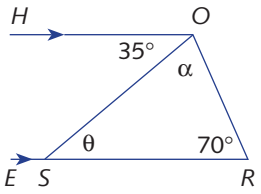




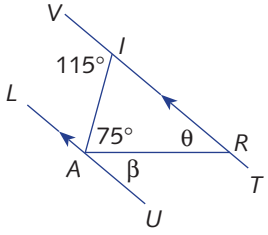

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**7** Find $\alpha$, $\beta$, $\gamma$ and $\theta$ in the following diagrams, giving reasons.

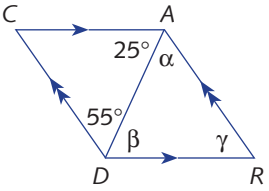
**a**



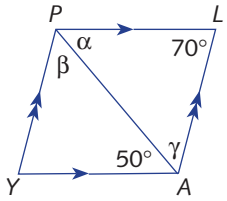

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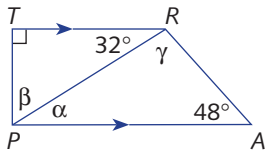

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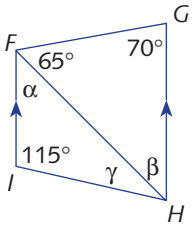

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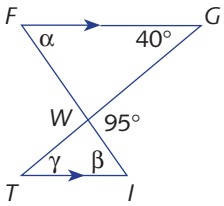

**e**



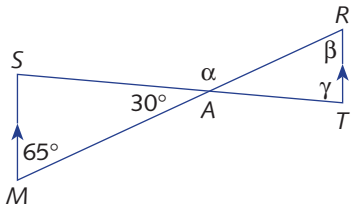

**f**




**g**

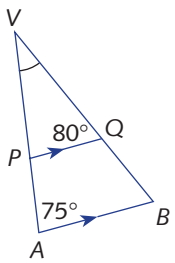



**h**

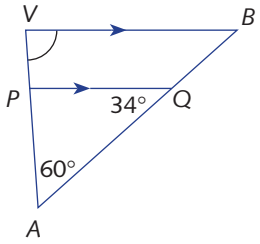



**8** In each case, find the size of the marked angle, $\angle AVB$, giving reasons.

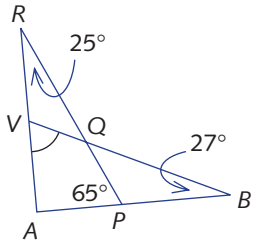
**a**



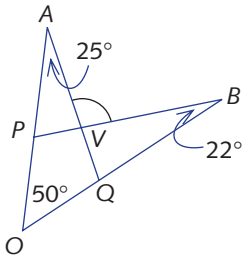

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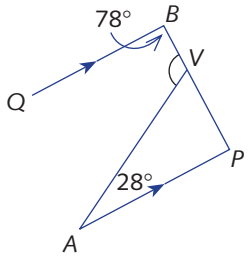

**c**



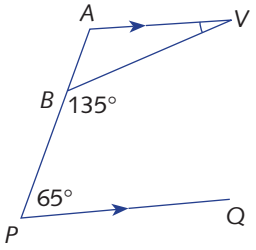

**d**



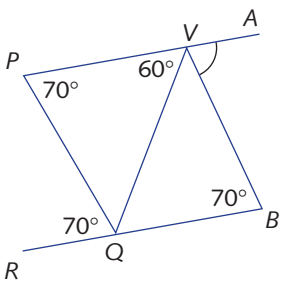

**e**



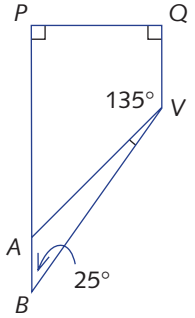

**f**



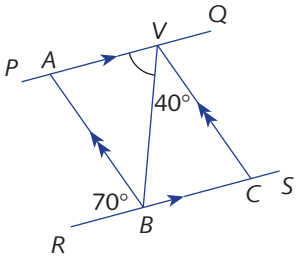

**g**




**h**




**i**







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# 13B Circles and compasses

From this section onwards, you will need compasses for your geometrical constructions. Make sure that your pencil is very sharp.

'Compasses' is a plural word. We use 'a pair of compasses', just as we wear 'a pair of trousers' and use 'a pair of scissors'. The singular word 'compass' is the instrument that navigators use to find magnetic north.

## Using compasses to draw a circle

You are probably used to drawing circles with a pair of compasses, but here is an exercise just to get the language sorted out.

Copy the interval $AB$ and point $O$ on your page. Open your compasses to the length of the interval $AB$. Then place the point of your compasses firmly into the point $O$, called the **centre**. Holding the compasses only by the very top, draw a circle.

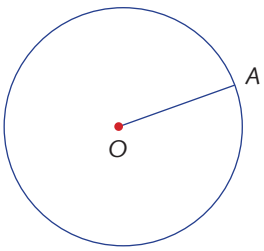



This is called **drawing a circle with centre $O$ and radius $AB$**. Notice that **every point on the circle is the same distance from the centre $O$**, because the distance between the point and the pencil lead never changes.

## Parts of circles

Here we will identify some important parts of a circle. We start by drawing a circle with centre $O$.

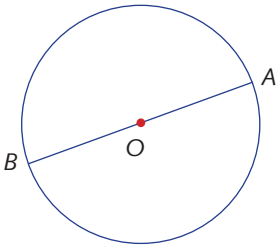
### Radius




Draw an interval from any point $A$ on the circle to the centre $O$. This interval $AO$ is called a **radius** of the circle. Every radius of the circle has the same length, because the setting of the compasses remained the same while the circle was being drawn.

The word 'radius' is used both for the interval $AO$ and for the length of the interval $AO$. 'Radius' is a Latin word meaning 'a spoke of a wheel'. Its plural is 'radii'.

### Diameter

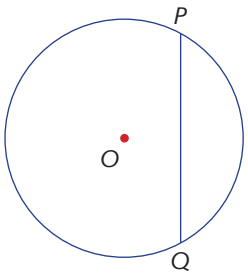



Draw a line through the centre $O$, cutting the circle at $A$ and $B$. The interval $AB$ is called a **diameter** of the circle. Every diameter has length twice that of any radius, because a diameter consists of two radii put together.

The word 'diameter' is used both for the interval $AB$ and for the length of the interval $AB$. It comes from Greek and means 'to measure through'.


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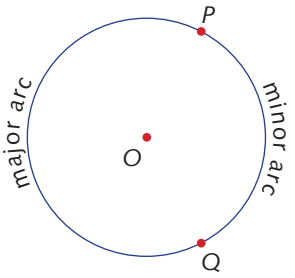
### Chord




Choose any two distinct points $P$ and $Q$ on the circle, and join the interval $PQ$. This interval is called a **chord** (from a Greek word meaning 'a cord or string').

A diameter is thus a chord passing through the centre.

### Arc




Choose two distinct points $P$ and $Q$ on the circle. These two points cut the circle into two curved parts called **arcs**.

There are two arcs $PQ$. The larger arc is called the **major arc** $PQ$ and the smaller arc is called the **minor arc** $PQ$. ('Major' and 'minor' are the Latin words for 'larger' and 'smaller'; 'arc' is from a Latin word for a bow or arch.)

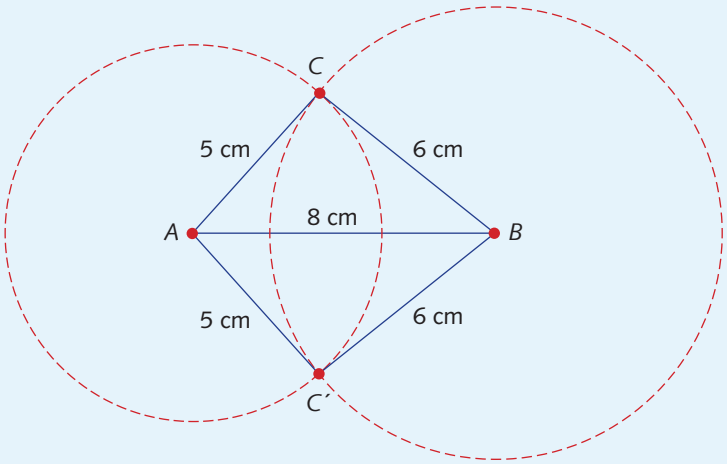
## Constructing a triangle with given measurements

In the exercises, you will construct triangles with different side lengths and angle sizes. Here is a simple way to construct a triangle whose three side lengths are given.

**Example 3**

Construct a triangle $ABC$ in which $AB = 8$ cm, $AC = 5$ cm and $BC = 6$ cm.

**Solution**




*Step 1*: Draw an interval $AB$ of length 8 cm.

*Step 2*: Draw a circle with centre $A$ and radius 5 cm.

*Step 3*: Draw a circle with centre $B$ and radius 6 cm.

Let $C$ and $C'$ be the two points where the circles intersect.

Then the triangles $ABC$ and $ABC'$ satisfy the conditions.


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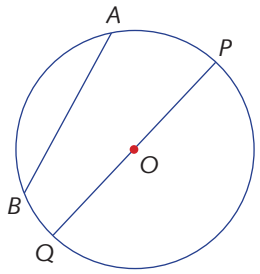
## Circles and constructions

- A circle is drawn by opening the arms of the compasses to some given radius and placing the point on some given centre.
- Some words associated with circles:
  **centre, radius, diameter, chord, major and minor arcs**
- All radii of a circle have equal length.
- Every diameter of a circle is twice the length of the radius of that circle.
- Compasses can be used to construct triangles with given side lengths.

## Exercise 13B

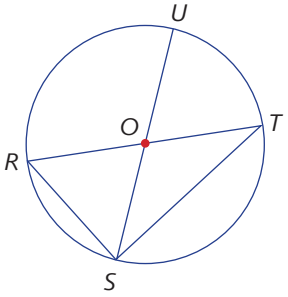
**1** In the diagram on the right, what parts of the circle are:

**a** the point $O$?
**b** the interval $OP$?
**c** the interval $PQ$?
**d** the interval $AB$?
**e** the curve $PAB$?
**f** the curve $PQB$?




**2** In the diagram on the right, name:

**a** the centre of the circle
**b** two diameters of the circle
**c** two chords that are not diameters
**d** four radii
**e** a major arc (there are four)
**f** a minor arc (there are four)




**3** **a** Set your compasses to a radius of 8 cm. Then choose a point $O$ in the middle of your page and draw a circle with centre $O$ and radius 8 cm.

**b** Change the setting of the compasses to 6 cm and draw a circle with the same centre $O$ and radius 6 cm.

**c** Repeat this process, drawing circles with centre $O$ and radii 4 cm and 2 cm.

These four circles are called **concentric circles** because they all have the same centre.

**d** Draw a horizontal line through the centre $O$. From left to right, label the eight points where the line intersects the circles with the letters $A$, $B$, $C$, $D$, $E$, $F$, $G$ and $H$.

**e** Check that the eight intervals $AB$, $BC$, $CD$, $DO$, $OE$, $EF$, $FG$ and $GH$ all have the same lengths.


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**4** **a** Set your compasses to a radius of 3 cm, and do not change the radius again until the final part of this question.

**b** Choose a point $O$ in the middle of your page and draw a circle with centre $O$.

**c** Choose any point $A$ on the circle, and draw a second circle with centre $A$.

**d** Let the two circles intersect at $B$ and $F$.

**e** With centre $B$, draw a third circle, cutting the first circle at $A$ and $C$.

**f** With centre $C$, draw a fourth circle, cutting the first circle at $B$ and $D$.

**g** With centre $D$, draw a fifth circle, cutting the first circle at $C$ and $E$.

**h** With centre $E$, draw a sixth circle this circle should cut the first circle at $D$ and $F$.

**i** With centre $F$, draw a seventh circle this circle should cut the first at $E$ and $A$.

**j** Change the setting of the compasses to 6 cm. Now draw a circle with centre $O$ that just touches the outsides of the six outer circles.

Example 3

**5** Construct a triangle $ABC$ in which the three side lengths are $AB = 9$ cm, $AC = 6$ cm and $BC = 5$ cm as follows.

**a** Draw a horizontal interval $AB$ of length 9 cm, leaving about 6 cm above.

**b** With radius 6 cm and centre $A$, draw an arc above the interval $AB$.

**c** With radius 5 cm and centre $B$, draw another arc above the interval $AB$.

**d** Let the two arcs meet at $C$, and join the intervals $AC$ and $BC$.

**e** Measure the sizes of the three angles with your protractor.

**6** **a** Construct a triangle $ABC$ in which:

- two of the side lengths are $AB = 7$ cm and $AC = 4$ cm
- the angle between these two sides is $\angle A = 110°$.

**b** Join the interval $BC$ and measure its length.

**7** **a** Construct a triangle $ABC$ in which:

- two of the angles are $\angle A = 40°$ and $\angle B = 30°$
- the side joining these vertices has length $AB = 8$ cm.

**b** Explain why the third angle $\angle C$ is 110°.

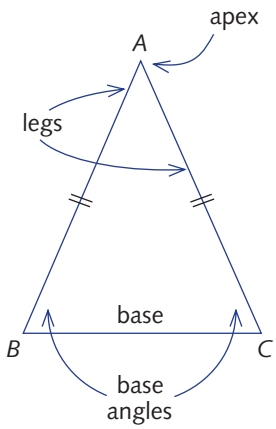
**c** Measure the lengths of the sides $AC$ and $BC$.


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# 13C Isosceles and equilateral triangles

Triangles with two or three sides equal have some interesting properties. At this stage, however, we can only give informal proofs of the results in this section.

## Isosceles triangles

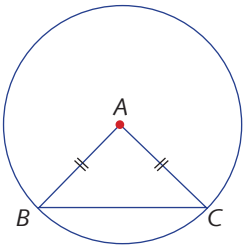



An **isosceles triangle** is a triangle with two (or more) sides equal.

- The equal sides $AB$ and $AC$ of the isosceles triangle $ABC$ to the right are called the **legs**. They have been marked with double dashes to indicate that they are equal in length.
- The vertex $A$ where the legs meet is called the **apex**.
- The third side $BC$ is called the **base**.
- The angles $\angle B$ and $\angle C$ at the base are called **base angles**.

The word *isosceles* is a Greek word meaning 'equal legs' – *iso* means 'equal', and *sceles* means 'leg'.

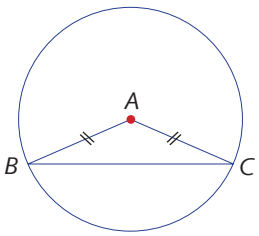
## Constructing an isosceles triangle using a circle




Any two radii $AB$ and $AC$ of the circle opposite are equal. Thus they form the equal legs of the isosceles triangle $ABC$. The chord $BC$ is the base and the centre $A$ is the apex.

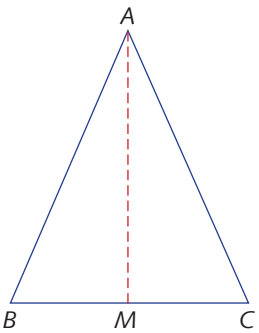
Using two radii of a circle is an easy way to construct an isosceles triangle. It is used in the investigation below.

## The base angles of an isosceles triangle are equal




The base angles of an isosceles triangle are equal, regardless of the size and shape of the triangle. This is illustrated in the opposite diagram, and is proved in the Year 8 textbook.

## A test for an isosceles triangle




If a triangle has two equal angles, then the two sides opposite those angles are equal and the triangle is isosceles.

It is possible to give a proper proof of this theorem now, provided that we use the previous result that the base angles of an isosceles triangle are equal. See the challenge questions for the details.


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## Converses of the two results

The two theorems discussed so far are **converses** of each other. The first (slightly reworded) says:

*If two sides of a triangle are equal, then the opposite angles are equal.*

The second says:

*If two angles of a triangle are equal, then the opposite sides are equal.*

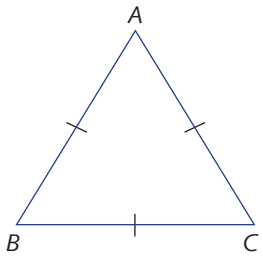
These ideas were introduced in Chapter 6.

### Isosceles triangles

- An **isosceles triangle** is a triangle with two (or more) sides equal.
  - The equal sides are called the **legs**.
  - The legs meet at the **apex**.
  - The third side is the **base**.
  - The angles opposite the legs are called **base angles**.
- Two radii of a circle and the chord joining them form an isosceles triangle.
- The base angles of an isosceles triangle are equal.
- Conversely, if two angles of a triangle are equal, then the sides opposite those angles are equal.

## Constructing an equilateral triangle using two circles

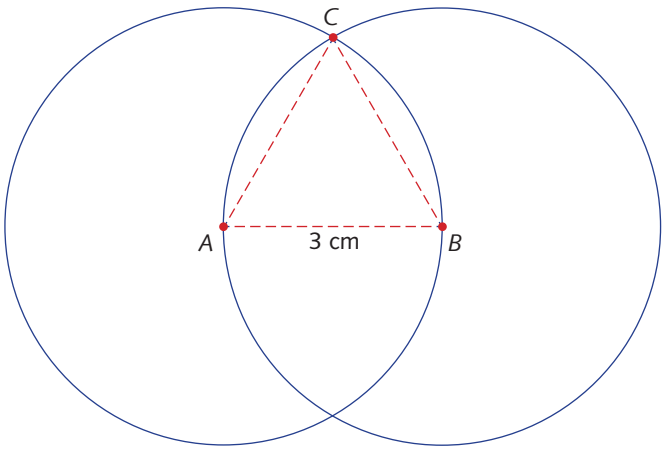
An **equilateral triangle** is a triangle in which all three sides have equal length.




The diagram to the right shows an equilateral triangle $ABC$. Notice that it is an isosceles triangle in three different ways, because the base could be taken as $AB$, $BC$ or $CA$.

The word *equilateral* comes from Latin – *equi* means 'equal' and *latus* means 'side'.

Here is a construction of an equilateral triangle with sides of length 3 cm.




Can you explain why all the sides of this triangle $ABC$ are 3 cm long?


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## The angles of an equilateral triangle are all 60°

Use a protractor to confirm that all three angles of the equilateral triangle that you drew above are 60°.

We can prove this result, provided that we use the earlier result that the base angles of an isosceles triangle are equal.

Any two angles in the triangle must be equal because they are opposite equal sides. Therefore all the angles must be equal. Since they add up to 180°, each angle must be $180° \div 3 = 60°$.

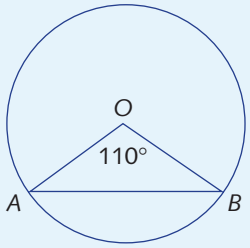
### Equilateral triangles

- An **equilateral triangle** is a triangle with all three sides equal.
- An equilateral triangle can be constructed using two circles, as described above.
- The interior angles of an equilateral triangle are all 60°.

### Example 4

$O$ is the centre of the circle.

Find $\angle B$ in the diagram opposite.




#### Solution

First, $AO = BO$ (radii)

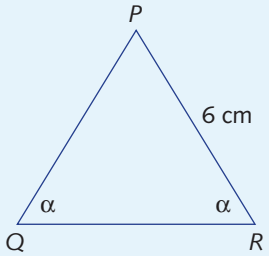
Hence, $\angle A = \angle B$ (isosceles)

$\angle A + \angle B = 180° - 110° = 70°$

Hence, $\angle B = 35°$ (base angles of isosceles $\Delta ABO$)

### Example 5

Find the length of $PQ$ in the diagram opposite.



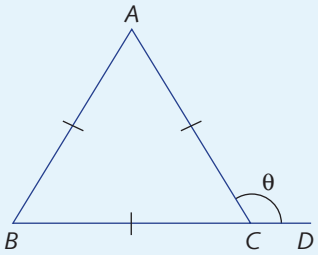

#### Solution

$PQ = 6\,\text{cm}$ (opposite angles $\angle Q$ and $\angle R$ are equal)


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### Example 6

Find θ in the diagram.




### Solution

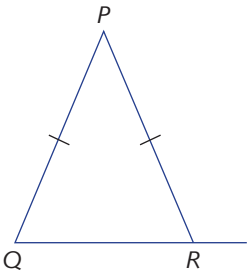
First, $\angle ACB = 60°$ (equilateral $\Delta ABC$)

Hence, $\theta + 60° = 180°$ (straight angle at $C$),

so $\theta = 120°$

## Exercise 13C

**1** In the isosceles triangle to the right, $PQ = PR$.




**a** Name the apex of the isosceles triangle.

**b** Name the legs and measure their lengths.

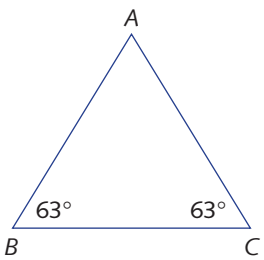
**c** Name the base of the isosceles triangle.

**d** Name the base angles and measure their sizes.

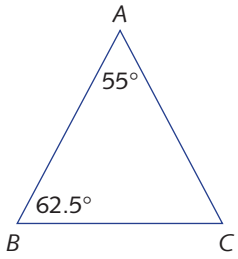
**e** What property of isosceles triangles do these angle sizes illustrate?

**2** Determine whether each triangle $\Delta ABC$ below is an isosceles triangle. Give reasons.

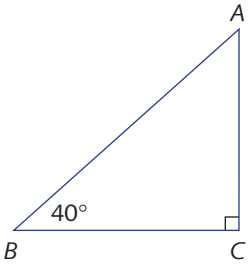
**a**



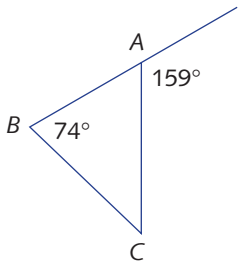

**b**



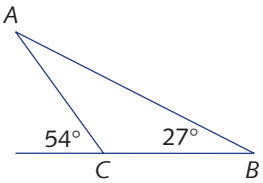

**c**



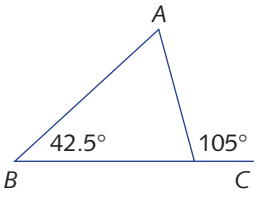

**d**




**e**




**f**






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**3** In each part, draw a diagram and mark equal intervals and the given sizes of angles.

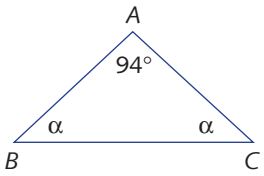
**a** In $\Delta ABC$, $AB = AC$. Mark $D$, a point on $BC$ on the opposite side of $C$ from $B$. If $\angle ACD = 110°$, calculate the size of $\angle ABC$.

**b** $P$ and $Q$ are points on a circle with centre $O$ such that $\angle POQ = 56°$. Calculate the size of $\angle OPQ$.

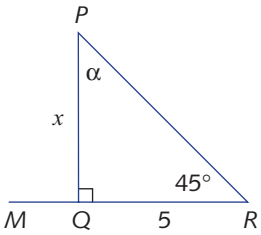
Example 4, 5, 6

**4** Find the values of the pronumerals in these diagrams.

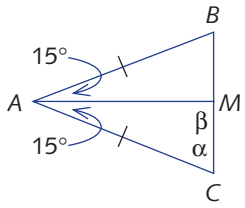
**a**



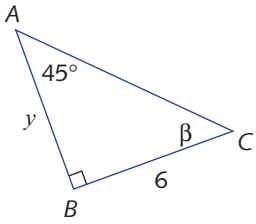

**b**



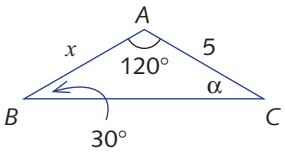

**c**



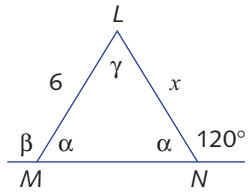

**d**



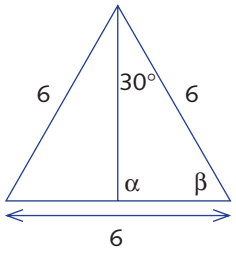

**e**



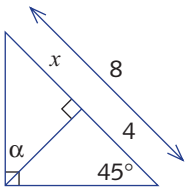

**f**



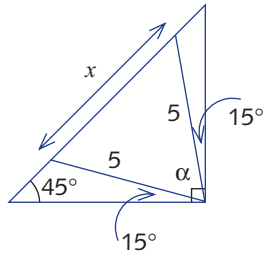

**g**




**h**

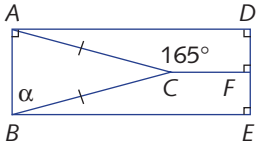



**i**

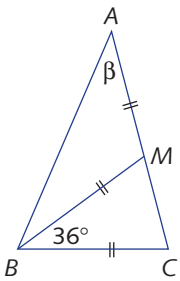



**5** Find the values of the pronumerals in these diagrams.

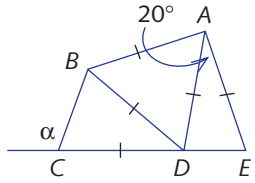
**a**



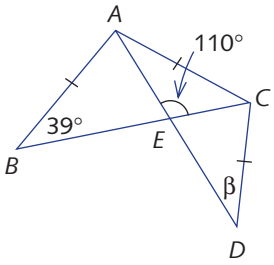

**b**



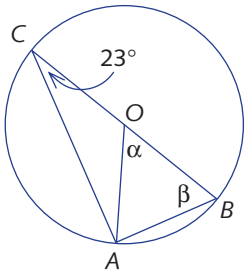

**c**



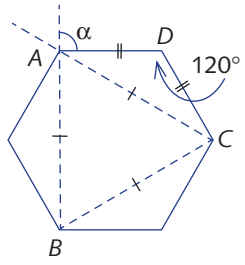

**d**




**e**




**f**





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# 13D Constructions with compasses and a straight edge

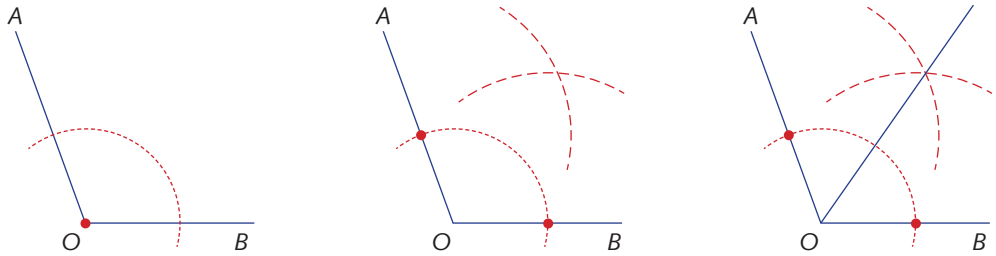
Careful constructions with compasses and a straight edge have always been an essential part of geometry. These constructions are based on a fundamental fact about circles:

*All radii of a circle are equal.*

## Construction – Bisecting an angle

The word **bisect** means to divide into two equal parts.

The diagram below shows the steps to follow to bisect a given angle $\angle AOB$.




Step 1 Step 2 Step 3

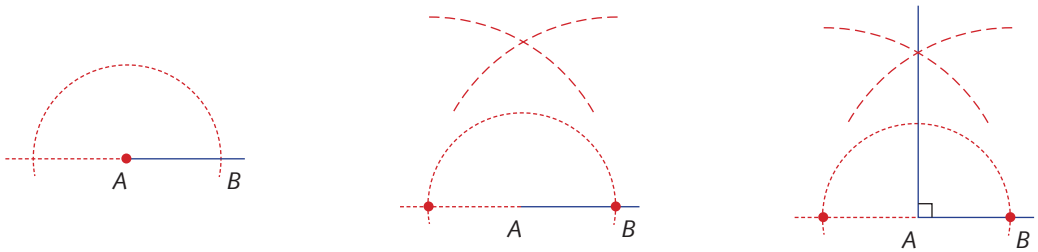
The two arcs in step 2 can have a different radius from the arc in step 1.

Folding the paper along the constructed line provides an informal proof that the construction works. The arms $OA$ and $OB$ then fall exactly on top of each other, so $\angle AOB$ has been cut into two equal angles.

The line you have constructed also bisects the reflex angle $\angle AOB$. (Can you prove this?)

## Construction – A right angle at the endpoint of an interval

A right angle is half a straight angle. Thus bisecting a straight angle using the previous construction will give a right angle. We begin by producing (extending) the interval $BA$.




Step 1 Step 2 Step 3

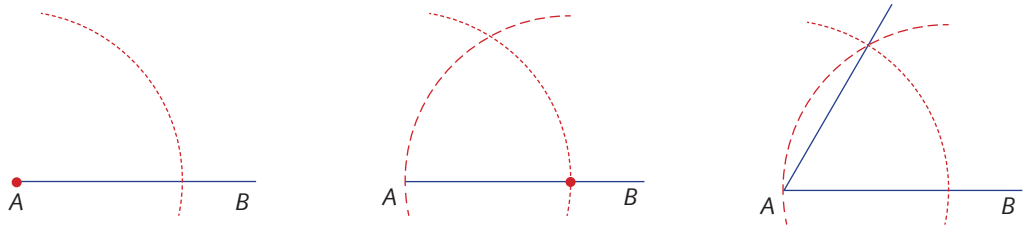
The two arcs in step 2 will need to have a larger radius than the arc in step 1.

To construct a right angle at a point within an interval, the same construction works.


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## Construction – An angle of 60° at the endpoint of an interval

The angles of an equilateral triangle are all 60°. Thus constructing an equilateral triangle will give an angle of 60°.




Step 1 Step 2 Step 3

This time the arcs in steps 1 and 2 must have the same radius.

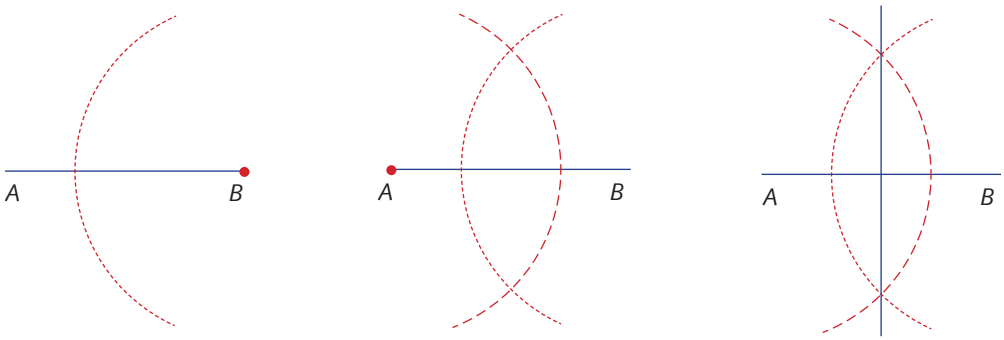
## Construction – Further angles by bisection

Many other angles can now be constructed by applying the angle bisection construction to angles already constructed. For example:

- Bisecting 90° will produce an angle of 45°.
  - Bisecting again leads to $22\frac{1}{2}°$ and $67\frac{1}{2}°$.
- Bisecting 60° produces an angle of 30°.
  - Bisecting again leads to 15° and 75°.

## Construction – The perpendicular bisector of an interval

Use these steps to bisect a given interval $AB$.




Step 1 Step 2 Step 3

The arcs in steps 1 and 2 must have the same radius.

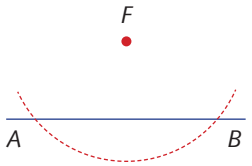
Informally speaking, the diagram is symmetric about the line you have constructed, as you can see by folding the paper along it. This means that the line bisects $AB$ and is perpendicular to $AB$.

*Note*: You can use the construction above to produce the midpoint of an interval, even if you don't actually need the perpendicular bisector.

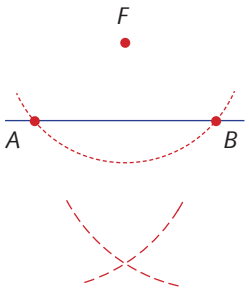

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## Construction – Dropping a perpendicular from a point to a line

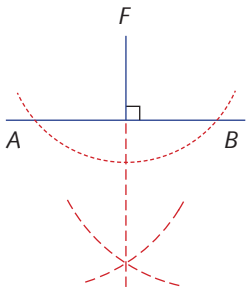
Use these steps to construct a line passing through a given point $F$ and perpendicular to a given line $AB$.




Step 1




Step 2




Step 3

The two arcs in step 2 can have a different radius from the arc in step 1.

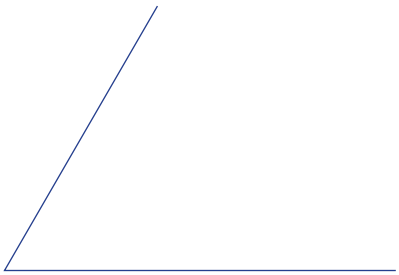
Informally speaking, the diagram is symmetric about the line you have constructed, so the line is perpendicular to $AB$.

## Exercise 13D

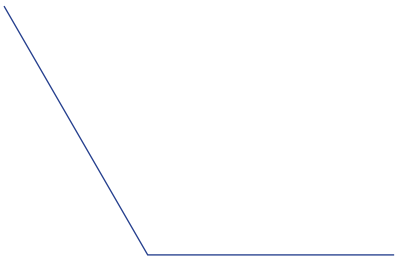
In Question 1, make an accurate copy of the diagrams. (You may need to extend the two rays.) Use only compasses, a straight edge and a sharpened pencil.

**1** Use the angle bisection construction to bisect each angle.

**a**



**b**



**2** Use the equilateral triangle construction to construct an angle of 60° at the endpoint $A$ of an interval $AB$.

**3** Use the construction for bisecting a straight angle to construct a right angle at the endpoint $A$ of an interval $AB$. (First produce the interval $BA$ to a point $X$.) Do this first with an interval that is on a slope, and then with one that is vertical.

**4** Each part below requires a combination of constructions. Draw an interval $AB$.

**a** Construct angles of 30° and 15° at point $A$.

**b** Construct angles of 45° and $22\frac{1}{2}°$ at $A$.

**c** Construct an angle of 150° at $A$.

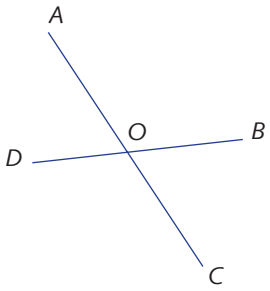
**d** Construct an angle of 135° at $A$.

**5** Draw three different intervals and construct the perpendicular bisector of each interval.

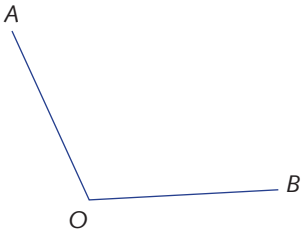
**6** Draw three different intervals and use the construction for dropping a perpendicular to construct the line a point $P$ perpendicular to $AB$.


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**7** **a** Construct the bisectors of $\angle AOB$ and $\angle BOC$. Check that the two bisectors are perpendicular. Do this on your page first with $AB = 8$ cm and $DB = 10$ cm.




**b** Use the angle bisection construction seven times to divide $\angle AOB$ into eight equal parts. Do this on your page with $OA = 10$ cm and $OB = 10$ cm.




**8** Draw an interval then construct a square that has that interval as one of its sides.

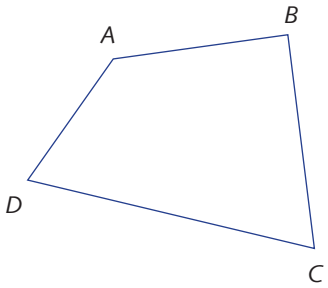
**9** **a** Draw a circle with centre $O$. Draw a square $ABCD$ whose vertices lie on the square.

**b** Bisect $\angle AOB$ and $\angle BOC$.

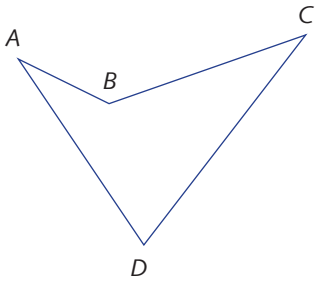
**c** Hence, find eight points equally spaced around the circle.

# 13E Quadrilaterals

A **quadrilateral** is a plane figure bounded by four straight sides.




The quadrilateral on the left is called a **convex quadrilateral** because none of its four interior angles is a reflex angle.



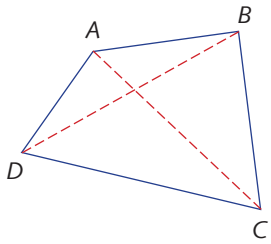

The quadrilateral on the right is called a **non-convex quadrilateral** because one of its interior angles is a reflex angle.

*Note*: We do not allow any interior angle of a quadrilateral to be 180° – such a figure is best described as a triangle.

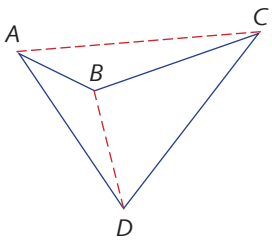

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The word quadrilateral comes from Latin – *quadri* means 'four' and *latus* means 'side'.

A quadrilateral has two **diagonals** that join opposite vertices.




Both diagonals of a convex quadrilateral are inside the quadrilateral.




The quadrilateral above is non-convex – one diagonal is inside the quadrilateral and the other is outside.

## The interior angles of a quadrilateral have sum 360°

The interior angles of a quadrilateral always add to 360°. Here are two ways to demonstrate this result for a particular quadrilateral:

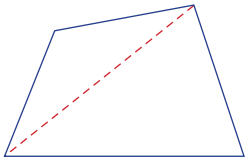
**1** Construct quadrilaterals of different shapes and measure their angles.

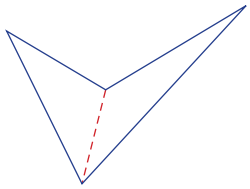
**2** Cut out a quadrilateral. Tear off the four corners and show that they fit together to form a revolution.

It is much better, however, and a lot less trouble, to give a proper proof.

*Theorem*: The sum of the interior angles of a quadrilateral is 360°.

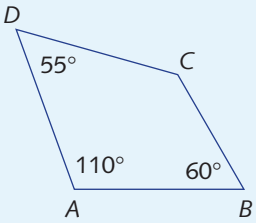
*Proof*: There are two kinds of quadrilaterals, as shown in the diagrams below. Join a pair of opposite vertices to form two triangles as shown. The angle sum of each triangle is 180°. Hence, the angle sum of the quadrilateral is 360°.





### Example 7

Find the size of $\angle C$ in the diagram opposite.




### Solution

$\angle C + 110° + 55° + 60° = 360°$ (angle sum of quadrilateral $ABCD$)

$\angle C + 225° = 360°$

so $\angle C = 135°$


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## Quadrilaterals

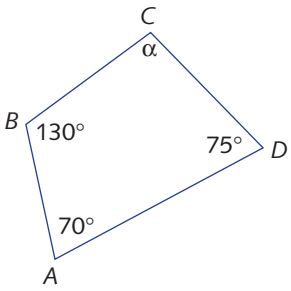
- A **quadrilateral** is a plane figure that is bounded by four straight sides and has four vertices.
- A quadrilateral is **convex** if each of its interior angles is less than 180°. Both diagonals of a convex quadrilateral lie inside the figure.
- A quadrilateral is **non-convex** if one of its interior angles is greater than 180°. One diagonal of a non-convex quadrilateral lies outside it, the other inside.
- The sum of the interior angles of a quadrilateral is 360°.

# Exercise 13E

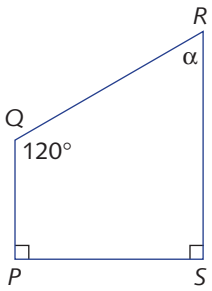
Example 7

**1** Find $\alpha$ in each diagram.

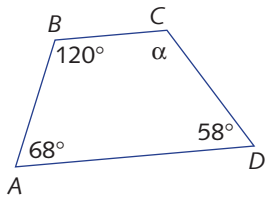
**a**



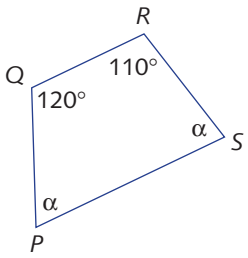

**b**



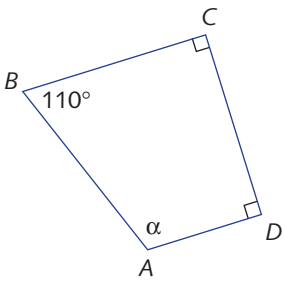

**c**



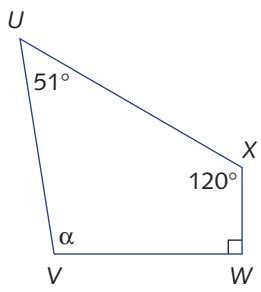

**d**




**e**




**f**

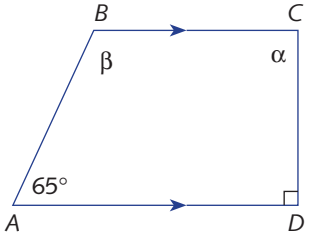



**2** In each part, three angles of a quadrilateral are given. Calculate the size of the fourth angle of the quadrilateral. Draw a quadrilateral with these angle sizes.

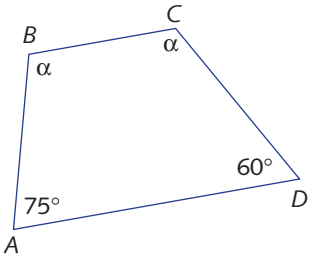
**a** 90°, 90°, 120°
**b** 60°, 90°, 120°
**c** 70°, 150°, 55°
**d** 75°, 75°, 75°
**e** 10°, 10°, 10°
**f** 80°, 90°, 80°

**3** Find the values of the pronumerals in these diagrams.

**a**



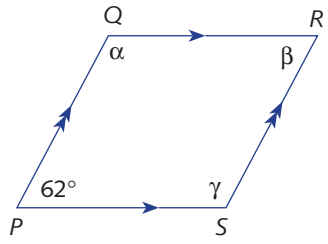

**b**



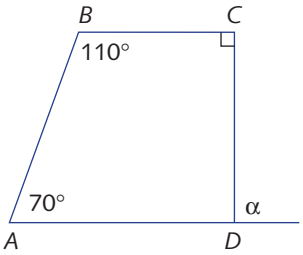


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**c**

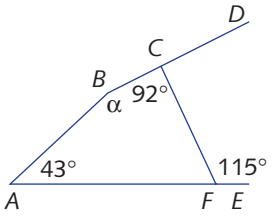



**d**

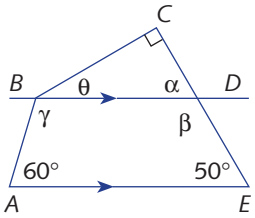



**4** Find the values of the pronumerals in these diagrams.

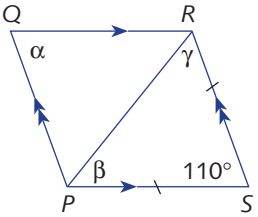
**a**



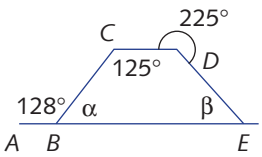

**b**



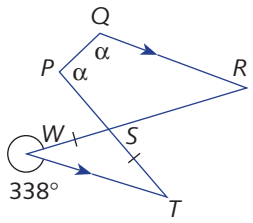

**c**



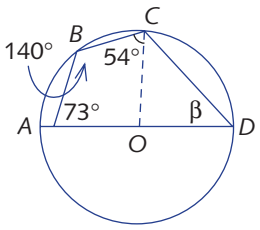

**d**




**e**

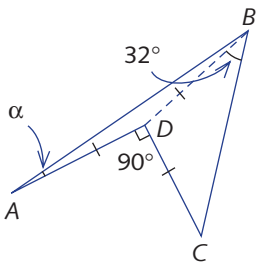



**f**

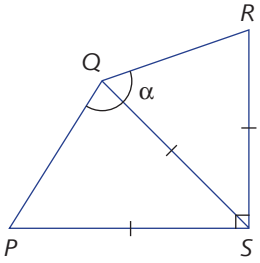



**5** Find the values of the pronumerals in these diagrams.

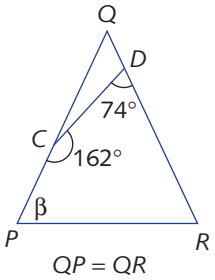
**a**



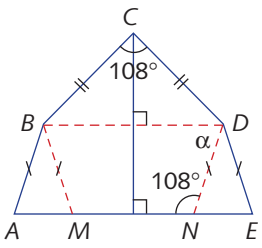

**b**



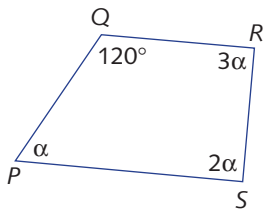

**c**



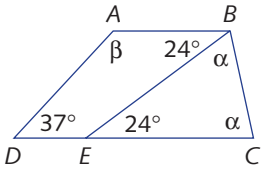

**d**



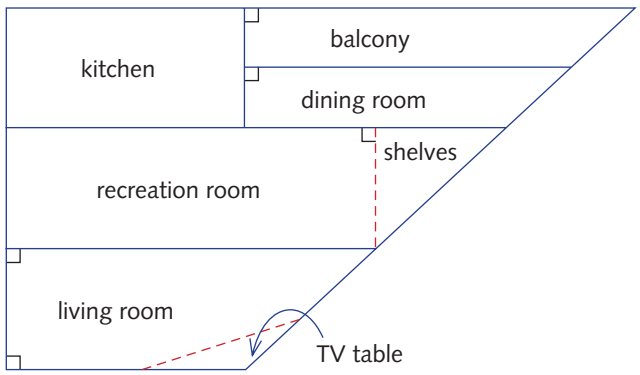

**e**




**f**




**6** The diagram below is a plan of the first floor of Tony's house.




**a** Tony would like to build some wooden shelves at the end of the recreation room. Each shelf would be an isosceles triangle. Find the internal angles of these shelves.

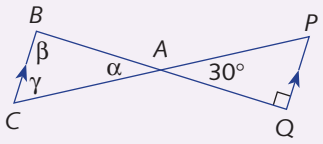
**b** Tony would also like to construct a TV table in the shape of an isosceles triangle in the living room. Evaluate the internal angles of the TV table.


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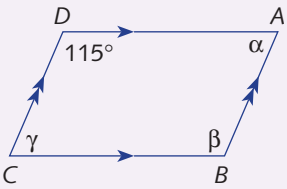
# Review exercise

**1** Find $\alpha$, $\beta$, $\gamma$ and $\theta$ in in each diagram, giving reasons.

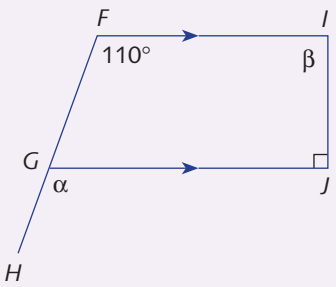
**a**



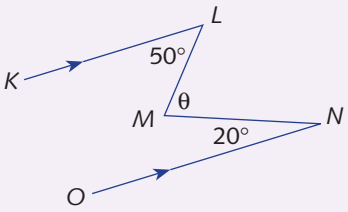

**b**




**c**

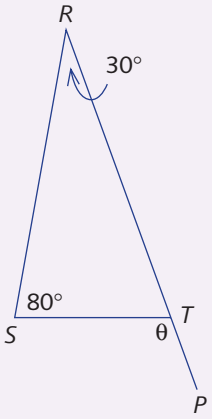



**d**

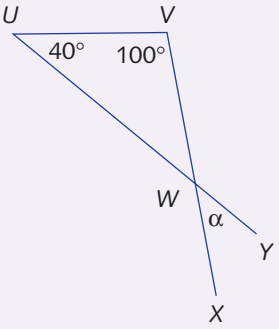



**2** Find $\alpha$, $\beta$, $\gamma$ and $\theta$ in each diagram, giving reasons.

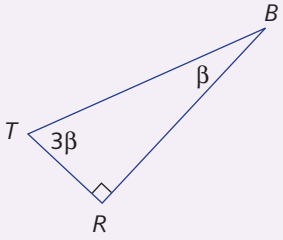
**a**



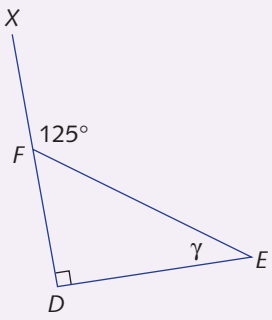

**b**



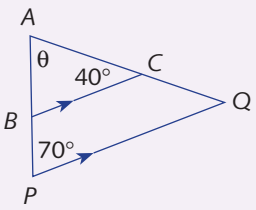

**c**



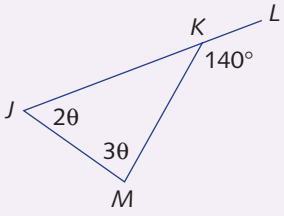

**d**



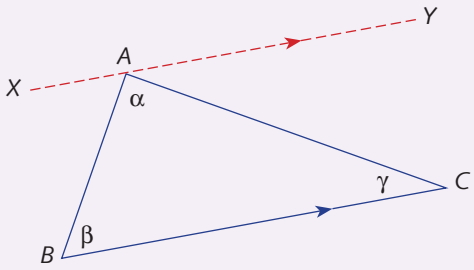

**e**




**f**




**3** Use the diagram opposite to prove that the sum of the interior angles of a triangle is 180°.

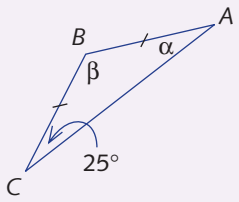




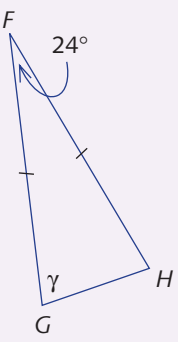
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**4** Find $\alpha$, $\beta$, $\gamma$ and $\theta$ in each diagram, giving reasons.

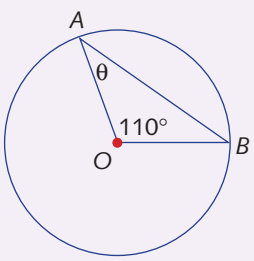
**a**



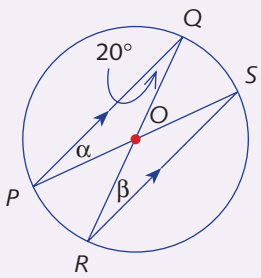

**b**



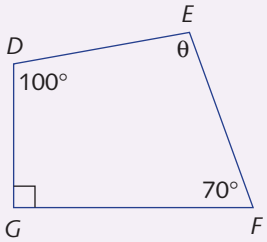

**c**



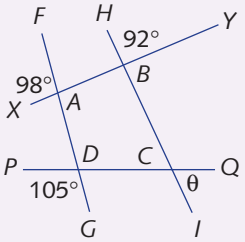

**d**




**e**

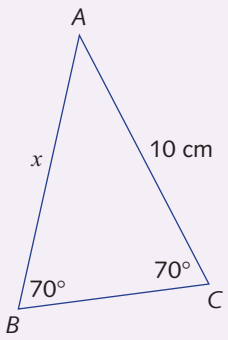



**f**

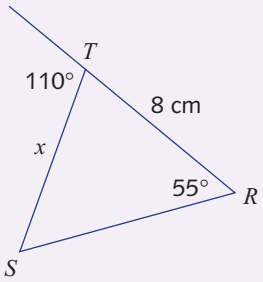



**5** Find the length $x$ in each diagram, giving reasons.

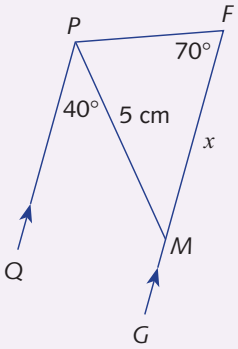
**a**




**b**

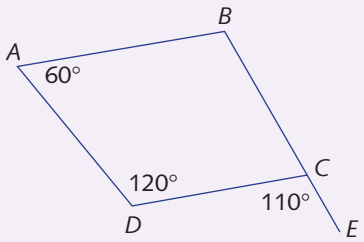



**c**

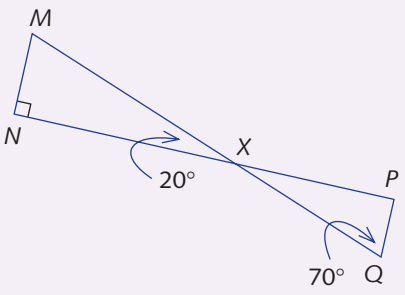



**6** Identify any pairs of parallel lines in each diagram, giving reasons.

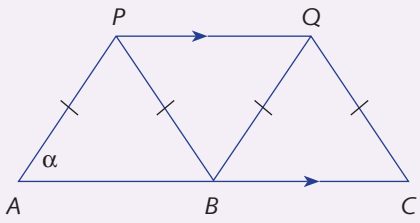
**a**




**b**




**c**







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**7** **a** Use a ruler and compasses to construct a triangle with side lengths 6 cm, 7 cm and 8 cm.

**b** Use a ruler, compasses and a protractor to construct an isosceles triangle with legs of length 6 cm and an apex angle of 110°.

**8** Use a ruler and compasses to construct an equilateral triangle of side length 5 cm.

**9** Use a ruler and compasses to construct angles of size:

**a** 60° **b** 30° **c** 15° **d** 90° **e** 45° **f** $22\frac{1}{2}°$

**10** **a** Draw a large obtuse-angled triangle.

**b** Use a ruler and compasses to construct the bisector of each vertex angle.

**c** These bisectors should be concurrent. Construct the circle with centre at this point that just touches each of the three sides.

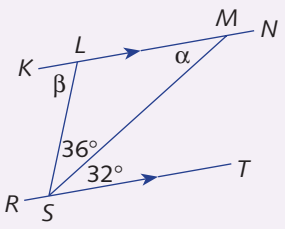
**11** **a** Draw a large triangle.

**b** Use a ruler and compasses to construct the perpendicular bisector of each side.

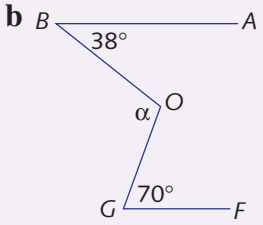
**c** These bisectors should be concurrent. Construct the circle with centre at this point that passes through each vertex.

**12** Find the values of the pronumerals.

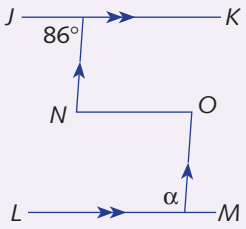
**a**



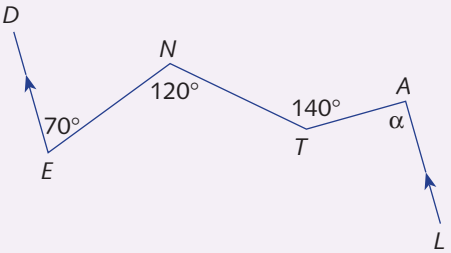

**b**



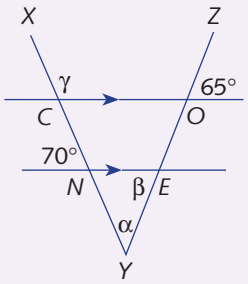

**c**



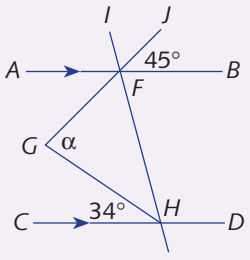

**d**



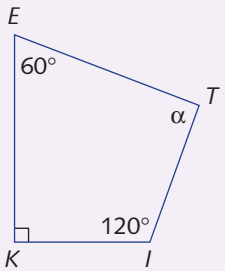

**e**



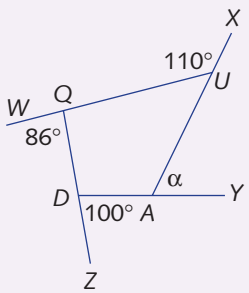

**f**




**g**




**h**





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# Challenge exercise

**1** This question provides a proof of the following theorem:

*If two angles of a triangle are equal, then the opposite sides are equal.*

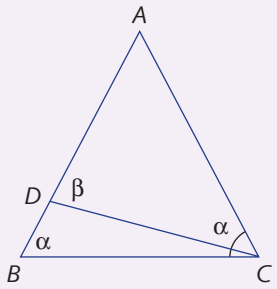
It uses the method of **proof by contradiction**, and relies on the theorem that the base angles of an isosceles triangle are equal.

If the sides $AB$ and $AC$ are not equal, then one of them is longer. Suppose, for example, that $AB$ is longer than $AC$. Mark the point $D$ on $AB$ so that $AD = AC$. Mark the angles in the two triangles as shown in the diagram below.

**a** Use angle $\angle ADC$ as an exterior angle of $\Delta BCD$ to show that $\beta > \alpha$.

**b** Show that $\angle ACD$ is also equal to $\beta$. Now look at the diagram to see that $\beta < \alpha$.

**c** Explain how the theorem follows from this contradiction.




**2** **a** Construct a large circle with centre $O$.

**b** Using ruler and compasses only, construct an equilateral triangle $ABC$ in which all three vertices $A$, $B$ and $C$ lie on the circle.

**3** **a** Construct a large circle with centre $O$ and draw a diameter $AOB$.

**b** Choose any point $P$, different from $A$ and $B$, on the circle and join $PA$ and $PB$.

**c** Prove that $\angle APB$ is a right angle. Do this by drawing the radius $OP$ and working with the angles in the two isosceles triangles $AOP$ and $BOP$.

**4** **a** On a new page of your exercise book, draw a large quadrilateral $ABCD$. Don't make it any special sort of quadrilateral.

**b** Use a straight edge and compasses to construct the midpoints of the four sides.

**c** Join these four midpoints to form a new, smaller quadrilateral inside $ABCD$.

**d** Use compasses to check that the opposite sides of the smaller quadrilateral are equal.


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**5** **a** Construct a large circle with centre $O$.

**b** Draw two diameters $AOB$ and $POQ$, not at right angles.

**c** Join up the quadrilateral $APBQ$.

**d** Prove that each angle of $APBQ$ is a right angle.

**6** **a** Construct a large circle with centre $O$.

**b** Using a ruler and compasses only, construct a square $ABCD$ with all four vertices $A$, $B$, $C$ and $D$ on the circle.

**7** **a** Draw an interval $AB$.

**b** Using a ruler and compasses only, construct an isosceles triangle $ABC$ in which the angle $\angle ACB$ is a right angle.

**8** **a** Construct a large circle with centre $O$.

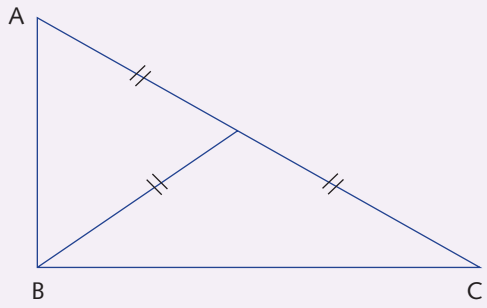
**b** Mark any four points $A$, $B$, $C$ and $D$ going clockwise around the circle and join up the quadrilateral $ABCD$. Make sure the centre $O$ is inside $ABCD$.

**c** Prove that these opposite angles are supplementary. Do this by drawing the radii $OA$, $OB$, $OC$ and $OD$, and working with the angles in the four isosceles triangles they form.

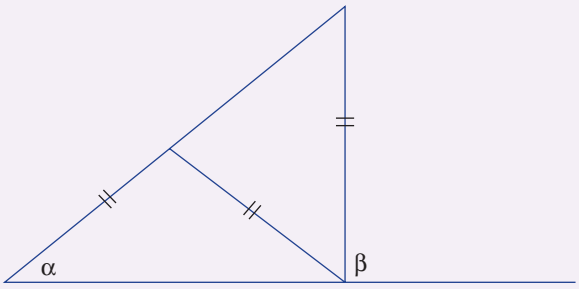
**d** Repeat part **b** but this time choose points $A$, $B$, $C$ and $D$ so that point $O$ is *outside* the quadrilateral $ABCD$. Now repeat part **c**. The proof this time will be a little different, because you will need to take differences as well as sums of angles.

**9** Explain why a quadrilateral cannot have more than one reflex angle.

**10** Let $\angle ACB = x$. In the diagram below show that $\angle ABC = 90°$.




**11** Use the exterior angle theorem to prove that $\beta = 3\alpha$ in the diagram below.





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CHAPTER

14

Number and Algebra

# Negative fractions and decimals

In Chapters 4, 5 and 8 we reviewed fractions and decimals. In the same way that we extended the whole numbers to negative integers in Chapter 11, we can also extend fractions to the negative fractions. Just as the integers can be divided into

- negative integers
- zero
- positive integers,

we have

- negative fractions
- zero
- positive fractions.

We can mark negative fractions on the number line to the left of zero.

A situation where we see negative fractions and decimals is in temperatures.

A temperature of –4.5° is 4.5°C below zero.

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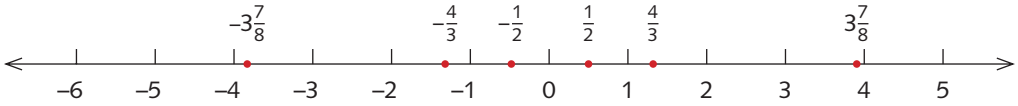


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Each positive fraction has an opposite fraction.

For example, $\frac{5}{4}$ has as its opposite $-\frac{5}{4}$.

Several negative fractions are marked on the number line below.




The arithmetic for the integers, which we have considered in the previous sections, also extends to the fractions.

# 14A Addition and subtraction of negative fractions

**Example 1**

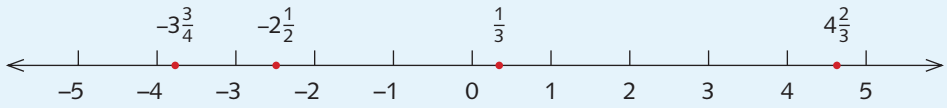
**a** Arrange the numbers $-2\frac{1}{2}$, $\frac{1}{3}$, $4\frac{2}{3}$ and $-3\frac{3}{4}$ in increasing order.

**b** Draw a number line from −5 to 5 and mark on it the numbers $-2\frac{1}{2}$, $\frac{1}{3}$, $4\frac{2}{3}$ and $-3\frac{3}{4}$.

**Solution**

**a** $-3\frac{3}{4}, -2\frac{1}{2}, \frac{1}{3}, 4\frac{2}{3}$

**b**




We combine the techniques for adding and subtracting integers with the techniques for adding and subtracting fractions.

For example, $-\frac{1}{2} + 2 = 2 - \frac{1}{2} = 1\frac{1}{2}$

**Example 2**

Write the answer to these additions.

**a** $-\frac{1}{3} + 3$ **b** $-1\frac{1}{2} + 2$ **c** $-\frac{1}{3} + \frac{2}{3}$

**Solution**

**a** $-\frac{1}{3} + 3 = 2\frac{2}{3}$ **b** $-1\frac{1}{2} + 2 = \frac{1}{2}$ **c** $-\frac{1}{3} + \frac{2}{3} = \frac{1}{3}$


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When adding or subtracting fractions, first find the lowest common denominator.

## Example 3

Evaluate:

**a** $-\frac{2}{3}+\frac{4}{5}$ **b** $-\frac{3}{5}+\left(-\frac{1}{3}\right)$ **c** $4-\left(-\frac{3}{5}\right)$ **d** $-\frac{3}{5}-\left(-\frac{2}{3}\right)$

### Solution

**a** $-\frac{2}{3}+\frac{4}{5}=-\frac{10}{15}+\frac{12}{15}$
$=\frac{2}{15}$

**b** $-\frac{3}{5}+\left(-\frac{1}{3}\right)=-\frac{9}{15}+\left(-\frac{5}{15}\right)$
$=-\frac{9}{15}-\frac{5}{15}$
$=-\frac{14}{15}$

**c** $4-\left(-\frac{3}{5}\right)=4+\frac{3}{5}$
$=4\frac{3}{5}$

**d** $-\frac{3}{5}-\left(-\frac{2}{3}\right)=-\frac{3}{5}+\frac{2}{3}$
$=-\frac{9}{15}+\frac{10}{15}$
$=\frac{1}{15}$

## Example 4

Find:

**a** $-2\frac{1}{2}-3\frac{3}{4}$ **b** $-2\frac{1}{3}-\left(-4\frac{1}{2}\right)$ **c** $-2\frac{3}{7}+\left(-4\frac{3}{4}\right)$ **d** $-17\frac{1}{3}-14\frac{1}{2}$

### Solution

**a** $-2\frac{1}{2}-3\frac{3}{4}=-2\frac{2}{4}-3\frac{3}{4}$
$=-2-3-\frac{2}{4}-\frac{3}{4}$
$=-5-\frac{5}{4}$
$=-6\frac{1}{4}$

**b** $-2\frac{1}{3}-\left(-4\frac{1}{2}\right)=-2\frac{1}{3}+4\frac{1}{2}$
$=-2\frac{2}{6}+4\frac{3}{6}$
$=2\frac{1}{6}$

**c** $-2\frac{3}{7}+\left(-4\frac{3}{4}\right)=-2-\frac{3}{7}-4-\frac{3}{4}$
$=-6-\frac{3}{7}-\frac{3}{4}$
$=-6-\frac{12}{28}-\frac{21}{28}$
$=-6-\frac{33}{28}$
$=-7\frac{5}{28}$

**d** $-17\frac{1}{3}-14\frac{1}{2}=-17-\frac{1}{3}-14-\frac{1}{2}$
$=-31-\frac{1}{3}-\frac{1}{2}$
$=-31-\frac{2}{6}-\frac{3}{6}$
$=-31-\frac{5}{6}$
$=-31\frac{5}{6}$


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## Exercise 14A

Example 1a

**1** Arrange each set of numbers in increasing order.

**a** $-2, \frac{1}{4}, -1, -\frac{7}{5}, \frac{1}{2}, 1$

**b** $-\frac{7}{4}, -\frac{11}{5}, \frac{1}{4}, \frac{7}{2}$

**c** $-\frac{2}{3}, -\frac{4}{5}, -\frac{11}{12}, \frac{12}{13}$

**d** $-1\frac{11}{13}, -\frac{15}{13}, -2, -1$

Example 1b

**2** Draw a number line from $-5$ to 5 and mark on it the numbers $-3\frac{1}{2}, -4\frac{3}{4}, -3\frac{1}{4}, -1\frac{1}{2}$ and $1\frac{1}{2}$.

Example 2

**3** Write the answer to these additions.

**a** $-\frac{1}{2} + 1$

**b** $-2\frac{1}{2} + 5$

**c** $-\frac{2}{5} + \frac{1}{5}$

**d** $-\frac{2}{7} + \frac{5}{7}$

**e** $-1 + \frac{1}{2}$

**f** $-4 + 1\frac{1}{2}$

**g** $-\frac{2}{3} + \frac{1}{3}$

**h** $-\frac{2}{5} + \frac{4}{5}$

Example 3a, b

**4** Write the answer to these additions.

**a** $-\frac{1}{4} + \frac{1}{2}$

**b** $-\frac{1}{2} + \frac{3}{5}$

**c** $\frac{1}{6} + \left(-\frac{3}{4}\right)$

**d** $-\frac{1}{7} + \frac{1}{3}$

**e** $\frac{1}{5} + \left(-\frac{1}{2}\right)$

**f** $-\frac{2}{3} + \left(-\frac{4}{7}\right)$

**g** $-\frac{2}{5} + \frac{1}{3}$

**h** $-\frac{4}{5} + \left(-\frac{2}{9}\right)$

**i** $-\frac{3}{5} + \frac{1}{2}$

**j** $-\frac{5}{6} + \frac{1}{4}$

**k** $-\frac{1}{3} + \frac{2}{5}$

**l** $\frac{2}{5} + \left(-\frac{1}{3}\right)$

Example 3c, d

**5** Write the answer to these subtractions.

**a** $-\frac{1}{4} - \frac{1}{2}$

**b** $-\frac{1}{2} - \frac{4}{5}$

**c** $-\frac{3}{4} - \frac{1}{6}$

**d** $-\frac{1}{7} - \frac{1}{3}$

**e** $-\frac{1}{5} - \left(-\frac{1}{2}\right)$

**f** $-\frac{2}{3} - \left(-\frac{3}{5}\right)$

**g** $-\frac{2}{5} - \left(-\frac{1}{4}\right)$

**h** $-\frac{4}{5} - \left(-\frac{1}{8}\right)$

**i** $-\frac{3}{5} - \frac{1}{2}$

**j** $-\frac{5}{6} - \left(-\frac{2}{5}\right)$

**k** $-\frac{1}{3} - \left(-\frac{3}{5}\right)$

**l** $-\frac{2}{5} - \frac{1}{3}$

**6** Evaluate:

**a** $-\frac{1}{2} + 3$

**b** $-1\frac{1}{2} + 4$

**c** $-\frac{2}{3} - \frac{1}{3}$

**d** $-\frac{2}{3} + \frac{1}{3}$

**e** $-\frac{3}{4} - \frac{3}{4}$

**f** $-\frac{2}{3} + \frac{4}{5}$

**g** $-\frac{3}{4} + \frac{1}{4}$

**h** $-\frac{1}{5} - \frac{2}{5}$

**i** $-\frac{3}{4} + 3$

**j** $-\frac{3}{5} - 3$

**k** $-1\frac{1}{2} + \left(-\frac{7}{9}\right)$

**l** $\frac{3}{7} - 2\frac{1}{2}$


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Example 4

**7** Write the answer to each of these additions and subtractions.

**a** $-\frac{3}{4}-\left(-3\frac{2}{11}\right)$ **b** $2\frac{1}{5}-\left(-2\frac{3}{4}\right)$ **c** $-2\frac{1}{5}-2\frac{3}{4}$

**d** $-3\frac{2}{5}-2\frac{3}{8}$ **e** $-2\frac{1}{4}+1\frac{1}{2}$ **f** $-1\frac{1}{2}-3\frac{4}{5}$

**g** $-1\frac{3}{4}+2\frac{1}{6}$ **h** $-2\frac{1}{7}-4\frac{1}{3}$ **i** $-3\frac{1}{5}+\left(-2\frac{1}{2}\right)$

**j** $-1\frac{2}{3}-\left(-5\frac{3}{5}\right)$ **k** $-4\frac{2}{5}+\left(-1\frac{1}{4}\right)$ **l** $-2\frac{4}{5}-\left(-3\frac{1}{8}\right)$

**8** The rules for the order of operations are the same as those for the whole numbers and the integers.

Evaluate:

**a** $\frac{1}{2}-\frac{1}{3}-\frac{1}{4}$ **b** $\frac{1}{2}-\left(-\frac{1}{3}\right)-\frac{1}{4}$

**c** $-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}$ **d** $-\frac{1}{2}-\frac{1}{3}-\left(-\frac{1}{4}\right)$

**e** $\frac{1}{2}-\frac{2}{3}-\frac{1}{4}$ **f** $-\frac{1}{2}-\frac{2}{3}-\frac{3}{4}$

# 14B Multiplication and division of negative fractions

The methods we have used with the positive fractions also work with the negative fractions. We also we use the rules for multiplying positive and negative numbers.

## Example 5

Evaluate:

**a** $-\frac{2}{3}\times\frac{1}{5}$ **b** $-\frac{2}{3}\times\left(-\frac{1}{2}\right)$ **c** $-\frac{3}{4}\times\left(-\frac{16}{21}\right)$

## Solution

**a** $-\frac{2}{3}\times\frac{1}{5}=-\frac{2}{15}$

**b** $-\frac{2}{3}\times\left(-\frac{1}{2}\right)=\frac{\cancel{2}}{3}\times\frac{1}{\cancel{2}}$

$=\frac{1}{3}$

**c** $-\frac{3}{4}\times\left(-\frac{16}{21}\right)=\frac{\cancel{3}}{\cancel{4}}\times\frac{\cancel{16}^{4}}{\cancel{21}_{7}}$

$=\frac{4}{7}$



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## Example 6

Evaluate:

**a** $3 \div \left(-\frac{2}{5}\right)$ **b** $\frac{1}{4} \div \left(-\frac{3}{5}\right)$ **c** $-\frac{5}{8} \div \left(-\frac{25}{24}\right)$

### Solution

**a** $3 \div \left(-\frac{2}{5}\right) = -\left(\frac{3}{1} \div \frac{2}{5}\right)$

$= -\left(\frac{3}{1} \times \frac{5}{2}\right)$

$= -\frac{15}{2}$

$= -7\frac{1}{2}$

**b** $\frac{1}{4} \div \left(-\frac{3}{5}\right) = -\left(\frac{1}{4} \div \frac{3}{5}\right)$

$= -\left(\frac{1}{4} \times \frac{5}{3}\right)$

$= -\frac{5}{12}$

**c** $-\frac{5}{8} \div \left(-\frac{25}{24}\right) = \frac{5}{8} \div \frac{25}{24}$

$= \frac{\cancel{5}}{\cancel{8}} \times \frac{\cancel{24}^{3}}{\cancel{25}_{5}}$

$= \frac{3}{5}$

In earlier work we saw that $16 \div 2$ can also be written as $\frac{16}{2}$. Similarly, $3 \div 4$ can be written as $\frac{3}{4}$.

This notation can also be used with negative numbers.

Thus, $-3 \div 4$ can be written as $\frac{-3}{4}$

$-16 \div (-2)$ can be written as $\frac{-16}{-2}$

$3 \div (-4)$ can be written as $\frac{3}{-4}$

Because of the rules for multiplying and dividing integers:

$\frac{-3}{4} = \frac{3}{-4} = -\frac{3}{4}$ and $\frac{-12}{3} = \frac{12}{-3} = -4$

Also, $\frac{-5}{-8} = \frac{5}{8}$ and $\frac{-16}{-2} = \frac{16}{2} = 8$

## Example 7

Carry out the following divisions.

**a** $\frac{36}{-4}$ **b** $\frac{-72}{-12}$ **c** $\frac{-14}{16}$ **d** $\frac{-27}{54}$

### Solution

**a** $\frac{36}{-4} = -9$

**b** $\frac{-72}{-12} = \frac{72}{12}$

$= 6$

**c** $\frac{-14}{16} = -\frac{\cancel{14}^{7}}{\cancel{16}_{8}}$

$= -\frac{7}{8}$

**d** $\frac{-27}{54} = -\frac{\cancel{27}^{}}{\cancel{54}_{2}}$

$= -\frac{1}{2}$


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### Example 8

Carry out each of these multiplications and divisions.

**a** $-1\frac{2}{7} \times \left(-1\frac{1}{2}\right)$ **b** $-2\frac{1}{3} \div 1\frac{1}{2}$

### Solution

**a** $-1\frac{2}{7} \times \left(-1\frac{1}{2}\right) = -\frac{9}{7} \times -\frac{3}{2}$

$= \frac{27}{14}$

$= 1\frac{13}{14}$

**b** $-2\frac{1}{3} \div 1\frac{1}{2} = \frac{-7}{3} \div \frac{3}{2}$

$= \frac{-7}{3} \times \frac{2}{3}$

$= \frac{-14}{9} = -1\frac{5}{9}$

## Signs of product and quotients

**The sign of the product of two numbers**

- The product of a negative number and a positive number is a negative number. For example:

$$-\frac{1}{4} \times \frac{2}{7} = -\frac{1}{14}$$

- The product of two negative numbers is a positive number. For example:

$$-\frac{1}{4} \times \left(-\frac{2}{7}\right) = \frac{1}{14}$$

The rules for division are similar.

**The sign of the quotient of two numbers**

- The quotient of a positive number and a negative number is a negative number. For example:

$$-\frac{1}{4} \div \frac{2}{7} = -\frac{7}{8}$$

- The quotient of a negative number and a positive number is a negative number. For example:

$$\frac{1}{4} \div \left(-\frac{2}{7}\right) = -\frac{7}{8}$$

- The quotient of two negative numbers is a positive number. For example:

$$-\frac{1}{4} \div \left(-\frac{2}{7}\right) = \frac{7}{8}$$


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### Example 9

Evaluate:

**a** $\left(-\frac{9}{8}\right)^2$ **b** $\left(-\frac{4}{3}\right)^3$

### Solution

**a** $\left(-\frac{9}{8}\right)^2 = \left(-\frac{9}{8}\right) \times \left(-\frac{9}{8}\right)$

$= \frac{81}{64}$

$= 1\frac{17}{64}$

**b** $\left(-\frac{4}{3}\right)^3 = -\frac{4}{3} \times \left(-\frac{4}{3}\right) \times \left(-\frac{4}{3}\right)$

$= -\frac{64}{27}$

$= -2\frac{10}{27}$

## Exercise 14B

Example 5

**1** Evaluate:

**a** $-\frac{1}{4} \times \frac{2}{5}$ **b** $-\frac{2}{7} \times \frac{1}{5}$ **c** $-\frac{3}{4} \times \frac{3}{5}$ **d** $-\frac{3}{7} \times \frac{5}{11}$

**e** $-\frac{3}{5} \times \frac{10}{21}$ **f** $-\frac{3}{8} \times \frac{4}{9}$ **g** $\frac{2}{11} \times \left(-\frac{3}{5}\right)$ **h** $\frac{5}{6} \times \left(-\frac{7}{13}\right)$

**i** $\frac{2}{3} \times \left(-\frac{1}{7}\right)$ **j** $\frac{1}{5} \times \left(-\frac{4}{9}\right)$ **k** $-\frac{3}{5} \times \left(-\frac{1}{2}\right)$ **l** $-\frac{1}{3} \times \left(-\frac{3}{5}\right)$

Example 6

**2** Evaluate:

**a** $-3 \div \frac{1}{4}$ **b** $5 \div \left(-\frac{3}{7}\right)$ **c** $\frac{1}{2} \div \left(-\frac{5}{7}\right)$

**d** $\frac{2}{9} \div \left(-\frac{14}{27}\right)$ **e** $-\frac{5}{6} \div \left(-\frac{1}{12}\right)$ **f** $-\frac{2}{13} \div \left(-\frac{38}{39}\right)$

Example 7

**3** Complete the following divisions.

**a** $\frac{28}{-7}$ **b** $\frac{-15}{5}$ **c** $\frac{-81}{-9}$ **d** $\frac{-24}{32}$

**e** $\frac{-18}{-4}$ **f** $\frac{-14}{42}$ **g** $\frac{-9}{-24}$ **h** $\frac{-52}{7}$

**i** $\frac{-34}{5}$ **j** $\frac{-456}{-7}$ **k** $\frac{-345}{20}$ **l** $\frac{87}{-7}$

Example 8

**4** Write the answers to these multiplications.

**a** $-2\frac{7}{8} \times \left(-1\frac{7}{11}\right)$ **b** $-1\frac{1}{3} \times 2\frac{1}{5}$ **c** $\frac{12}{13} \times (-4) \times \frac{3}{8}$

**d** $\frac{-5}{6} \times 7 \times \left(\frac{-2}{9}\right)$ **e** $1\frac{2}{3} \times \left(-2\frac{1}{2}\right) \times 1\frac{1}{4}$ **f** $1\frac{1}{2} \times \left(-3\frac{5}{8}\right) \times \frac{2}{5}$


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**5** Write the answers to these divisions.

**a** $2 \div \left(-\frac{2}{9}\right)$ **b** $-2\frac{1}{3} \div \left(-1\frac{2}{9}\right)$ **c** $-3\frac{1}{8} \div 2\frac{2}{11}$

**d** $-1\frac{4}{5} \div \left(-1\frac{2}{3}\right)$ **e** $-6\frac{1}{3} \div \left(-1\frac{2}{3}\right)$ **f** $-1\frac{1}{6} \div \left(-2\frac{4}{9}\right)$

Example 9

**6** Evaluate:

**a** $\left(-\frac{1}{3}\right)^2$ **b** $\left(-\frac{1}{3}\right)^3$ **c** $\left(-\frac{2}{3}\right)^2$ **d** $\left(-\frac{3}{2}\right)^2$

**e** $\left(-\frac{3}{2}\right)^3$ **f** $\left(-\frac{2}{7}\right)^3$ **g** $\left(-\frac{1}{3}\right)^2 \times \frac{3}{4}$ **h** $\left(-\frac{2}{3}\right)^2 \times \left(-\frac{1}{3}\right)^2$

# 14C Negative decimals

The methods we have used with positive decimals also work with negative decimals.

## Example 10

Complete the following calculations. Give your answers as decimals.

**a** $-0.2 + 2$ **b** $-0.2 - 0.4$ **c** $-0.6 \times 0.7$

**d** $-1.2 \div 0.3$ **e** $-12.6 \div (-0.3)$

## Solution

**a** $-0.2 + 2 = 1.8$

**b** $-0.2 - 0.4 = -0.6$

**c** $-0.6 \times 0.7 = -\frac{6}{10} \times \frac{7}{10}$
$= -\frac{42}{100}$
$= -0.42$

**d** $-1.2 \div 0.3 = -\frac{1.2}{0.3}$
$= -\frac{12}{3}$
$= -4$

**e** $-12.6 \div (-0.3) = -\frac{126}{10} \div \left(-\frac{3}{10}\right)$
$= \frac{126}{\cancel{10}} \times \frac{\cancel{10}}{3}$
$= \frac{126}{3}$
$= 42$

or $\frac{-12.6}{-0.3} = \frac{12.6}{0.3}$
$= \frac{126}{3}$
$= 42$


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### Example 11

Complete the following calculations. Give your answers as decimals.

**a** $(-0.2)^3$ **b** $(-0.1)^3$ **c** $(-0.2)^3 + (-0.1)^3$

### Solution

**a** $(-0.2)^3 = -0.2 \times (-0.2) \times (-0.2)$
$= -(0.2 \times 0.2 \times 0.2)$
$= -\left(\frac{2}{10} \times \frac{2}{10} \times \frac{2}{10}\right)$
$= -\frac{8}{1000}$
$= -0.008$

**b** $(-0.1)^3 = -0.1 \times (-0.1) \times (-0.1)$
$= -(0.1 \times 0.1 \times 0.1)$
$= -\left(\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}\right)$
$= -\frac{1}{1000}$
$= -0.01$

**c** $(-0.2)^3 + (-0.1)^3 = -0.008 + (-0.001)$
$= -0.008 - 0.001$
$= -0.009$

## Exercise 14C

Example 10a

**1** Complete the following calculations. Give your answers in decimal form.

**a** $-0.4 + 3$ **b** $-6 + 11.25$ **c** $-4.8 + 6$ **d** $-7.2 + 6$
**e** $-3.5 + 7$ **f** $-2.5 + 3.2$ **g** $-4.6 + 2.1$ **h** $-3.75 + 2.2$

Example 10b

**2** Complete the following calculations. Give your answers in decimal form.

**a** $-0.5 - 0.2$ **b** $-3.2 - 5$ **c** $3.62 - (-2.5)$ **d** $4.6 - 5$
**e** $-2.7 - 3.1$ **f** $-4.2 - 6.1$ **g** $3.72 - (-1.21)$ **h** $7.16 - (-2.31)$

Example 10c

**3** Complete the following calculations. Give your answers in decimal form.

**a** $-0.3 \times 0.8$ **b** $-0.4 \times (-0.7)$ **c** $-0.3 \times (-0.8)$ **d** $-0.2 \times (-0.91)$
**e** $2.5 \times (-0.3)$ **f** $-0.01 \times (-0.2)$ **g** $-0.12 \times (-0.2)$ **h** $-2.1 \times (-0.3)$

Example 10d, e

**4** Complete the following calculations. Give your answers in decimal form.

**a** $-2.5 \div 0.5$ **b** $-16.4 \div (-0.4)$ **c** $-12.3 \div (-0.3)$ **d** $12.5 \div (-0.5)$
**e** $-6 \div 10$ **f** $-6.3 \div (-3)$ **g** $-5.5 \div (-5)$ **h** $-21.7 \div (-0.07)$

Example 11

**5** Complete the following calculations. Give your answers in decimal form.

**a** $(-0.3)^3$ **b** $(-0.4)^3$ **c** $(-0.3)^3 + (-0.4)^3$
**d** $(-0.1)^3 + (-0.1)^2$ **e** $(-0.2)^3 + (-0.1)^5$ **f** $(-0.2)^3 + (-0.2)^4$

**6** Complete the following calculations. Give your answers in decimal form.

**a** $-56 \div 11$ **b** $-26.3 + (-4.1)$ **c** $-15.72 + (-0.63)$
**d** $-2.7 + (-5.06)$ **e** $-2.07 + (-0.96)$ **f** $-17.01 + 2.34$
**g** $-23.56 - 2.7$ **h** $67 \div (-11)$ **i** $-4.5 \div 3 + 6$


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**7** Complete the following calculations. Give your answers in decimal form.

**a** $-0.07 \times (-0.3)$ **b** $-0.025 \times (-0.3)$ **c** $-0.525 \times (-0.4)$

**d** $-0.525 \times 0.05$ **e** $-6.25 \times (-0.05)$ **f** $-5.75 \times 0.001$

**8** Complete the following calculations. Give your answers in decimal form.

**a** $0.1 - 0.6 - 0.9$ **b** $0.2 - 0.6 - (-0.7)$ **c** $-0.1 + 0.6 - 0.9$

**d** $-0.1 - 0.6 - 0.9$ **e** $-2.2 - 0.6 - (-1.7)$ **f** $-0.2 - 0.6 - (-0.7)$

**g** $-0.1 \times 0.6 \times 0.9$ **h** $-0.1 \times (-0.6) \times 0.9$ **i** $-0.3 \times 1.1 \times (-0.1)$

# 14D Substitution involving negative fractions and decimals

In earlier chapters we substituted whole number and positive fractions and negative integers for pronumerals. In this section we extend this to include negative fractions.

## Example 12

Evaluate each expression for $x = -\frac{3}{4}$.

**a** $4x + 3$ **b** $-2x + 4$ **c** $5(x + 2)$

**d** $-6\left(x + \frac{4}{7}\right)$ **e** $-\frac{1}{2}x^2$ **f** $\left(-\frac{1}{2}x\right)^2$

## Solution

**a** $4x + 3 = 4 \times \left(-\frac{3}{4}\right) + 3$
$= -3 + 3$
$= 0$

**b** $-2x + 4 = -2 \times \left(-\frac{3}{4}\right) + 4$
$= \frac{3}{2} + 4$
$= 5\frac{1}{2}$

**c** $5(x + 2) = 5 \times \left(-\frac{3}{4} + 2\right)$
$= 5 \times 1\frac{1}{4}$
$= 5 \times \frac{5}{4}$
$= \frac{25}{4}$
$= 6\frac{1}{4}$

**d** $-6\left(x + \frac{4}{7}\right) = -6 \times \left(-\frac{3}{4} + \frac{4}{7}\right)$
$= -6 \times \left(-\frac{21}{28} + \frac{16}{28}\right)$
$= -6 \times \left(-\frac{5}{28}\right)$
$= \frac{15}{14}$
$= 1\frac{1}{14}$

*(continued over page)*


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**e** $-\frac{1}{2}x^2 = -\frac{1}{2} \times \left(-\frac{3}{4}\right)^2$

$= -\frac{1}{2} \times \frac{9}{16}$

$= -\frac{9}{32}$

**f** $\left(-\frac{1}{2}x\right)^2 = \left(-\frac{1}{2} \times \left(-\frac{3}{4}\right)\right)^2$

$= \left(\frac{3}{8}\right)^2$

$= \frac{9}{64}$

## Exercise 14D

Example 12

**1** Evaluate each expression for $x = -\frac{1}{2}$.

**a** $2x$ **b** $-x$ **c** $x + 2$ **d** $x - 3$

**e** $2x + 3$ **f** $x^3$ **g** $-x^3$ **h** $(-x)^2$

**i** $3 - x$ **j** $3 - 2x$ **k** $5 + 2x$ **l** $5 + 6x$

**2** Substitute $m = -4$, $n = \frac{1}{3}$ and $p = -\frac{1}{2}$ to evaluate.

**a** $m + n$ **b** $m + p$ **c** $m - p$ **d** $mp$

**e** $np$ **f** $\frac{p}{m}$ **g** $mnp$ **h** $\frac{p}{n}$

**3** Evaluate each expression for $x = -0.1$

**a** $5x + 4$ **b** $-5x + 4$ **c** $5(x + 4)$

**d** $-5(x + 4)$ **e** $-5x^2$ **f** $(-5x)^2$

**4** Evaluate each expression for $x = -\frac{2}{5}$.

**a** $5x + 6$ **b** $-5x + 6$ **c** $5(x + 6)$

**d** $-5(x + 6)$ **e** $-5x^2$ **f** $\frac{5}{x}$

**5** Evaluate each expression for $x = -\frac{3}{4}$.

**a** $6 - x$ **b** $6 + x$ **c** $x^3$ **d** $x^5$

**e** $-x^2$ **f** $(-2x)^2$ **g** $-2x^2$ **h** $5 - 2x$

**6** Substitute $a = -0.1$, $b = -0.9$ and $c = -5$ to evaluate.

**a** $a + b$ **b** $c + a$ **c** $b - c$

**d** $bc$ **e** $ac$ **f** $\frac{a}{c}$

**7** Substitute $m = -4\frac{1}{6}$, $n = 5\frac{1}{3}$ and $p = -1\frac{1}{2}$ to evaluate.

**a** $m + n$ **b** $m + p$ **c** $m - p$ **d** $mp$

**e** $np$ **f** $\frac{p}{m}$ **g** $mnp$ **h** $\frac{p}{n}$


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**8** Evaluate each expression for $x = -1.1$.

**a** $10 - x$ **b** $10 + x$ **c** $x^3$ **d** $x^5$

**e** $-x^2$ **f** $(-4x)^2$ **g** $-5x^2$ **h** $10 - 2x$

**9** Substitute $a = -0.4$, $b = -0.15$ and $c = -4$ to evaluate.

**a** $a + b + c$ **b** $a - b - c$ **c** $c - b - a$

**d** $abc$ **e** $\frac{ab}{c}$ **f** $\frac{bc}{a}$

**10** Given that $p = -0.001$, $q = 0.02$ and $r = -0.14$, evaluate:

**a** $p + q - r$ **b** $p - q + r$ **c** $\frac{pr}{q}$

**d** $\frac{qr}{p}$ **e** $\frac{r}{pq}$ **f** $\frac{q}{p} + r$

# Review exercise

**1** Arrange these numbers in increasing order.

**a** $\frac{3}{4}, -3, -1\frac{1}{2}, 2\frac{3}{4}, -1\frac{3}{4}, 2$

**b** $-6\frac{3}{7}, \frac{19}{3}, \frac{-7}{3}, \frac{4}{7}, \frac{-5}{-7}, -\frac{11}{3}$

**2** Evaluate the following, giving your answers in simplest form.

**a** $-\frac{1}{3} + \frac{5}{3}$ **b** $3\frac{1}{2} - \frac{6}{2}$ **c** $-\frac{4}{5} + 2$ **d** $\frac{5}{7} + \left(-\frac{3}{5}\right)$

**e** $2\frac{5}{8} + \left(-\frac{7}{2}\right)$ **f** $-\frac{9}{4} - \left(-\frac{2}{5}\right)$ **g** $\frac{11}{13} + \left(-\frac{9}{2}\right)$ **h** $-3\frac{1}{4} - \left(-2\frac{4}{9}\right)$

**3** Evaluate the following, giving your answers in simplest form.

**a** $5 \times \left(-\frac{7}{3}\right)$ **b** $-\frac{3}{5} \times \left(-\frac{2}{5}\right)$ **c** $\frac{7}{9} \times \left(-2\frac{2}{5}\right)$ **d** $5 \div \left(-\frac{2}{7}\right)$

**e** $-3\frac{4}{5} \div 5\frac{3}{8}$ **f** $-\frac{8}{3} \div \left(-\frac{4}{11}\right)$ **g** $-\frac{1}{4} \times \frac{2}{3} \div \frac{5}{6}$ **h** $-\frac{1}{2} \times \frac{5}{6} + \left(-\frac{2}{3}\right)$

**4** Evaluate the following, giving your answers in decimal form.

**a** $4 + (-5.75)$ **b** $-7.42 - 9$ **c** $-14.6 + (-12)$

**d** $2.5 - 3.7$ **e** $-19.65 - (-13.21)$ **f** $-9.5 + 3.7 \times (-2.1)$

**g** $6 \times (-2.9)$ **h** $-42 \div 6$ **i** $-7 \times (-3.2) + 4$

**j** $(16 + (-5.6)) \div 2$ **k** $-4.5 \times (-4) \div (-12)$


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**5** Complete the following calculations. Give your answers in decimal form.

**a** $(-0.7)^2$ **b** $(-0.5)^3 + (0.4)^2$ **c** $4.5 \div (-3) - (-2)$

**d** $0.6 + 0.9 \div (-9)$ **e** $4.3 - (-0.01) \times 44$ **f** $-5.75 + (-0.5)^2$

**6** Evaluate each expression for $x = -\frac{1}{3}$.

**a** $5x$ **b** $1 - x$ **c** $x^2$ **d** $-7x - 1$

**7** Evaluate each expression for $x = -0.4$.

**a** $-x$ **b** $5x + 7$ **c** $5x - 7.8$ **d** $6x^2$

**8** Evaluate each expression for $x = -2$, $y = 0.1$ and $z = -\frac{1}{2}$.

**a** $xyz$ **b** $x + y$ **c** $y \div z$ **d** $2x + z - y$

**e** $\frac{yz}{2x}$ **f** $y(5.5 + x)$ **g** $-z - 3y$ **h** $\frac{x}{z} - y$

# Challenge exercise

**1** Complete the magic square on the right, in which each row, column and diagonal must add to the same sum.

| | | |
|---|---|---|
| $-3$ | | |
| | $-1\frac{2}{3}$ | |
| $-4$ | $-\frac{1}{3}$ | |

**2** Find two fractions between $-\frac{1}{3}$ and $-\frac{1}{2}$.

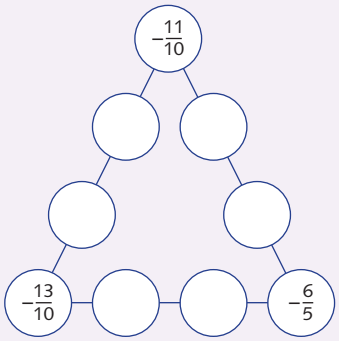
**3** Arrange the following numbers in decreasing order.

**a** $-3\frac{7}{11}$, $-3.6\dot{3}$, $-3.63$, $-3.\dot{6}$

**b** $-1.43\dot{5}$, $-1.4\dot{3}\dot{5}$, $-1\frac{4}{9}$, $-1.\dot{4}3\dot{5}$

**4** The average of four numbers is $-5\frac{1}{3}$. The average of the same four numbers and a fifth number is $6\frac{2}{3}$. Find the fifth number.

**5** Place the numbers $-\frac{7}{5}$, $-\frac{3}{2}$, $-\frac{8}{5}$, $-\frac{17}{10}$, $-\frac{9}{5}$, $-\frac{19}{10}$ in the circle so that the sum along any of the lines is $-\frac{57}{10}$.




**6** The sum of two numbers is $-3\frac{1}{5}$. Their difference is $2\frac{1}{3}$. What are the two numbers?


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CHAPTER

# 15

Number and Algebra

# Percentages and ratios

We encounter percentages often and they can be very useful in many aspects of our lives. For example, we see percentage discounts offered in shops everywhere – we definitely need to know how to compare prices if we want the best deal.

The word 'percentage' comes from the Latin *per centum*, meaning 'per hundred'. A percentage is another way of writing a fraction with a denominator of 100. The symbol for percentage is %. For example:

$8\% = \frac{8}{100}$, $25\% = \frac{25}{100}$, $99\% = \frac{99}{100}$, $150\% = \frac{150}{100}$

In this chapter, you will learn how to do several different types of practical calculations involving percentages.

Ratios are usually used to compare two related quantities. For example, salad dressing may be made using a ratio of one part vinegar to two parts oil.

We will see that many practical problems can be solved by working with ratios in appropriate ways. Many problems about percentages and ratios can be solved by using a familiar and simple idea – the unitary method.

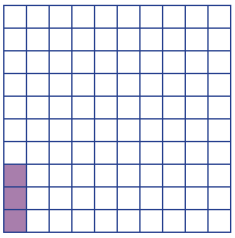


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# 15A Percentages, fractions and decimals

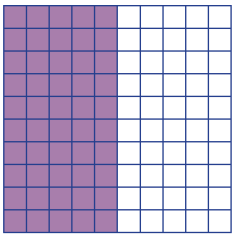
'Percentage' means 'out of a hundred'. A square that has been divided into 100 smaller squares can be used to model percentages.

If we colour in three of them, we say that 'three out of a hundred' or '3 per cent' are coloured in.



If we colour in 50 of them, we say that '50 out of a hundred' or '50 per cent' are coloured in.

In this case, the fraction coloured in is $\frac{50}{100} = \frac{1}{2}$, so half of the smaller squares are coloured in.



## Converting percentages to fractions

A **percentage** is a fraction that has a denominator of 100. To convert a percentage to its fraction equivalent, write it as a fraction with a denominator of 100 and then simplify it. For example:

$$65\% = \frac{65}{100} = \frac{13}{20} \qquad 150\% = \frac{150}{100} = 1\frac{1}{2}$$

## Converting percentages to decimals

Similarly, a percentage can be converted to a decimal by first writing it as a fraction with a denominator of 100, and then converting this to a decimal.

$$65\% = \frac{65}{100} = 0.65 \qquad 150\% = \frac{150}{100} = 1.5 \qquad 37.5\% = \frac{37.5}{100} = 0.375$$

## Converting fractions and decimals to percentages using equivalent fractions

Fractions with denominator 100 can be converted easily to percentages. For example:

$$\frac{2}{100} = 2\% \qquad \frac{37}{100} = 37\% \qquad \frac{175}{100} = 175\% \qquad 1 = \frac{100}{100} = 100\%$$

Equivalent fractions can be used for some fractions with denominators that are not 100:

$$\frac{2}{10} = \frac{20}{100} = 20\% \qquad \frac{3}{20} = \frac{15}{100} = 15\% \qquad \frac{3}{5} = \frac{60}{100} = 60\% \qquad \frac{5}{2} = \frac{250}{100} = 250\%$$


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Decimals can also be changed to percentages in this way.

$0.6 = \frac{6}{10} = \frac{60}{100} = 60\%$ $\quad 3.2 = \frac{32}{10} = \frac{320}{100} = 320\%$ $\quad 0.03 = \frac{3}{100} = 3\%$

## Converting fractions and decimals to percentages using the rule

To convert a fraction or a decimal to a percentage, multiply by 100%, which is the same as multiplying by 1:

$$\frac{2}{5} = \frac{2}{5} \times 100\% = \frac{2}{5} \times \frac{100}{1}\% = \frac{200}{5}\% = 40\%$$

$$\frac{1}{3} = \frac{1}{3} \times 100\% = \frac{100}{3}\% = 33\tfrac{1}{3}\%$$

$$3.2 = 3.2 \times 100\% = 320\%$$

$$0.6 = 0.6 \times 100\% = 60\%$$

Here are some commonly used percentages and their fraction equivalents. It is very useful to know these.

| Fraction | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{3}$ | $\frac{3}{4}$ | $\frac{1}{5}$ | $\frac{1}{10}$ | $\frac{2}{5}$ | $\frac{3}{5}$ | $\frac{4}{5}$ |
|---|---|---|---|---|---|---|---|---|---|
| Percentage | 50% | 25% | $33\frac{1}{3}\%$ | 75% | 20% | 10% | 40% | 60% | 80% |

### Percentages

- A percentage is another way of writing a fraction that has a denominator of 100. For example: $5\% = \frac{5}{100}$
- To convert a percentage to a fraction, write the percentage as a fraction with a denominator of 100 and then simplify.
- To convert a percentage to a decimal, write the percentage as a fraction with a denominator of 100 and then convert to a decimal.
- To convert a fraction or a decimal to a percentage, multiply by 100%.

### Example 1

Convert each percentage to a fraction.

**a** 4% **b** 85% **c** 235% **d** $7\frac{1}{4}\%$

### Solution

**a** $4\% = \frac{4}{100} = \frac{1}{25}$

**b** $85\% = \frac{85}{100} = \frac{17}{20}$

**c** $235\% = \frac{235}{100} = 2\frac{35}{100} = 2\frac{7}{20}$

**d** $7\frac{1}{4}\% = \frac{7\frac{1}{4}}{100} = \frac{29}{400}$


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## Example 2

Convert each percentage to a decimal.

**a** 55% **b** 5% **c** 235% **d** 6.5%

### Solution

**a** $55\% = \frac{55}{100}$
$= 0.55$

**b** $5\% = \frac{5}{100}$
$= 0.05$

**c** $235\% = \frac{235}{100}$
$= 2\frac{35}{100}$
$= 2.35$

**d** $6.5\% = \frac{6.5}{100}$
$= 0.065$

## Example 3

Express each fraction as a percentage.

**a** $\frac{33}{100}$ **b** $\frac{3}{10}$ **c** $\frac{33}{20}$

### Solution

**a** $\frac{33}{100} = 33\%$

**b** $\frac{3}{10} = \frac{3 \times 10}{10 \times 10}$
$= \frac{30}{100}$
$= 30\%$

**c** $\frac{23}{20} = \frac{23 \times 5}{20 \times 5}$
$= \frac{115}{100}$
$= 115\%$

or

$\frac{23}{20} = \frac{23}{\cancel{20}} \times \frac{\cancel{100}^{5}}{1}\%$
$= 115\%$

## Example 4

Write each decimal as a percentage.

**a** 0.35 **b** 1.03

### Solution

**a** $0.35 = \frac{35}{100}$
$= 35\%$

or

$0.35 = 0.35 \times 100\%$
$= 35\%$

**b** $1.03 = \frac{103}{100}$
$= 103\%$

or

$1.03 = 1.03 \times 100\%$
$= 103\%$


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### Example 5

Write each fraction as a percentage.

**a** $\frac{2}{3}$ **b** $\frac{5}{6}$ **c** $\frac{1}{7}$

### Solution

**a** $\frac{2}{3} = \frac{2}{3} \times \frac{100}{1}\%$
$= \frac{200}{3}\%$
$= 66\frac{2}{3}\%$

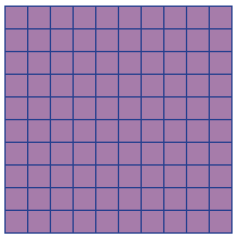
**b** $\frac{5}{6} = \frac{5}{6} \times \frac{100}{1}\%$
$= \frac{500}{6}\%$
$= 83\frac{1}{3}\%$

**c** $\frac{1}{7} = \frac{1}{7} \times \frac{100}{1}\%$
$= \frac{100}{7}\%$
$= 14\frac{2}{7}\%$

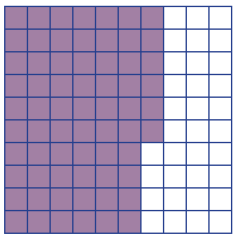
## Exercise 15A

**1** What percentage of each square is shaded?

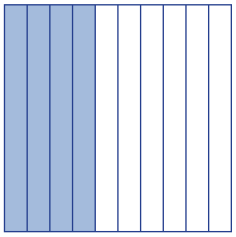
**a**



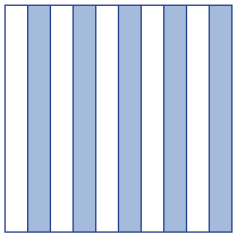
**b**



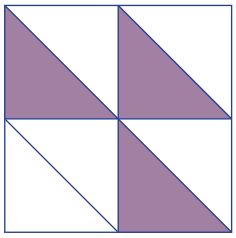
**c**



**d**



**e**



Example 1

**2** Convert each percentage to a fraction or mixed numeral.

**a** 8% **b** 15% **c** 75%
**d** 60% **e** 30% **f** 80%
**g** 120% **h** 165% **i** 210%
**j** 450% **k** 125% **l** 448%

Example 2

**3** Convert each percentage to a decimal.

**a** 7% **b** 25% **c** 75%
**d** 35% **e** 90% **f** 127%
**g** 410% **h** 460% **i** 170%
**j** 12.5% **k** 6.2% **l** 37.5%


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Example 3a, b

**4** Express each fraction as a percentage.

**a** $\frac{27}{100}$ **b** $\frac{97}{100}$ **c** $\frac{13}{100}$ **d** $\frac{229}{100}$

**e** $\frac{7}{10}$ **f** $\frac{9}{10}$ **g** $\frac{7}{1000}$ **h** $\frac{527}{1000}$

Example 3c

**5** Express each fraction as a percentage.

**a** $\frac{3}{5}$ **b** $\frac{17}{20}$ **c** $\frac{11}{25}$ **d** $\frac{23}{50}$

**e** $\frac{19}{25}$ **f** $\frac{19}{20}$ **g** $\frac{53}{50}$ **h** $\frac{52}{25}$

**i** $\frac{87}{20}$ **j** $\frac{3}{40}$ **k** $\frac{7}{40}$ **l** $\frac{17}{40}$

Example 4

**6** Write each decimal as a percentage.

**a** 0.35 **b** 0.27 **c** 0.73 **d** 1.3

**e** 5.6 **f** 1.29 **g** 0.125 **h** 0.375

Example 1d

**7** Write each percentage as a fraction.

**a** $2\frac{1}{2}\%$ **b** $5\frac{1}{2}\%$ **c** $6\frac{1}{4}\%$ **d** $87\frac{1}{2}\%$

**8** Complete the table, using decimal, fraction and percentage equivalents for each value.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Decimal** | 0.5 | | | 0.4 | | 0.457 | | 0.403 | |
| **Fraction** | | $\frac{1}{4}$ | | | | | $\frac{23}{100}$ | | $\frac{5}{8}$ |
| **Percentage** | 50% | | 75% | | 100% | | | | |

Example 5

**9** Write as percentages.

**a** $\frac{1}{6}$ **b** $\frac{1}{8}$ **c** $\frac{3}{8}$ **d** $\frac{2}{7}$ **e** $\frac{5}{6}$

**f** $\frac{2}{9}$ **g** $\frac{5}{11}$ **h** $\frac{7}{12}$ **i** $\frac{5}{7}$ **j** $\frac{11}{12}$


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# 15B One quantity as a percentage of another

Percentages can be used to compare quantities.

### Example 6

There are 50 people in a swimming club, and 35 of these people go to squad training. Calculate the number of people who go to squad training as a percentage of the number of swimming club members.

#### Solution

The fraction going to squad training is $\frac{35}{50}$.

$$\begin{aligned}\text{Percentage going to squad} &= \frac{35}{50} \times 100\% \\ &= \frac{35}{50} \times \frac{100}{1}\% \\ &= 70\%\end{aligned}$$

So 70% of the swimming club members go to squad training.

### One quantity as a percentage of another

To express one quantity as a percentage of another, write the first as a fraction of the second, and then convert to a percentage by multiplying by 100%.

### Example 7

Express the first number as a percentage of the second.

**a** 30, 50 **b** 35, 40

#### Solution

**a** $\frac{30}{50} = \frac{60}{100}$
$= 60\%$

or $\frac{30}{50} = \frac{30}{\cancel{50}} \times \frac{\cancel{100}^{2}}{1}\%$
$= 60\%$

**b** $\frac{35}{40} = \frac{7}{8}$
$= 87\frac{1}{2}\%$

or $\frac{35}{40} = \frac{35}{\cancel{40}_{2}} \times \frac{\cancel{100}^{5}}{1}\%$
$= \frac{175}{2}\%$
$= 87\frac{1}{2}\%$


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### Example 8

Express each quantity as a percentage of the second.

**a** 70 cents, \$1 **b** 350 m, 1 km **c** 340 grams, 2 kg

### Solution

**a** Using 100 cents = \$1:

$$\frac{70}{100} = 70\%$$

**b** Using 1 km = 1000 m:

$$\frac{350}{1000} = \frac{35}{100} = 35\% \quad \text{or} \quad \frac{350}{1000} = \frac{350}{1000} \times \frac{100}{1}\% = 35\%$$

**c** Using 1000 grams = 1 kg:

$$\frac{340}{2000} = \frac{17}{100} = 17\% \quad \text{or} \quad \frac{340}{2000} = \frac{340}{2000} \times \frac{100}{1}\% = 17\%$$

## Exercise 15B

Example 7

**1** Express the first number as a percentage of the second.

**a** 40, 50 **b** 25, 40 **c** 21, 75

**d** 17, 20 **e** 3, 10 **f** 15, 40

**g** 100, 80 **h** 75, 80 **i** 80, 40

**j** 56, 64 **k** 30, 150 **l** 25, 8

Example 8

**2** Express each quantity as a percentage of the second.

**a** 60 cents, \$1 **b** 750 m, 1 km **c** 340 grams, 4 kg

**d** \$1.20, \$5 **e** 23 mm, 5 cm **f** \$1.50, \$10

**g** \$4.80, \$20 **h** \$3.60, \$8.00 **i** 1250 m, 2 km

**j** 200 $cm^2$, 5 $m^2$ **k** \$23 500, \$100 000 **l** \$34.20, \$60

Example 6

**3** Express each amount as a percentage of the total.

**a** 18 boys in a class of 25 students

**b** 160 women in a train carrying 200 people

**c** 15 people out of 125 people in a restaurant ordering lasagne

**d** 54 marks out of a total of 80 marks on a test

**e** 100 marks out of a total of 160 marks on a test


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# 15C Percentage of a quantity

In Chapter 5, we found fractions of a quantity. In this section, we use percentages for the same purpose.

## Example 9

25% of people in a small town watched the tennis final. Calculate how many people watched the tennis final if there are 3220 people in town.

### Solution

25% of 3220 people watched the final.

$$\begin{aligned}\text{Number of people watching} &= 25\% \text{ of } 3220 \\ &= \frac{25}{100} \times 3220 \\ &= \frac{1}{4} \times 3220 \\ &= 805\end{aligned}$$

## Example 10

Calculate:

**a** 20% of 415 **b** 63% of 200 **c** 150% of 600 **d** 200% of 5.2

### Solution

**a** $$\begin{aligned}20\% \text{ of } 415 &= \frac{20}{100} \times \frac{415}{1} \\ &= \frac{1}{5} \times \frac{415}{1} \\ &= 83\end{aligned}$$

**b** $$\begin{aligned}63\% \text{ of } 200 &= \frac{63}{\cancel{100}^{1}} \times \frac{\cancel{200}^{2}}{1} \\ &= 126\end{aligned}$$

**c** $$\begin{aligned}150\% \text{ of } 600 &= \frac{150}{100} \times \frac{600}{1} \\ &= \frac{3}{2} \times \frac{600}{1} \\ &= 900\end{aligned}$$

**d** $$\begin{aligned}200\% \text{ of } 5.2 &= 2 \times 5.2 \\ &= 10.4\end{aligned}$$


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## Exercise 15C

Example 9

**1** Johan spends 25% of his wages on rent. He earns $888 a week. How much rent does he pay a week?

Example 10

**2** Calculate:

| | | |
|---|---|---|
| **a** 20% of 100 | **b** 20% of 200 | **c** 20% of 1000 |
| **d** 50% of 300 | **e** 50% of 30 | **f** 50% of 3 |
| **g** 150% of 100 | **h** 150% of 500 | **i** 150% of 30 |
| **j** 200% of 100 | **k** 200% of 118 | **l** 200% of 3.5 |

**3** Use your knowledge of fractions equivalent to given percentages to calculate these amounts.

| | | |
|---|---|---|
| **a** 50% of 368 | **b** 25% of 144 | **c** 12.5% of 328 |
| **d** $33\frac{1}{3}$% of 621 | **e** 20% of 750 | **f** 40% of 255 |

**4** Celine receives $200 as a gift from her grandparents. She decides that she will spend 13% of the money purchasing CDs. How much does she spend on CDs?

**5** Daniel earns $400 a week. He pays 15% of this in tax. How much tax does he pay every week?

**6** Nick works out that there are 168 hours in each week. He spends 37.5% of this time sleeping, 20% at school, 2% at his part-time job and 5% doing homework. The rest of his time is leisure time.

**a** How many hours each week does Nick work at his part-time job?

**b** How many hours homework does he do each week?

**c** What percentage of his week does Nick have as leisure time?

# 15D Ratios

Ratios provide a way of comparing two or more related quantities. Ratios are closely connected to fractions, but in many problems they are more convenient to use than fractions.

Suppose I have 4 red jelly beans and 11 yellow jelly beans. The ratio of the number of red jelly beans to the number of yellow jelly beans is written as:

number of red jelly beans : number of yellow jelly beans = 4 : 11


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### Example 11

A family has 3 males and 7 females. Write down:

**a** the ratio of the number of males to the number of females

**b** the ratio of the number of females to the number of males

**c** the ratio of the number of females to the total number of people in the family

#### Solution

**a** number of males : number of females = 3 : 7

**b** number of females : number of males = 7 : 3

**c** number of females : total number of family members = 3 : 10

Now, suppose that I mix 200 mL of cordial and 700 mL of water in a jug. The mixture then contains two parts of cordial to every seven parts of water. This is written as:

cordial : water = 2 : 7 (Read this as 'The ratio of cordial to water is 2 to 7.')

or as:

= 7 : 2 (Read this as 'The ratio of water to cordial is 7 to 2.')

In this example, 'one part' is 100 mL. Another mixture of identical strength could be made by taking 'one part' to be 1 L, and mixing 2 L of cordial with 7 L of water. This idea of parts can be very useful in dealing with problems involving ratios.

### Example 12

A mixture contains 30 litres of fruit juice and 50 litres of water. What is the ratio of fruit juice to water?

#### Solution

We will solve this using the language of parts.

Take 10 litres as 1 part.

30 litres of fruit juice = 3 parts each of 10 litres

50 litres of water = 5 parts each of 10 litres

Hence, the ratio of fruit juice to water = 3 : 5.

## Ratios and fractions

If you cut a rope into two equal lengths, the ratio of the two parts is 1 : 1. In this case, each of the shorter lengths is half the total length. Can you see a connection between the ratio 1 : 1 and the fraction $\frac{1}{2}$?

In the following examples, we explore this link between ratios and fractions.


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**Example 13**

The ratio of butter to to flour in a biscuit dough is 2 : 7. What fraction of the biscuit dough is flour?

**Solution**

There are $2 + 7 = 9$ equal parts, 7 of which are flour.

Hence, the fraction of flour in the dough $= \frac{7}{9}$.

## Reducing a ratio to simplest form

A ratio involving whole numbers can be reduced to simplest form – just like a fraction – by dividing all the terms by their highest common factor (HCF). Sometimes doing this can make it easier to understand the situation we are looking at, as in the next example.

**Example 14**

The number of students enrolled at a school is 1200. Of these, 625 are male. What is the ratio, expressed in simplest form, of the number of males to the number of females in the school?

**Solution**

There are $1200 - 625 = 575$ females in the school.

We simplify the ratio in stages.

Number of males : number of females $= 625 : 575$
$= 125 : 115$ (Divide by 5.)
$= 25 : 23$ (Divide by 5.)

The HCF of 625 and 575 is 25 and we could have divided through by 25 directly.

Equivalent ratios behave like equivalent fractions. $1 : 3 = 2 : 6 = 3 : 9 = 4 : 12$ and so on.



## Exercise 15D

Example 11, 12

**1** In a bowl of fruit, there are 11 oranges and 7 apples. Write down:

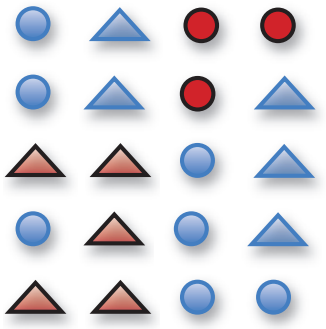
**a** the ratio of the number of apples to the number of oranges

**b** the ratio of the number of oranges to the number of apples

**c** the ratio of the number of oranges to the number of pieces of fruit.


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**2** For the diagram opposite, write down:



**a** the ratio of the number of triangles to the number of circles

**b** the ratio of the number of red circles to the number of blue circles

**c** the ratio of the number of blue triangles to the number of red triangles

**d** the ratio of the number of blue triangles to the number of red circles.

Example 14

**3** Reduce each ratio to simplest form.

**a** 3 : 6 **b** 8 : 12 **c** 64 : 48 **d** 96 : 144

**e** 12 : 3 **f** 33 : 22 **g** 60 : 70 **h** 88 : 96

Example 13

**4** A rope is cut into sections so that the resulting lengths are in the ratio 3 : 2.

**a** Express the length of the shorter piece of rope as a fraction of the total length of rope.

**b** Express the length of the longer piece of rope as a fraction of the total length.

**5** A bowl contains green and blue marbles. There are 24 green marbles and 72 blue marbles. What is the ratio of the number of green marbles to the number of blue marbles?

**6** In a bus, 27 of the 63 passengers are male. What is the ratio of the number of male passengers to the number of female passengers?

**7** One-third of the flowers in a garden are blue. What is the ratio of the number of blue flowers to the number of flowers of other colours?

**8** In February Jennifer measured her height to be 125 cm. By the end of December she was 130 cm tall.

**a** What is the ratio of her height in February to her height in December?

**b** Find the ratio of the amount Jennifer had grown in this period to her height in February.

**9** A rectangle has length 15 cm and width 6 cm. A second rectangle has length 18 cm and width 24 cm. Find the ratio of:

**a** the lengths of the rectangles

**b** the widths of the rectangles

**c** the perimeters of the rectangles

**d** the areas of the rectangles


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# 15E Solving problems with ratios

Ratio problems are often best solved using the language of parts. The rest of the working is just another form of the unitary method.

### Example 15

There are 48 blue and red marbles in a box. The ratio of red marbles to blue marbles is 3 : 5.

How many blue marbles are there?

#### Solution

There are $3 + 5 = 8$ parts.

8 parts = 48 marbles

Thus, 1 part = 6 marbles

The red marbles are 3 of the parts and the blue marbles 5 of the parts.

There are $3 \times 6 = 18$ red marbles and $5 \times 6 = 30$ blue marbles.

### Example 16

There are sheep and goats in a paddock. The ratio of the number of sheep to the number of goats is 7 : 3. There are 24 goats.

**a** How many sheep are there in the paddock?

**b** How many sheep and goats are there in the paddock?

#### Solution

There are 10 parts in total.

Three parts are goats and there are 24 goats.

So 3 parts = 24
1 part = 8

**a** There are $7 \times 8 = 56$ sheep.

**b** There are $10 \times 8 = 80$ sheep and goats.

## Exercise 15E

Example 15

**1** **a** Divide 45 in the ratio 4 : 5

**b** Divide 96 in the ratio 9 : 7.

**c** Divide 72 in the ratio 3 : 5.

**d** Divide 144 in the ratio 5 : 7.

**e** Divide 100 in the ratio 3 : 2

**f** Divide 132 in the ratio 9 : 2.

**g** Divide 256 in the ratio 3 : 5.

**h** Divide 99 in the ratio 3 : 8.


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Example 16

**2** It is known that the ratio of petunias to pansies in a garden bed is 3 : 5. If there are 138 petunias in the garden bed, how many pansies are there?

**3** There are 64 blue and red marbles in a box. The ratio of red marbles to blue marbles is 3 : 5. How many blue marbles are there?

**4** A bowl contains green and red glass balls. The ratio of the number of green balls to the number of red balls is 1 : 4. If there are 28 red balls, how many green balls are there?

**5** The ratio of the number of girls to the number of boys in a class is 3 : 4. If there are 20 boys, how many girls are there?

**6** A piece of string 256 cm long is to be divided in the ratio 5 : 3. How long is each part?

**7** A piece of string is to be divided in the ratio 5 : 3. The shorter length is 222 cm. What is the length of the other part?

**8** An interval $AB$ is 10 cm in length. $C$ is a point on $AB$ such that the ratio of the length $AC$ to the length $CB$ is 3 : 2. Find the lengths of $AC$ and $CB$.

**9** A rectangle has perimeter 56 cm. The ratio of the length to the width is 5 : 2. Find the length and the width.

**10** A piece of string is cut into two lengths so that the longer length is 25 cm. The ratio of the lengths is 5 : 2. What is the length of the string?

**11** The length of a rectangle is 69 cm. The ratio of the length to the width is 3 : 5. Find the perimeter of the rectangle.

**12** The ratio of length to width in a rectangle is 5 : 3. The length is 6 cm longer than the width. Find the area of the rectangle.

# 15F Best buys

In this section we apply the unitary method to help us compare prices.

**Example 17**

Packet 1 of lentils contains 240 grams of lentils and costs $6.

Packet 2 of lentils contains 220 grams of lentils and costs $5.

Which packet is the cheaper?


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**Solution**

We take 1 part to be 10 grams.

**For the first packet**

There are $240 \div 10 = 24$ parts.

The cost of 1 part $= 6.00 \div 24 = 0.25$,

so the cost of one part is 25 cents.

**For the second packet**

There are $220 \div 10 = 22$ parts.

The cost of 1 part $= 5.00 \div 22 = 0.2272 \ldots$ ,

so the cost of one part is approximately 23 cents, correct to the nearest cent.

The second packet is cheaper, but only by a little more than 2 cents per 10 grams.

In Australia supermarkets are required to display **unit prices** on some items. For volume, the unit price is given per 100 millilitres. For items sold by weight, it is usually per 100 grams. For the above example, the unit price is \$2.50 per 100 grams for the first packet, and approximately \$2.27 per 100 grams for the second packet. The calculation above could have been carried out with 100 grams as the unit.

Calculating unit prices is an application of the unitary method.

## Exercise 15F

**1** A box of mangoes contains 24 mangoes. The cost of the box is \$20. What is the cost per mango (rounding correct to the nearest cent)?

**2** One store at the market advertises oranges at 15 for \$2.00 and another at 20 for \$2.40. Calculate the price per orange for each store (rounding correct to the nearest cent).

**3** Detergent is offered at a cost of:

**a** \$6 for 2.4 litres

**b** \$12 for 5 litres

What is the cost per litre of each offer?

Example 17

**4** Shop A has a special offer on a type of confectionery: \$7.50 for 400 grams

Shop B also has a special offer: \$16 for a kilogram of the same type of confectionary.

What is the cost per 100 grams from each shop?

**5** A stationery dealer advertises 400 A4 pages for \$5.60 and also 1000 A4 pages at \$12. Which is the better offer?


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**6** Brand A of fruit juice is priced at \$4.50 for a bottle containing 1000 millilitres = 1 litre of juice. Brand B of juice costs \$4.00 for a bottle containing 750 millilitres of the same type of juice. Calculate the unit price (rounding correct to the nearest cent) using:

**a** the price of each per 50 millilitres

**b** the price of each per 100 millilitres

**7** In the schoolyard marble market, Alice and Carol offer different exchange rates:

Alice: 3 blue marbles are worth 5 red marbles

Carol: 5 blue marbles are worth 8 red marbles

You have 15 blue marbles. Explain how to get a free red marble.

# Review exercise

**1** Convert each percentage to a fraction or mixed numeral.

**a** 25% **b** 56% **c** 3% **d** 140%

**e** 108% **f** 999% **g** $2\frac{1}{2}\%$ **h** $12\frac{1}{2}\%$

**2** Convert each percentage to a decimal.

**a** 34% **b** 99% **c** 23% **d** 2%

**e** 9% **f** 123% **g** 250% **h** 383%

**3** Convert each number to a percentage.

**a** $\frac{2}{10}$ **b** 0.45 **c** $\frac{98}{100}$ **d** 0.2 **e** $\frac{23}{10}$

**f** $\frac{100}{10}$ **g** 1.8 **h** $\frac{4}{5}$ **i** 3.7 **j** $\frac{8}{20}$

**k** 0.03 **l** $\frac{2}{40}$ **m** $\frac{1}{4}$ **n** $\frac{9}{4}$ **o** 0.0004

**4** Calculate:

**a** 33% of 100 **b** 89% of 100 **c** 24% of 50

**d** 12% of 150 **e** 15% of 120 **f** 50% of 18

**g** 15% of 488 **h** 39% of 285 **i** 87% of 34

**j** 12.5% of 1888 **k** $2\frac{1}{2}\%$ of 1200 **l** 37.5% of 4000

**5** **a** Express \$2.40 as a percentage of \$5.00.

**b** Express 250 metres as a percentage of 4.5 kilometres.

**c** Express \$23 678 as a percentage of \$200 000.

**d** Express 345 grams as a percentage of 4 kg.


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**6** Twenty-three of the students in a maths class of 25 students completed their test. What percentage is this?

**7** At the final of a tennis tournament, 15% of the reserved seats are vacant. If there is a total of 860 reserve seats, how many are vacant?

**8** There are 750 students in the school and 200 are in Year 7. What percentage of the school population is in Year 7?

**9** In a bowl of fruit, there are 11 pears and 8 apples. Write down:

**a** the ratio of the number of pears to the number of apples

**b** the ratio of the number of apples to the number of pears

**c** the ratio of the number of pears to the number of pieces of fruit

**10** A bowl contains green and blue marbles. There are 14 green marbles and 70 blue marbles. What is the ratio of the number of green marbles to the number of blue marbles?

**11** Reduce each ratio to simplest form.

**a** 7 : 14 **b** 9 : 12 **c** 108 : 63 **d** 108 : 144

**e** 24 : 3 **f** 55 : 33 **g** 80 : 70 **h** 56 : 96

**12** **a** Divide 108 in the ratio 4 : 5. **b** Divide 90 in the ratio 8 : 7.

**c** Divide 144 in the ratio 1 : 2. **d** Divide 144 in the ratio 5 : 7.

**13** It is known that the ratio of carrots to turnips in a vegetable garden is 3 : 2. If there are 138 turnips in the vegetable garden, how many carrots are there?

**14** There are 132 blue and red marbles in a box. The ratio of red marbles to blue marbles is 3 : 8. How many blue marbles are there?

**15** A bowl contains green and red balls. The ratio of the number of green balls to the number of red balls is 7 : 4. If there are 244 red balls, how many green balls are there?

**16** The ratio of the number of boys to the number of girls in a class is 3 : 4. If there are 21 boys, how many girls are there?

**17** A box of apples contains 20 apples. The cost of the box is $15. What is the cost per apple?

**18** One store at the market advertises pears at 20 for $6.00 and another store 15 for $5.00. Calculate the price per pear at each store. (Round to the nearest cent.)


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# Challenge exercise

**1** Raylee gives 60% of her weekly wage to her mother, and 25% of the remainder to her brother. She still has $240 left. How much does she earn in one week?

**2** Felicity spent 20% of her savings on a bicycle and 15% of the remainder on a book. What percentage of her savings did she have left?

**3** Calculate:

**a** 20% of 30% of 100

**b** 10% of 50% of 100

**c** 5% of 30% of 200

**4** Ana went on a shopping spree. She spent $24 of her pocket money on a dress and 20% of the remainder on a shirt. She still had $\frac{2}{3}$ of her money left. How much did she have before she began spending?

**5** Judith's salary is $800 per week. She receives a pay increase of 12% in January but then has a pay decrease in July of 8%. What is her salary after this decrease?

**6** In a lottery, only 0.0008% of tickets won prizes. If there were five prizes, how many tickets were sold?

**7** Gary makes up a drink so that 10% of the drink is pure orange juice. The remaining 90% of the fluid has no orange juice in it. Gary has 400 mL of the drink in a jug. He wants to add orange juice to the drink so that 12% of the drink is orange juice. How much extra orange juice does he need to add to the jug?

**8** Researchers are trying to estimate the number of penguins in a colony. One day, they catch 40 penguins, tag their legs, then set them free. The next day, they catch 33 penguins and 5 of these have tagged legs. Estimate the number of penguins in the colony.

**9** Bottle 1 contains water and cordial in the ratio 2 : 5. Bottle 2 has the same total volume and has water and cordial in the ratio 3 : 7. If the contents are combined, what is the ratio of water to cordial? Hint: It is not 5 : 12.

**10** In the previous question, what would the combined ratio of water to cordial be if bottle 2 had twice the volume of bottle 1?


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CHAPTER

# 16

Number and Algebra

# Solving equations

What number plus 5 gives 18?

We can write this question as $x + 5 = 18$, where the pronumeral $x$ is the unknown number. This is called an equation.

While we can guess the solution is $x = 13$, more complicated questions give rise to harder equations.

For example, in the introduction to Chapter 12 we discussed the use of algebra to convert a temperature from the Celsius to the Fahrenheit scale.

We substituted $C = -5$ in the expression $\frac{9}{5}C + 32$. The result was $\frac{9}{5} \times (-5) + 32 = 23$, which means that $-5$°C is the same as 23°F.

In Australia, where we use the Celsius scale, it is more likely we would want to convert a temperature from the Fahrenheit to the Celsius scale. For example, what does a temperature of 86°F in the United States mean to us in Australia?

We ask: what value of $C$ makes $\frac{9}{5}C + 32 = 86$ a true statement? This statement is an **equation**, and 30 is the solution of the equation.

So 86°F corresponds to 30°C, a fairly warm day!

In this chapter, we develop a systematic approach to solving equations.



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# 16A An introduction to equations

Joe has a pencil case that contains a certain number of pencils. He has three other pencils, and in total he has 11 pencils. How many pencils are in the pencil case?





Let $x$ be the number of pencils in the pencil case. We know that:

$$x + 3 = 11$$

This statement is called an **equation**.

The solution is $x = 8$, because $\mathbf{8} + 3 = 11$ and no other number makes the statement true.

This means that there are 8 other pencils in the pencil case.

The process of finding the pronumeral in an equation is called **solving the equation**.

## Finding a solution by trial and error

Here is another equation:

$$2x + 4 = 10$$

We can find a solution to this equation by trying a few numbers as values of $x$. For example:

$$2 \times 1 + 4 = 6$$
$$2 \times 2 + 4 = 8$$
$$2 \times 3 + 4 = 10$$

So the solution of this equation is $x = 3$.

We have found this solution by **trial and error**. This is an unsystematic way to solve equations and only very simple equations can be solved in this way. In the rest of this chapter, we begin to develop systematic methods for solving equations.

### Example 1

Solve each equation mentally.

**a** $x + 4 = 6$

**b** $10 - a = 6$

**c** $6x + 7 = 19$

**d** $\frac{x}{3} = 5$

### Solution

**a** For the equation $x + 4 = 6$, the solution is $x = 2$, because $\mathbf{2} + 4 = 6$.

**b** For the equation $10 - a = 6$, the solution is $a = 4$, because $10 - \mathbf{4} = 6$.

**c** For the equation $6x + 7 = 19$, the solution is $x = 2$, because $6 \times \mathbf{2} + 7 = 19$.

**d** For the equation $\frac{x}{3} = 5$, the solution is $x = 15$, because $\frac{\mathbf{15}}{3} = 5$.


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## Exercise 16A

**1** Three pencil cases each have $x$ pencils in them, and there are four loose pencils. We know that there are 31 pencils in total.



**a** Write down an equation for $x$.

**b** Write down the solution of this equation.

**c** How many pencils are there in each pencil case?

Example 1

**2** Solve each equation for $x$ mentally.

**a** $x + 3 = 10$ **b** $x - 4 = 11$ **c** $x - 5 = 10$ **d** $2x = 10$

**e** $x - 14 = 7$ **f** $\frac{x}{3} = 7$ **g** $2x + 3 = 11$ **h** $17 - x = 10$

**i** $2x + 6 = 15$ **j** $18 - 2x = 0$ **k** $\frac{x}{4} = 6$ **l** $\frac{x}{5} = 20$

**3** Solve each equation for $a$ mentally.

**a** $a + 3 = 11$ **b** $a - 7 = 23$ **c** $2a = 30$ **d** $5a = 30$

**e** $\frac{a}{5} = 7$ **f** $5a + 7 = 22$ **g** $15 - a = 8$ **h** $2a + 10 = 30$

**4** Twenty-three boxes of chocolates each have $a$ chocolates in them. There are also 17 loose chocolates. In total there are 707 chocolates.

**a** Write an equation for $a$. **b** Solve the equation for $a$.

**5** Tom has 10 bags of marbles, each containing $x$ marbles. He tidies his room and finds 23 more marbles under his bed. He knows that he now has a total of 523 marbles.

**a** Write an equation for $x$. **b** Solve the equation for $x$.

**6** A company runs minibuses, each of which carries $n$ passengers. There are six minibuses and they hold a total of 72 passengers.

**a** Write an equation for $n$. **b** Solve the equation for $n$.

**7** Craig has six packets of chocolates, each of which holds $c$ chocolates. He also has seven chocolates that are not in a packet. He has a total of 127 chocolates.

**a** Write an equation for $c$. **b** Solve the equation for $c$.

**8** Sally has 19 stickers and gives $x$ stickers to her sister. She counts the remaining stickers and discovers she now has 12.

**a** Write an equation for $x$. **b** Solve the equation for $x$.

**9** If $y = 3x$, what is the value of $x$ when $y = 33$?

**10** If $y = 2x + 1$, what is the value of $x$ when $y = 11$?

**11** If $y = x - 5$, what is the value of $x$ when $y = 19$?



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# 16B Equivalent equations

Consider the equation:

$2x + 3 = 9$ (1)

Suppose we add 2 to each side.

$2x + 5 = 11$ (2)

Equation (2) is obtained from equation (1) by adding 2 to each side of the equation.

Equation (1) is obtained from equation (2) by subtracting 2 from each side of the equation.

Equations (1) and (2) are said to be **equivalent** equations.

Here are same more equations equivalent to equation (1):

$2x = 6$ (3)

$x = 3$ (4)

Equation (3) is obtained from equation (1) by subtracting 3 from each side of the equation.

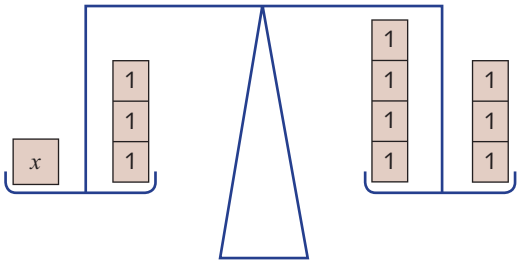
The value $x = 3$ satisfies each of the above equations.

Equation (4) is obtained from equation (3) by dividing each side of the equation by 2. You can obtain equation (3) from equation (4) by multiplying each side by 2. Once again, we say that equations (3) and (4) are equivalent. So equations (1), (2), (3) and (4) are all equivalent.

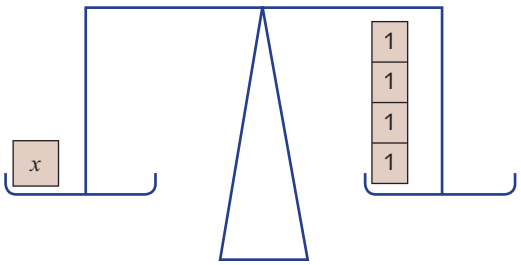
## Intepretation with scales

Imagine a pair of balanced scales, as shown in the diagrams below. Two different weights are used: 1 and $x$. You could think of the numbers as representing weights in kilograms, so 1 means 1 kg. When we have simple scales like the ones shown, in which the arms are of equal length, then they are balanced when the weights are equal.

The equation $x + 3 = 7$ can be used to represent the fact that the scales in the first diagram are balanced.



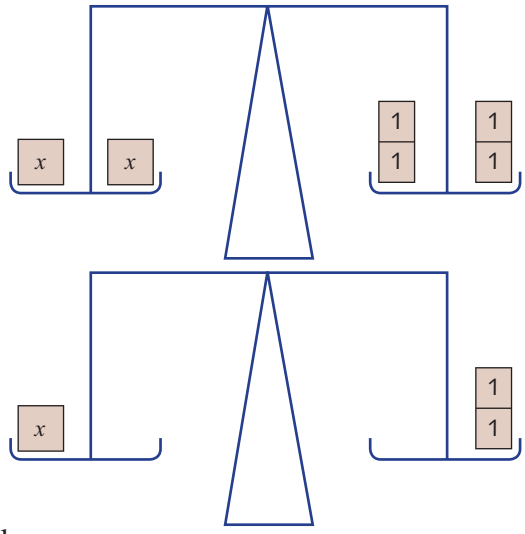

Subtract 3 from both sides to show that $x = 4$ is the solution to the equation $x + 3 = 7$.





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Consider another example. In the diagram below, the equation $2x = 4$ represents the fact that the scales are balanced.




Divide both sides by 2 to show that $x = 2$ is the solution of the equation $2x = 4$.

From the examples above, we can see the following general rules.

## Equivalent equations

Given an equation we can form an equivalent equation by:

- adding the same number to, or subtracting the same number from, both sides of an equation.
- multiplying or dividing both sides of an equation by the same non-zero number.

Equivalent equations have the same solution.

## Example 2

Solve each equation for $x$.

**a** $x + 3 = 5$ **b** $x - 4 = 7$ **c** $3x = 23$ **d** $\frac{x}{4} = 7$

### Solution

**a**
$$x + 3 = 5$$
$\boxed{-3}$ $x + 3 - 3 = 5 - 3$ (Subtract 3 from both sides of the equation.)
$$x = 2$$

**b**
$$x - 4 = 7$$
$\boxed{+4}$ $x - 4 + 4 = 7 + 4$ (Add 4 to both sides of the equation.)
$$x = 11$$

**c**
$$3x = 23$$
$\boxed{\div 3}$ $\frac{3x}{3} = \frac{23}{3}$ (Divide both sides of the equation by 3.)
$$x = 7\frac{2}{3}$$

**d**
$$\frac{x}{4} = 7$$
$\boxed{\times 4}$ $\frac{x}{4} \times 4 = 7 \times 4$ (Multiply both sides of the equation by 4.)
$$x = 28$$

The notation $\boxed{\times 4}$, $\boxed{\div 3}$ and so on is recommended as a way of recording your work. You do not need both the box and the comment in parentheses.


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### Example 3

In each case below, write an equation and solve it.

**a** A number $x$ has 7 added to it and the result is 35.

**b** A number $x$ is multiplied by 15 and the result is 165.

**c** A number $x$ has 10 subtracted from it and the result is 3.

**d** A number $x$ is divided by 9 and the result is 12.

### Solution

**a**

$$x + 7 = 35$$

$\boxed{-7}$

$$x = 35 - 7$$

$$= 28$$

(Subtract 7 from both sides of the equation.)

The number is 28.

**b**

$$15x = 35$$

$\boxed{\div 15}$

$$x = \frac{165}{15}$$

$$= 11$$

(Divide both sides of the equation by 15.)

The number is 11.

**c**

$$x - 10 = 3$$

$\boxed{+10}$

$$x = 3 + 10$$

$$= 13$$

(Add 10 to both sides of the equation.)

The number is 13.

**d**

$$\frac{x}{9} = 12$$

$\boxed{\times 9}$

$$x = 12 \times 9$$

$$= 108$$

(Multiply both sides of the equation by 9.)

The number is 108.

## Exercise 16B

Example 2a, b

**1** Solve each equation for $m$.

**a** $m + 3 = 6$ **b** $m + 5 = 16$ **c** $m + 8 = 10$

**d** $m - 5 = 11$ **e** $m - 6 = 11$ **f** $m - 10 = 6$

Example 2c

**2** Solve each equation for $n$.

**a** $2n = 6$ **b** $3n = 9$ **c** $5n = 25$

**d** $3n = 16$ **e** $12n = 100$ **f** $18n = 46$

**g** $5n = 17$ **h** $6n = 51$ **i** $3n = 17$


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Example 2d

**3** Solve each equation for $x$.

**a** $\frac{x}{3} = 12$ **b** $\frac{x}{2} = 15$ **c** $\frac{x}{5} = 16$

**d** $\frac{x}{10} = 2$ **e** $\frac{x}{3} = 6$ **f** $\frac{x}{4} = 8$

**4** Solve each equation for $x$.

**a** $x + 3 = 7$ **b** $5x = 27$ **c** $\frac{x}{4} = 5$

**d** $x - 4 = 5$ **e** $3x = 36$ **f** $x + 2 = 6$

**g** $2x = 18$ **h** $x + 5 = 11$ **i** $\frac{x}{5} = 2$

**j** $x - 3 = 11$ **k** $x - 5 = 2$ **l** $3x = 7$

**m** $4x = 9$ **n** $7x = 15$ **o** $6x = 17$

Example 3

**5** In each part below, write an equation and solve it.

**a** A number $x$ has 5 added to it and the result is 21.

**b** A number $x$ is multiplied by 7 and the result is 35.

**c** A number $x$ is multiplied by 5 and the result is 37.

**d** A number $x$ is divided by 3 and the result is 23.

**e** A number $x$ has 15 subtracted from it and the result is 37.

**6** In each part below, write an equation and solve it to find the number.

**a** A number $z$ has 7 added to it and the result is 12.

**b** Twelve is added to a number $z$ to give 19.

**c** Six is subtracted from a number $z$ and the result is 14.

**d** A number $z$ is taken away from 9 and the result is 6.

**e** A number $z$ is multiplied by 3 and the result is 5.

**f** Five is multiplied by a number $z$ and the result is 45.

**g** A number $z$ is divided by 6 and the result is 7.


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# 16C Solving equations involving more than one step

We cannot solve $2x + 3 = 7$ in one step, although mentally the solution $x = 2$ is easy to find. We need two steps to solve the equation.

First, subtract 3 from both sides of the equation.

$$\begin{aligned} 2x + 3 - 3 &= 7 - 3 \\ 2x &= 4 \end{aligned}$$

Now, divide both sides of the equation by 2.

$$\begin{aligned} \frac{2x}{2} &= \frac{4}{2} \\ x &= 2 \end{aligned}$$

A similar method is used to solve the equation $3x - 5 = 13$.

First, add 5 to both sides of the equation.

$$3x = 18$$

Now, divide both sides of the equation by 3.

$$x = 6$$

## Example 4

Solve each equation for $x$ and check the result by substitution.

**a** $2x - 3 = 11$

**b** $4x + 3 = 10$

### Solution

**a**

$$\begin{aligned} 2x - 3 &= 11 \\ \boxed{+3} \quad 2x - 3 + 3 &= 11 + 3 \\ 2x &= 14 \\ \boxed{\div 2} \quad x &= 7 \end{aligned}$$

*Check:* $\text{LHS} = 2 \times 7 - 3$
$= 11$
$= \text{RHS}$

**b**

$$\begin{aligned} 4x + 3 &= 10 \\ \boxed{-3} \quad 4x &= 7 \\ \boxed{\div 4} \quad x &= 1\tfrac{3}{4} \end{aligned}$$

*Check:* $\text{LHS} = 4 \times 1\frac{3}{4} + 3$
$= 4 \times \frac{7}{4} + 3$
$= 10$
$= \text{RHS}$

## Example 5

Solve each equation for $x$.

**a** $6 + 4x = 22$

**b** $\frac{x}{5} + 7 = 22$


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### Solution

**a**

$$\begin{array}{rl} & 6 + 4x = 22 \\ \boxed{-6} & 4x = 16 \\ \boxed{\div 4} & x = 4 \end{array}$$

**b**

$$\begin{array}{rl} & \frac{x}{5} + 7 = 22 \\ \boxed{-7} & \frac{x}{5} = 15 \\ \boxed{\times 5} & x = 15 \times 5 \\ & \quad = 75 \end{array}$$

## Example 6

In each part below, write an equation and solve it.

**a** A number $x$ is multiplied by 15, and 5 is added. The result is 170.

**b** A number $x$ is divided by 11, and 6 is added. The result is 13.

### Solution

**a** The equation is $15x + 5 = 170$.

$$\begin{array}{rl} \boxed{-5} & 15x = 165 \\ \boxed{\div 15} & x = \frac{165}{15} \\ & \quad = 11 \end{array}$$

The number is 11.

**b** The equation is $\frac{x}{11} + 6 = 13$.

$$\begin{array}{rl} \boxed{-6} & \frac{x}{11} = 7 \\ \boxed{\times 11} & x = 11 \times 7 \\ & \quad = 77 \end{array}$$

The number is 77.

## Exercise 16C

Example 4, 5a

**1** Solve each equation for $x$. Check your solutions to parts **c**, **f** and **i**.

**a** $2x + 1 = 7$ **b** $5x - 1 = 11$ **c** $7x + 3 = 17$

**d** $4x + 2 = 18$ **e** $1 + 5x = 21$ **f** $5 + 20x = 100$

**g** $2 + 10x = 44$ **h** $5x - 11 = 30$ **i** $10x + 23 = 100$

Example 5b

**2** Solve each equation for $z$. Check your solutions to parts **c** and **f**.

**a** $\frac{z}{4} + 5 = 11$ **b** $\frac{z}{3} - 5 = 10$ **c** $\frac{z}{4} - 7 = 8$

**d** $\frac{z}{11} - 7 = 10$ **e** $\frac{z}{8} + 11 = 20$ **f** $\frac{z}{8} - 12 = 8$

**g** $\frac{z}{9} + 13 = 16$ **h** $\frac{z}{7} - 8 = 17$ **i** $\frac{z}{12} + 73 = 84$

**3** Solve each equation.

**a** $2x - 4 = 5$ **b** $3a - 6 = 36$ **c** $11b + 4 = 121$

**d** $4x + 18 = 30$ **e** $2b - 6 = 12$ **f** $3a - 16 = 19$

**g** $12z - 18 = 12$ **h** $11k + 22 = 43$ **i** $10a + 16 = 42$

**j** $\frac{x}{12} - 2 = 15$ **k** $\frac{x}{16} - 3 = 12$ **l** $\frac{z}{7} + 6 = 18$



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Example 6

**4** In each part below, write an equation and solve it to find the number.

**a** A number $x$ is multiplied by 7, and 6 is then added. The result is 20.

**b** A number $x$ is divided by 4, and 11 is then added. The result is 20.

**c** A number $x$ is divided by 3, and 4 is then subtracted. The result is 23.

**d** A number $x$ is multiplied by 4, and 6 is then added. The result is 30.

**e** A number $m$ is divided by 7 and 11 is subtracted. The result is 20.

**f** A number $m$ is multiplied by 6 and 14 is subtracted. The result is 10.

# 16D Equations with negative solutions

The methods we have developed in the previous section can be used to solve equations whose solutions are negative numbers.

## Example 7

Solve each equation for $x$.

**a** $x + 3 = -2$

**b** $-2x = 10$

**c** $\frac{x}{3} = -5$

**d** $2x + 5 = -6$

## Solution

**a**
$$\begin{array}{rl} x + 3 = -2 & \\ \boxed{-3} & x = -2 - 3 \\ & = -5 \end{array}$$

**b**
$$\begin{array}{rl} -2x = 10 & \\ \boxed{\div (-2)} & x = \frac{10}{-2} \\ & = -5 \end{array}$$

**c**
$$\begin{array}{rl} \frac{x}{3} = -5 & \\ \boxed{\times 3} & x = -5 \times 3 \\ & = -15 \end{array}$$

**d**
$$\begin{array}{rl} 2x + 5 = -6 & \\ \boxed{-5} & 2x = -6 - 5 \\ & 2x = -11 \\ \boxed{\div 2} & x = \frac{-11}{2} \\ & = -5\frac{1}{2} \end{array}$$

## Exercise 16D

Example 7a

**1** Solve each equation and check your solution.

**a** $x + 3 = -6$ **b** $x - 5 = -11$ **c** $x + 6 = -11$ **d** $x - 10 = -5$

**e** $x - 5 = -5$ **f** $x - 15 = -10$ **g** $x - 6 = -10$ **h** $x + 10 = 10$

**i** $x - 3 = -9$ **j** $x + 2 = -5$ **k** $x - 6 = -15$ **l** $m - 15 = -15$


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Example 7b, c

**2** Solve and check.

**a** $2x = -6$ **b** $-3x = 15$ **c** $-3x = -9$

**d** $\frac{x}{3} = -4$ **e** $\frac{x}{5} = -10$ **f** $\frac{x}{7} = -4$

**g** $-5x = -15$ **h** $-3m = -27$ **i** $7m = -98$

**j** $\frac{z}{4} = -25$ **k** $\frac{a}{8} = -9$ **l** $\frac{p}{9} = -11$

Example 7d

**3** Solve:

**a** $2x + 8 = -6$ **b** $3x - 6 = -15$ **c** $5x - 10 = -5$

**d** $4x + 27 = 21$ **e** $20 - 4x = 30$ **f** $5x - 11 = -60$

**g** $15 - 2x = -30$ **h** $7 - 12x = -77$ **i** $-15x + 30 = -45$

**j** $23x + 13 = -33$ **k** $-5m + 7 = 13$ **l** $3p + 19 = 12$

**4** Solve:

**a** $\frac{x}{3} = -4$ **b** $\frac{x}{2} = -6$ **c** $\frac{x}{3} - 4 = -5$

**d** $\frac{x}{8} + 12 = -16$ **e** $\frac{x}{7} + 10 = 5$ **f** $\frac{x}{6} - 5 = -11$

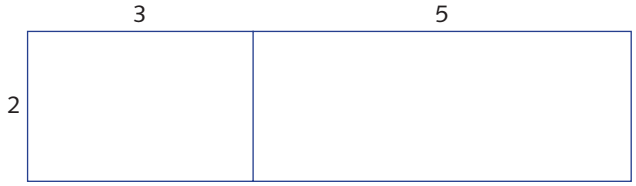
**g** $\frac{m}{12} - 15 = -30$ **h** $10 - \frac{m}{3} = 20$ **i** $-\frac{m}{6} - 15 = 17$

# 16E Expanding brackets and solving equations

In Chapter 1, we saw that:

$$2(3 + 5) = 2 \times 3 + 2 \times 5$$

This can be illustrated with a diagram of a rectangle, broken into two rectangles as shown below.



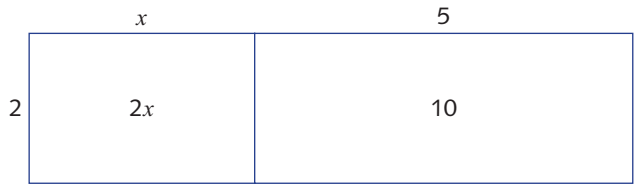

The total area can be found by multiplying 2 by (3 + 5) or by finding the area of each of the smaller rectangles and adding them to give $2 \times 3 + 2 \times 5$.

This idea can also be used in algebra. For example:

$$\begin{aligned} 2(x + 5) &= 2 \times x + 2 \times 5 \\ &= 2x + 10 \end{aligned}$$


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Again, we can use a diagram with rectangles to illustrate this.




Area $= 2(x + 5)$
$= 2x + 10$

The process is called '**expanding the brackets**'.

## Example 8

Expand the brackets.

**a** $4(x + 5)$ **b** $3(x - 4)$ **c** $6(2x - 4)$ **d** $x(x + 2)$

### Solution

**a** $4(x + 5) = 4 \times x + 4 \times 5$
$= 4x + 20$

**b** $3(x - 4) = 3 \times x - 3 \times 4$
$= 3x - 12$

**c** $6(2x - 4) = 6 \times 2x - 6 \times 4$
$= 12x - 24$

**d** $x(x + 2) = x \times x + x \times 2$
$= x^2 + 2x$

From now on we will drop the middle step in the expansion of brackets.

Brackets can appear in equations. In this section, we expand the brackets first, then solve the equation.

## Example 9

Expand the brackets and solve the equations for $z$.

**a** $2(z + 4) = 11$ **b** $2(z - 5) = 13$

### Solution

**a**
$2(z + 4) = 11$
$2z + 8 = 11$ (Expand)
$\boxed{-8}$ $2z = 3$
$\boxed{\div 2}$ $z = 1\frac{1}{2}$

**b**
$2(z - 5) = 13$
$2z - 10 = 13$ (Expand)
$\boxed{+10}$ $2z = 23$
$\boxed{\div 2}$ $z = 11\frac{1}{2}$

## Example 10

In each case below, write an equation and solve it.

**a** 5 is added to the number $x$, and the result is multiplied by 5. The result is 32.

**b** 3 is subtracted from the number $x$, and the result is multiplied by 7. The result of this is 47.


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**Solution**

**a** The equation is $5(x + 5) = 32$.

$5x + 25 = 32$ (Expand.)

$\boxed{-25}$

$5x = 7$

$\boxed{\div 5}$

$x = 1\frac{2}{5}$

**b** The equation is $7(x - 3) = 47$.

$7x - 21 = 47$ (Expand.)

$\boxed{+21}$

$7x = 47 + 21$

$\boxed{\div 7}$

$7x = 68$

$x = 9\frac{5}{7}$

## Exercise 16E

Example 8

**1** Expand the brackets.

**a** $4(x + 5)$ **b** $3(x - 4)$ **c** $6(2x - 4)$

**d** $5(3x - 5)$ **e** $7(2a - 4)$ **f** $6(4 + 3x)$

**g** $9(6x - 11)$ **h** $x(x - 3)$ **i** $x(x + 4)$

**j** $x(2x - 1)$ **k** $x(3 - x)$ **l** $x(2x + 1)$

Example 9

**2** Expand the brackets and solve each equation for $z$.

**a** $3(z + 2) = 11$ **b** $5(z - 6) = 21$ **c** $3(2z - 11) = 10$

**d** $4(z - 1) = 11$ **e** $7(z + 6) = 13$ **f** $2(3z + 1) = 15$

**3** Expand the brackets and solve each equation.

**a** $5(2x - 4) = 37$ **b** $6(3m + 5) = 61$ **c** $5(7m - 11) = 21$

**d** $6(x - 7) = 12$ **e** $12(12m - 4) = 52$ **f** $7(4n - 10) = 11$

**g** $7(2m - 11) = 30$ **h** $14(7n + 1) = 100$ **i** $11(5m + 2) = 30$

Example 10

**4** In each part below, write an equation and solve it.

**a** A number $x$ has 4 added to it and the result is multiplied by 3. The final result of this is 32.

**b** A number $x$ has 2 subtracted from it and the result is multiplied by 5. The final result of this is 42.

**c** A number $x$ has 3 subtracted from it and the result is multiplied by 2. The final result of this is 15.

**d** A number $z$ has 5 added to it and the result is multiplied by 6. The final result of this is 42.

**e** A number $n$ has 11 added to it and the result is multiplied by 12. The final result of this is 150.

**f** A number $z$ has 8 subtracted from it and the result is multiplied by 6. The final result of this is 100.

**5** A farmer has an unknown $x$ number of lambs in a paddock. When 4 more lambs join the paddock, he counts 100 legs of lamb. Find the value of $x$.


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# 16F Collecting like terms and solving equations

If Tim has 3 pencil cases with the same number, $x$, of pencils in each, he has $3x$ pencils in total.



If Sarah then gives him 2 more pencil cases with $x$ pencils in each, then he has $3x + 2x = 5x$ pencils in total. This is because the number of pencils in each case is the same.

## Like terms

The terms $2x$ and $3x$ in the above example are called **like terms** and they have been collected together. (In contrast, the terms $2x$ and $3y$ are not like terms, because the pronumerals are different.) Like terms have been discussed in Chapter 3.

## Adding and subtracting like terms

Like terms can be added and subtracted, as shown in the examples below.

### Example 11

Simplify each expression by adding or subtracting like terms.

**a** $2x + 3x + 5x$ **b** $3x + 2y + 5x + 7y$

**c** $3x + 4x + 7y - 3y$ **d** $2x + 1 + 3y - 3 - x + 4y$

#### Solution

**a** $2x + 3x + 5x = 10x$ **b** $3x + 2y + 5x + 7y = 8x + 9y$

**c** $3x + 4x + 7y - 3y = 7x + 4y$ **d** $2x + 1 + 3y - 3 - x + 4y = x + 7y - 2$

### Example 12

Simplify each expression by first expanding the brackets and then adding or subtracting like terms.

**a** $2(x + 4) + 3x$ **b** $4(5x - 3) + 10$ **c** $2(3x - 4) + 3(4x - 7)$

#### Solution

**a** $2(x + 4) + 3x = 2x + 8 + 3x$
$= 5x + 8$

**b** $4(5x - 3) + 10 = 20x - 12 + 10$
$= 20x - 2$

**c** $2(3x - 4) + 3(4x - 7) = 6x - 8 + 12x - 21$
$= 18x - 29$


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## Example 13

Collect like terms and solve each equation for $x$.

**a** $5x + 3x + 4 = 36$ **b** $10x - 4x + 6 = 24$

### Solution

**a**

$$\begin{aligned} 5x + 3x + 4 &= 36 \\ 8x + 4 &= 36 \quad \text{(Collect like terms.)} \\ \boxed{-4} \quad 8x &= 32 \\ \boxed{\div 8} \quad x &= 4 \end{aligned}$$

**b**

$$\begin{aligned} 10x - 4x + 6 &= 24 \\ 6x + 6 &= 24 \quad \text{(Collect like terms.)} \\ \boxed{-6} \quad 6x &= 18 \\ \boxed{\div 6} \quad x &= 3 \end{aligned}$$

## Example 14

Expand the brackets, collect like terms and solve each equation for $z$.

**a** $2(3z + 4) + 5 = 20$ **b** $5z + 2(z - 4) = 20$

### Solution

**a**

$$\begin{aligned} 2(3z + 4) + 5 &= 20 \\ 6z + 8 + 5 &= 20 \quad \text{(Expand brackets.)} \\ 6z + 13 &= 20 \quad \text{(Collect like terms.)} \\ \boxed{-13} \quad 6z &= 7 \\ \boxed{\div 6} \quad z &= 1\tfrac{1}{6} \end{aligned}$$

**b**

$$\begin{aligned} 5z + 2(z - 4) &= 20 \\ 5z + 2z - 8 &= 20 \quad \text{(Expand brackets.)} \\ 7z - 8 &= 20 \quad \text{(Collect like terms.)} \\ \boxed{+8} \quad 7z &= 28 \\ \boxed{\div 7} \quad z &= 4 \end{aligned}$$

Checking your answer in the original equation is advised.


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## Exercise 16F

Example 11

**1** Simplify each expression by adding or subtracting like terms.

**a** $3x + 4x + 7x$ **b** $2x + 3x + 7x + 4x$ **c** $2x + 3x + 4x + 7x - 3x$

**d** $2x + 3x - 2x + 4x$ **e** $11x - 3x + 12x - 4x$ **f** $2 + 3x + 5 + 6x$

**g** $5x - 2x + 2 + 7x$ **h** $11x + 2y + 3x + 5y$ **i** $4x + 3x + 3y - y$

**j** $x + 2x + 5x + 7y - y$

Example 12

**2** Simplify each expression by first expanding brackets and then adding or subtracting like terms.

**a** $3(x + 2) + 4x$ **b** $6(4x - 3) + 12$ **c** $4(5x + 2) + 7(x + 2)$

**d** $5(x + 6) - 5x$ **e** $7(6x + 2) + 4x$ **f** $3(2x + 6) + 5(x - 6)$

**g** $4(2x + 3) + 5(x - 4)$ **h** $3(4x + 2) + 7(x + 4)$

Example 13

**3** Collect like terms and solve each equation for $x$.

**a** $3x + 2x + 6 = 36$ **b** $11x - 5x + 7 = 25$ **c** $6x + x - 2 = 35$

**d** $4x + 11x + 3 = 33$ **e** $2x - 7x + 12 = 36$ **f** $7x - 2 + 8x = 28$

**g** $6x + 9x - 4 = 41$ **h** $8x - 11x + 12x - 18 = 0$

Example 14

**4** Expand the brackets, collect like terms and solve each equation for $z$.

**a** $4(3z + 4) + 10 = 40$ **b** $4(z + 4) + 2z = 36$

**c** $5z + 2(z - 4) = 27$ **d** $3(5z + 1) + 1 = 25$

**5** Solve:

**a** $5m + 2(3m - 4) = 25$ **b** $6(3m - 4) + 10 = 12$

**c** $4x + 5(3x + 8) = 15$ **d** $3m + 5(2 + 4m) = 30$

**e** $6x + 4x + 2(x - 4) = 100$ **f** $5n + 15n + 3(n + 4) = 50$

**6** A pencil costs $\$x$. A pen costs \$1 more. Sam bought one pencil and one pen, paying \$1.10. How much did the pencil cost? Hint: Not 10 cents!


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# 16G Equations with pronumerals on both sides

When there are pronumerals on both sides of the equation, the first step is to get all terms involving the pronumeral on the same side. We can then solve the equation as before.

## Example 15

Solve each equation by collecting like terms.

**a** $2x + 3 = 5x + 1$ **b** $15x - 10 = 6x + 3$

## Solution

**a**

$$\begin{aligned} & 2x + 3 = 5x + 1 \\ \boxed{-2x} \quad & 2x + 3 - 2x = 5x + 1 - 2x \quad \text{(Subtract } 2x \text{ from both sides of the equation.)} \\ & 3 = 3x + 1 \quad \text{(Collect like terms.)} \\ \boxed{-1} \quad & 2 = 3x \\ \boxed{\div 3} \quad & x = \frac{2}{3} \end{aligned}$$

**b**

$$\begin{aligned} & 15x - 10 = 6x + 3 \\ \boxed{-6x} \quad & 15x - 10 - 6x = 6x + 3 - 6x \quad \text{(Subtract } 6x \text{ from both sides of the equation.)} \\ & 9x - 10 = 3 \quad \text{(Collect like terms.)} \\ \boxed{-10} \quad & 9x = 13 \\ \boxed{\div 9} \quad & x = 1\tfrac{4}{9} \end{aligned}$$

## Exercise 16G

Example 15

**1** Solve each equation for $x$. Check your answers for parts **c**, **f**, **i** and **l**.

**a** $2x + 6 = x + 9$ **b** $3x + 4 = x + 10$ **c** $5x + 6 = 9x + 3$

**d** $6x - 4 = 4x + 6$ **e** $2x + 10 = x + 12$ **f** $5x + 4 = x + 9$

**g** $2x + 6 = 11x - 3$ **h** $6x - 6 = 2x + 6$ **i** $2x - 8 = x + 12$

**2** Solve:

**a** $2m + 5 = m$ **b** $13z - 2 = 11z$ **c** $20n + 5 = 10n$

**d** $7m - 4 = 20m$ **e** $20z + 6 = 8z$ **f** $10m - 6 = 5m$

**3** Solve each equation by first collecting like terms.

**a** $50 + 10x = 150 + 4x$ **b** $60 - 2x + 8x = 4x + 100$

**c** $20x + 30x - 10x = 5x + 100$ **d** $60 = 7x + 45 - 4x$

**e** $5x - 12 = -6x + 23 + 4x$ **f** $9x - 125 + 16x = 8x - 100$


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# 16H Solving problems using equations

The algebra introduced so far can be used to solve problems that would otherwise be quite difficult. In each problem, we can use the following procedure:

*Step 1*: Use a pronumeral to represent an unknown.

*Step 2*: Construct an equation from the given information.

*Step 3*: Use the methods we have learned to solve the equation.

*Step 4*: Check the answer with the given information.

The three examples below show how to apply this procedure.

## Example 16

A number is multiplied by 11 and 6 is added to the result. The final result is 24. What is the number?

### Solution

Let $x$ be the number. The resulting equation is:

$$\begin{aligned} 11x + 6 &= 24 \\ \boxed{-6} \quad 11x &= 18 \\ \boxed{\div 11} \quad x &= 1\tfrac{7}{11} \end{aligned}$$

The number is $1\frac{7}{11}$.

*Check:*
$$\begin{aligned} \text{LHS} &= 11 \times 1\tfrac{7}{11} + 6 \\ &= 11 \times \frac{18}{11} + 6 \\ &= 18 + 6 \\ &= 24 \\ &= \text{RHS} \end{aligned}$$

## Example 17

The length of a rectangle is twice its width. The perimeter of the rectangle is 45 cm. Find the length and width of the rectangle.

### Solution

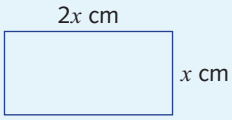
Let the width of the rectangle be $x$ cm.

The length of the rectangle is $2 \times x = 2x$ cm.

The perimeter is $2x + 2x + x + x = 6x$ cm.

But it is given that the perimeter is 45 cm,

so
$$\begin{aligned} 6x &= 45 \\ \boxed{\div 6} \quad x &= \frac{45}{6} \\ &= \frac{15}{2} \end{aligned}$$




*(continued over page)*


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The width of the rectangle is $\frac{15}{2} = 7\frac{1}{2}$ cm.

The length of the rectangle is $2 \times \frac{15}{2} = 15$ cm.

### Example 18

One number is 6 less than another number. The sum of the two numbers is $8\frac{1}{4}$. Find each of the numbers.

#### Solution

Let the larger number be $x$. Then the other number is $x - 6$.

Therefore $x + x - 6 = 8\frac{1}{4}$

$2x - 6 = 8\frac{1}{4}$ (Collect like terms.)

$+ 6$ $2x = 14\frac{1}{4}$

$\div 2$ $x = 7\frac{1}{8}$

The larger number is $7\frac{1}{8}$ and the smaller number is $7\frac{1}{8} - 6 = 1\frac{1}{8}$.

## Exercise 16H

For each question, introduce a pronumeral for the unknown and follow the steps given on page 422.

Example 16

**1** In each part, find the number.

**a** A number has 7 added to it and the result is 15.

**b** A number has 11 subtracted from it and the result is 23.

**c** A number is divided by 7 and the result is 14.

**d** A number is multiplied by 4 and the result is 48.

**e** A number is multiplied by 6, and 3 is added to the result. The final result of this is 39.

**f** A number is multiplied by 7, and 6 is subtracted from the result. The final result of this is 23.

**g** A number has 3 added to it and the result is multiplied by 4. The final result of this is 40.

**h** A number has 7 subtracted from it and the result is multiplied by 8. The final result of this is 32.


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Example 17

**2** A square has perimeter 84 cm. Find the length of each side of the square.

**3** The width of a rectangle is three times the length of the rectangle. The perimeter of the rectangle is 96 cm. Find the length and the width of the rectangle.

Example 18

**4** One number is three more than another number. The sum of the two numbers is 33. Find the numbers.

**5** The sum of two consecutive even numbers is 110. Find the numbers.

**6** The difference of two numbers is $\frac{3}{4}$. The sum of the two numbers is $9\frac{1}{4}$. Find the numbers.

**7** An equilateral triangle has perimeter 96 cm. Find the length of each side.

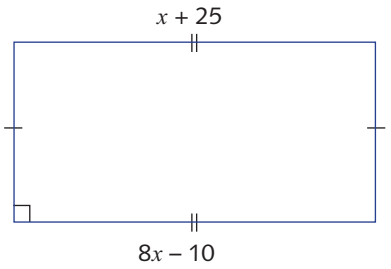
**8** A crop of 2181 bananas is packed into a number of cases. There are 27 cases, each holding the same number of bananas. There are also 21 loose bananas. How many bananas are there in each case?

**9** **a** Multiplying a number by 4 and subtracting 3 gives the same result as multiplying the number by 6 and subtracting 7. Find the number.

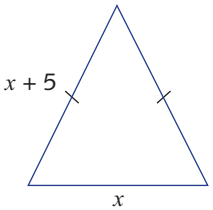
**b** Multiplying a number by 11 and subtracting 14 gives the same result as multiplying the number by 9 and adding 6. Find the number.

**c** Multiplying a number by 3 and subtracting 8 gives the same result as subtracting twice the number from 12. Find the number.

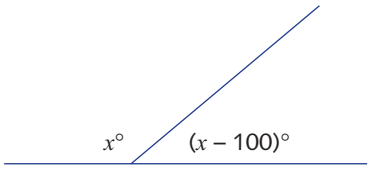
**10** Find $x$.




**11** The perimeter of the isosceles triangle is 20. Find $x$.




**12** Find $x$.






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# Review exercise

**1** Twelve recycling boxes each have $x$ old newspapers in them. Three current newspapers are loose on the table. There are 363 newspapers in total.

**a** Write an equation for $x$.

**b** Solve the equation for $x$.

**2** Solve:

**a** $x - 7 = 5$ **b** $5a = 27$ **c** $3z - 5 = 119$ **d** $11b + 22 = 121$

**e** $\frac{5x}{4} = 12$ **f** $\frac{x}{7} + 1 = 11$ **g** $n - 17 = -3$ **h** $2p = 6$

**i** $-4y = -18$ **j** $\frac{x}{6} = 18$ **k** $\frac{x}{10} = -2$ **l** $\frac{x}{4} - 11 = 29$

**m** $\frac{5x}{3} = 25$ **n** $9 - m = -5$ **o** $2(x + 6) = 23$ **p** $5(3x - 5) = 17$

**3** In each case below, write an equation and solve it to find the number.

**a** The number $x$ has 11 added to it and the result is 23.

**b** The number $x$ is multiplied by 7 and the result is 25.

**c** The number $x$ is divided by 7, and 3 is subtracted from it. The result is 6.

**4** A rectangle is four times as long as it is wide. The perimeter of the rectangle is 120 cm. Find the length and the width of the rectangle.

**5** The difference between two numbers is 12. The sum of the two numbers is 44. Find the two numbers.

**6** Solve:

**a** $x + 4 = -12$ **b** $x - 9 = -14$ **c** $3x = -12$

**d** $-8x = -24$ **e** $-9x = 9$ **f** $\frac{x}{4} = -7$

**g** $\frac{-3x}{2} = -8$ **h** $4x + 1 = -8$ **i** $-2x + 10 = -12$

**7** Solve each equation for $x$.

**a** $3x + 4 = 2x + 5$ **b** $4x + 7 = x + 13$ **c** $7x + 2 = 3x + 1$

**d** $5x - 2 = 8x + 1$ **e** $x + 14 = 4x + 10$ **f** $13x + 4 = 2x + 7$

**8** Solve:

**a** $5(2x - 4) + 3x = 10$ **b** $6x - 2x + 7x = 42$

**c** $50 + 2(4x - 5) = 100$ **d** $10(2x - 10) + 15 = 84$

In each of the following questions, first introduce a pronumeral and then solve the equation.

**9** I think of a number and divide it by 5. The answer is 4. What is the number?

**10** In 29 years' time a man will reach the retiring age of 65 years. How old is he now?


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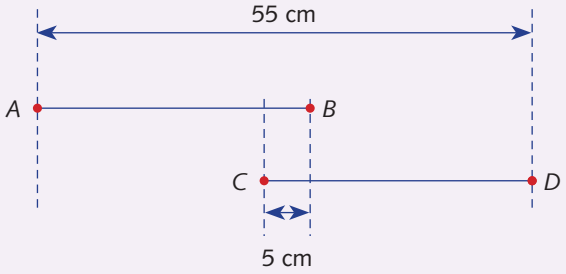
**11** I have read 163 pages of a book. How many pages are left to be read if the book contains 390 pages?

**12** A boy throws away one-third of the cakes he has cooked. If he has 34 left, how many did he cook in the first place?

**13** Multiplying a number by 6 and subtracting 6 gives the same result as multiplying the number by 3 and subtracting 4. Find the number.

**14** \$400 is to be shared among three friends. David gets \$8 more than Jennifer who gets \$16 more than Brett. How much does each of the friends receive?

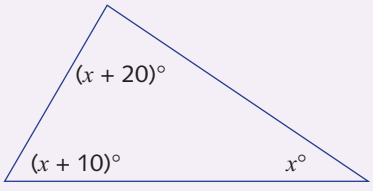
**15** The two rods $AB$ and $CD$ have the same length and overlap by 5 cm. Find the length of each rod.




**16** Three athletes are training for a race. In a particular week Jennifer ran 12 km less than Anne, and Carl ran half as far as Anne and the same distance as Jennifer. How far did each of the athletes run?

**17** Subtracting 10 from 6 times a number gives the same result as multiplying the number by 7 and adding 6. What is the number?

**18** Find the value of $x$.



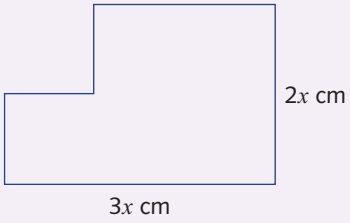

**19** Four hundred and fifty dollars is divided among three students, David, Isabel and Jason. David receives three times as much as Jason, and Isabel receives twice as much as Jason. How much does each student receive?

**20** Alice, Betty and Clair share \$28. Betty receives \$2 more than Alice and Clair receives three times as much as Betty. How much does Alice receive?


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# Challenge exercise

**1** **a** Find the perimeter of the figure shown below in terms of $x$. (All angles are right angles.)




**b** If the perimeter is 64 cm, find the value of $x$.

**2** Think of a number. Let this number be $x$.

**a** Write the following using algebra to see what you get.

- Add 6 to the number, then
- multiply the result by 10, then
- subtract the result from 20, then
- multiply this result by 2.

**b** If this final result is twice the original number, find the original number, $x$.

**c** If the number obtained in part **a** is 1000 less than the original number, find the original number $x$.

**3** Humphrey leaves Tantown at 2 p.m. and travels at 60 km/h along the Trans Highway. He passes through Sintown, which is 16 km from Tantown along the Trans Highway. Petro leaves Sintown at 3 p.m. and travels at 80 km/h along this highway in the same direction that Humphrey is travelling in. When and where does Petro catch up to Humphrey?

**4** Let $n$ be the age of a student (in years). The student's sister is 3 years older than the student, and the student's father is 25 years older than the student. The sum of the ages of the student and his sister is 11 less than the age of their father. Find the ages of the student, his sister and their father.

**5** David is 30 years older than his son Edward. In 6 years time he will be 4 times Edward's age. How old is Edward currently?

**6** Renae and James have \$60 between them. Renae gives James \$12. She now has twice as much as James. How much did she have previously?


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CHAPTER

# 17

Statistics and Probability

# Probability

In this chapter, we introduce probability, which deals with how likely it is that something will happen. This is an area of mathematics with many diverse applications.

The study of probability began in seventeenth-century France, when the two great French mathematicians Blaise Pascal (1623–1662) and Pierre de Fermat (1601–1665) corresponded about problems from games of chance. Problems such as these continued to influence the early development of the subject.

Nowadays probability is used in areas ranging from weather forecasting and insurance, where it is used to calculate risk factors and premiums, to predicting the risks and benefits of new medical treatments.


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# 17A An introduction to probability

Most people would agree with the following statements:

- It is certain that the sun will rise tomorrow.
- If I toss a coin, getting a head and getting a tail are equally likely.
- There is no chance of finding a plant that speaks English.




## Everyday language

Using everyday language to discuss probabilities can cause problems, because people do not always agree on the interpretation of words such as 'likely', 'probable' and 'certain'. Consider these two examples:

**1** Two farmers are discussing the prospects of getting a good wheat crop this year. Farmer Bill says, 'I don't think it is likely to rain for the next two weeks. I'm not going to plant wheat yet.' Farmer Tony says, 'I reckon you're wrong. I'm certain we'll have rain. It can't go on the way it has. I'm getting the tractor out tomorrow.'

**2** Alanna is captain of the Platypus Netball Team. They are going to play the Echidnas, whose captain is Maria. Each captain says to her team before the match, 'I think we'll probably win. Just follow the plans we've practised all week.' Alanna and Maria cannot both be right! It would be futile to try to assign a probability that the Echidnas (or the Platypuses) are going to win on the basis of what the captains told their teams.

On the other hand, there are many situations in which it would be useful to be able to measure how likely, or unlikely, it is that an event will occur. We can do this in mathematics by using the idea of **probability**, which we define as a number between 0 and 1 that we assign to any event we are interested in.

A probability of 1 represents an event that is 'certain' or 'guaranteed to happen'.

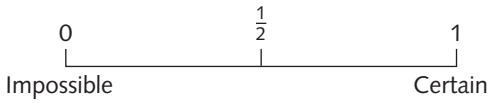
A probability of 0 represents an event that we would describe as 'impossible' or one that 'cannot possibly occur'.

An event that has a probability $\frac{1}{2}$ is as likely to occur as not to occur.


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An event that has a probability close to 0 is unlikely to occur.

An event that has a probability close to 1 is likely to occur.




Using these ideas, let us look at the three statements at the start of this section and express them in the new language of probability.

| Original statement | Statement in terms of probability |
|---|---|
| It is certain that the sun will rise tomorrow. | The probability that the sun will rise tomorrow is 1. |
| If I toss a coin, getting a head and getting a tail are equally likely. | If I toss a coin, the probability of getting a head is $\frac{1}{2}$ and the probability of getting a tail is $\frac{1}{2}$. |
| There is no chance of finding a plant that speaks English. | The probability of finding a plant that speaks English is 0. |

## Example 1

A TV game show contestant is shown three closed doors and told that there is a prize behind only one of the doors. If the contestant opens one of the doors, what is his probability of winning the prize?




### Solution

From the point of view of the quiz contestant the prize is equally likely to be behind each of the doors. So the probability of the contestant winning the prize is $\frac{1}{3}$.


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## Exercise 17A

Example 1

**1** Complete each of the following probability statements.

**a** It is certain that an iceblock will melt in the sun in Sydney.
The probability that an iceblock will melt in the sun in Sydney is _____

**b** There is no chance of finding a dog that speaks German.
The probability of finding a dog that speaks German is _____

**c** If there are three red discs and three blue discs in a bag and you take one out without looking in the bag, you are equally likely to get a red disc or a blue disc.
If there are three red discs and three blue discs in a bag and you take one out without looking in the bag, the probability of getting a red disc is _____ and the probability of getting a blue disc is _____

**d** If it is Thursday today, tomorrow will be Friday.
If it is Thursday today, the probability that it will be Friday tomorrow is _____

**e** If today is the 31st of January, there is no chance that tomorrow will be the 1st of May. If today is the 31st of January, the probability that tomorrow is the 1st of May is _____

**2** Make up some statements for which there is a corresponding probability statement involving the probabilities 0, 1 or $\frac{1}{2}$.

# 17B Experiments and counting

A standard die is a cube with each face marked with a number from 1 to 6 or with marks as shown in the diagram.



Suppose we roll a standard die. Since the outcomes 1, 2, ..., 6, are equally likely (assuming that the die is fair; that is, it is not 'loaded'), the probability of getting a 1 is $\frac{1}{6}$, the probability of getting a 2 is $\frac{1}{6}$, and so on.

### The total probability is 1

*Question*: If we roll a die, what is the chance of getting 1 or 2 or 3 or 4 or 5 or 6?

*Answer*: It is certain that we will get one of the numbers 1, 2, 3, 4, 5, 6, so we say that the **total probability** is 1.


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## Sample space

Rolling a die and recording the face that shows up is an example of doing an **experiment**. The numbers 1, 2, ..., 6, are called the **outcomes** of this experiment. The complete set of possible outcomes for any experiment is called the **sample space** of that experiment. For example, we can write down the sample space for the experiment of rolling a die as:

1 2 3 4 5 6

In this case, each outcome is equally likely and has probability $\frac{1}{6}$.

## Events

An **event** is something that happens. In everyday life, we speak of sporting events, or we may say that the school concert was a memorable event. We use the word 'event' in probability in a similar way.

For example, suppose that we roll a die and we are interested in getting a prime number. In this case 'the number is prime' is the event that interests us. Some of the outcomes will give rise to this event. For instance, if the outcome is 2, then the event 'the number is prime' has occurred. We say that the outcome 2 is **favourable to the event** 'the number is prime'. If the outcome is 4, then the event 'the number is prime' does not occur. The outcome 4 is **not favourable to the event**.

Of the six outcomes, three – 2, 3 and 5 – are favourable to the event 'the number is prime'. Three outcomes – 1, 4 and 6 – are not favourable to the event 'the number is prime'.

In many situations 'success' means favourable to the event and 'failure' means not favourable to the event.

### Example 2

Hassan rolls a die. Hassan is hoping for a perfect square number.

**a** What is the sample space?

**b** What are the outcomes that are favourable to the event 'the result is a perfect square'?

#### Solution

**a** The sample space is:

1 2 3 4 5 6

**b** The only perfect squares in the sample space are 1 and 4, so the outcomes favourable to the event are:

1 4

## Probability of an event

Suppose I roll a die. A natural question to ask is: What is the probability that I get a 1 or a 6?

To make sense of this, we have to decide how the question is to be interpreted.

There are six possible outcomes:

1 2 3 4 5 6


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All are equally likely. Of these, only two are favourable to the event 'the result is 1 or 6'. They are:

1 6

We say that the probability of getting a 1 or a 6 is $\frac{2}{6} = \frac{1}{3}$ because all of the outcomes 1, 2, 3, 4, 5 and 6 are equally likely. In general, the situation is as follows.

### Probability of an event

For an experiment in which all of the outcomes are equally likely:

$$\text{Probability of an event} = \frac{\text{number of outcomes favourable to that event}}{\text{total number of outcomes}}$$

Here is a good way of setting out the solution to a question about the probability of an event.

### Example 3

I roll a die. What is the probability that the result is a perfect square?

#### Solution

The sample space of all possible outcomes is:

1 2 3 4 5 6

The outcomes that are favourable to the event 'the result is a perfect square' can be circled, as shown.

(1) 2 3 (4) 5 6

$$\text{Probability that the result is a perfect square} = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$
$$= \frac{2}{6}$$
$$= \frac{1}{3}$$

## Notation

From now on, we will often write '$P$' instead of 'probability of'. Thus we write the previous result as:

$$P(\text{perfect square}) = \frac{1}{3}$$


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**Example 4**

I roll a die. What is the probability of getting a 2 or a 3 or a 4 or a 5?

**Solution**

The sample space, with the favourable outcomes circled, is:

1 (2) (3) (4) (5) 6

$$P(2 \text{ or } 3 \text{ or } 4 \text{ or } 5) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$
$$= \frac{4}{6}$$
$$= \frac{2}{3}$$

## Class discussion

$P(2 \text{ or } 3 \text{ or } 4 \text{ or } 5) = 1 - P(1 \text{ or } 6)$. Why is this so?

## The word 'not'

Sometimes we need to recognise the word 'not' hidden in the question. The next example gives a simple illustration of what we mean by this.

**Example 5**

Joe is playing a board game. If he tosses a 1 with the die, he goes to jail. What is the probability that he does not go to jail?

**Solution**

This is the same as the probability of *not* getting a 1, which means that Joe gets a 2, 3, 4, 5 or 6.

So $P(\text{not a } 1) = P(2, 3, 4, 5 \text{ or } 6)$
$$= \frac{5}{6}$$

Alternatively, we can consider the probability of getting anything except a 1.

Then $P(\text{not a } 1) = 1 - P(1)$
$$= 1 - \frac{1}{6}$$
$$= \frac{5}{6}, \text{ as before.}$$


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## The words 'random' and 'randomly'

In probability, we frequently hear expressions such as 'chosen randomly' or 'chosen at random', as in the following situations:

- A classroom contains 23 students. A teacher comes into the room and chooses a student at random to answer a question about history.

  What does this mean? It means that the teacher chose the student as if she knew nothing at all about the students. Another way of interpreting this is to imagine that the teacher had her eyes closed and had no idea who was in the class when she chose a student.

- In a TV quiz show, a prize wheel is spun. The contestant is asked to guess which number the wheel will stop at.

  The results of the spins are random and the contestant has no idea where the wheel will stop.

### Example 6

In a pick-a-box show, there are five closed boxes, each containing one snooker ball. Two of the boxes each contain yellow balls and the others contain a green ball, a black ball and a red ball, respectively. The contestant will win a prize if she chooses a yellow ball. She chooses a box at random and opens it. What is the probability that she wins a prize?

#### Solution

Here is the sample space with the favourable outcomes circled. They are equally likely, because she chooses at random.

(Y) (Y) G B R

$P(\text{prize}) = \frac{2}{5}$

### Example 7

A school has 1200 students. The table below gives information about whether or not each student plays a musical instrument.

| | Boys | Girls |
|---|---|---|
| **Plays a musical instrument** | 325 | 450 |
| **Does not play an instrument** | 225 | 200 |

Each student has a school number between 1 and 1200.

Each student's number is written on a card and the 1200 cards are placed in a hat. One card is chosen randomly from the hat. What is the probability that the number pulled out belongs to:

**a** a boy?

**b** a girl?

**c** a student who does not play an instrument?

**d** a boy who plays a musical instrument?


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**Solution**

**a** Number of student = 1200

Number of boys = 325 + 225

= 550

$P(\text{boy's number}) = \frac{550}{1200}$

$= \frac{11}{24}$

**b** Number of students = 1200

Number of girls = 450 + 200

= 650

$P(\text{girl's number}) = \frac{650}{1200}$

$= \frac{13}{24}$

**c** Number of students = 1200

Number who do not play a musical instrument = 225 + 200

= 425

$P(\text{student does not play a musical instrument}) = \frac{425}{1200}$

$= \frac{17}{48}$

**d** Number of students = 1200

Number of boys who play a musical instrument = 325

$P(\text{boy who plays a musical instrument}) = \frac{325}{1200}$

$= \frac{13}{48}$

## Exercise 17B

Example 2

**1** A bag contains 10 marbles numbered 1 to 10. A marble is taken out.

**a** Give the sample space for this experiment.

**b** Write down the outcomes favourable to the event 'a marble with an odd number is taken out'.

**2** A die is rolled. Write down the outcomes favourable to each given event.

**a** An even number is rolled.

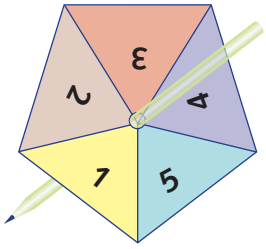
**b** An odd number is rolled.

**c** A number divisible by 3 is rolled.

**d** A number greater than 1 is rolled.

**3** **a** For the spinner shown opposite, write down the sample space of the experiment 'spinning the spinner'.

**b** Write down the outcomes favourable to the event 'the number obtained is less than 4'.





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Example 3, 4

**4** A die is rolled. Work out the probability that:

**a** an even number is rolled

**b** an odd number is rolled

**c** a number divisible by 3 is rolled

**d** a number greater than 1 is rolled

**5** A bag contains 10 marbles numbered 1 to 10. If a marble is taken out at random, what is the probability of getting:

**a** a 3?

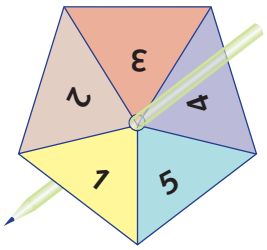
**b** an even number?

**c** a number greater than 2?

**d** a number divisible by 3?

**e** a number greater than 1?

**6** The spinner shown opposite is spun. Write down the probability of:




**a** obtaining a 4

**b** obtaining an odd number

**7** The letter tiles for the word MELBOURNE are placed in a box.




One piece is withdrawn at random. What is the probability of obtaining:

**a** an M? **b** an E? **c** a vowel? **d** a consonant?

Example 5

**8** A die is rolled. What is the probability of:

**a** obtaining a 3?

**b** not obtaining a 3?

**c** obtaining a number divisible by 3?

**d** obtaining a number not divisible by 3?

**9** A bag contains three red marbles numbered 1 to 3, five green marbles numbered 4 to 8, and two yellow marbles numbered 9 and 10. A single marble is withdrawn at random. Find the probability that:

**a** the marble is numbered 3

**b** the marble is green

**c** the marble is yellow

**d** the number is odd

**e** the number is greater than 6

**f** the number is green and even

**10** The letters of the alphabet are written on flashcards and put into a bag. A card is drawn out at random. Find the probability that the letter is:

**a** D

**b** not W, X, Y or Z

**c** a vowel

**d** in the word 'PERTH'

**e** in the word 'CANBERRA'


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Example 6

**11** There are three green apples and two red apples in a bowl. Georgia is blindfolded and randomly chooses one of the apples. What is the probability that she chooses a green apple?

**12** A bag of sweets contains 10 red ones, nine green ones, six yellow ones and five blue ones. They are all the same size and shape. You reach in and take one out at random. Find the probability that the sweet you pick is:

**a** blue **b** green or yellow **c** not red

**d** purple **e** not yellow or blue

**13** Cards with numbers 1–100 written on them are placed in a box. One card is randomly chosen. What is the probability of obtaining a number divisible by 5?

Example 7

**14** A bowl contains green and red normal jelly beans and green and red double-flavoured jelly beans. The numbers of the different colours and types are given in the table below.

| | Green | Red |
|---|---|---|
| **Normal jelly beans** | 250 | 400 |
| **Double-flavoured jelly beans** | 175 | 75 |

A jelly bean is randomly taken out of the bowl. Find the probability that:

**a** it is a double-flavoured jelly bean **b** it is a green jelly bean

**c** it is a green normal jelly bean **d** it is a red double-flavoured jelly bean

**e** it is a red jelly bean

**15** In a traffic survey taken during a 30-minute period, the number of people in each passing car was noted, and the results tabulated as follows.

| **Number of people in car** | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| **Number of cars** | 60 | 50 | 40 | 10 | 5 |

What is the probability that:

**a** there was only one person in a car during this period?

**b** there was more than one person in a car during this period?

**c** there were fewer than four people in a car during this period?

**d** there were five people in a car during this period?

**16** A door prize is to be awarded randomly at the end of a concert. The numbers of people at the concert in different age groups are given in the table below.

| **Age group** | 0–5 | 6–11 | 12–18 | 19–30 | 30–40 | 40+ |
|---|---|---|---|---|---|---|
| **Number of people in age group** | 10 | 150 | 350 | 420 | 125 | 85 |

What is the probability of the prize winner being:

**a** in the 0–5 age group? **b** in the 19–30 age group?

**c** aged 40 or less? **d** aged between 12 and 30?

**e** older than 5? **f** older than 40?


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# Review exercise

**1** There are eight marbles, numbered 1 to 8, in a bowl. A marble is randomly taken out. What is the probability of getting:

**a** an 8? **b** an even number? **c** a number less than 6? **d** a number divisible by 3?

**2** Twelve cards are put into a hat. Five of the cards are red and are numbered 1 to 5, and the other seven cards are blue and are numbered 6 to 12. A card is taken randomly from the hat. Find the probability of taking out:

**a** a blue card

**b** a card with an even number

**c** a card with a prime number

**d** a red card

**e** a card with a number greater than 13

**3** The letter tile pieces for the letters of the word 'PROBABILITY' are put into a box. One piece is chosen at random. What is the probability of choosing:

**a** the P? **b** a B? **c** a vowel? **d** a consonant?

**4** Seven red marbles numbered 1 to 7 and six blue marbles numbered 8 to 13 are placed in a box. A marble is taken randomly from the box. Find the probability of taking out:

**a** a red marble

**b** a blue marble

**c** a marble with a prime number

**d** a marble with a multiple of 3

**e** a red marble with an even number

**f** a blue marble with an odd number

**g** a red marble with a number less than 6 on it

**5** A large bowl contains two types of lollies: chocolates and toffees. The chocolates and toffees are manufactured by the Yummy Sweet Company and the ACE Confectionary Company. The number of chocolates and toffees in the bowl of the two different brands are given in the table below.

| | Yummy Sweet Company | ACE Confectionary Company |
|---|---|---|
| **Chocolates** | 250 | 170 |
| **Toffees** | 300 | 140 |

A lolly is randomly taken out of the bowl. Find the probability that:

**a** it is a chocolate manufactured by the Yummy Sweet Company

**b** it is a toffee manufactured by the ACE Confectionary Company

**c** it is a chocolate

**d** it is a toffee

**6** The 11 letter tiles for the letters of the word 'PROSPECTIVE' are put into a box.

One piece is chosen at random. What is the probability of choosing:

**a** the I? **b** a P? **c** an E? **d** a vowel?


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**7** Each number from 1 to 500 is written on a card and put into a box. A card is withdrawn at random. What is the probability of obtaining a card with:

**a** a number less than 100? **b** a number divisible by 10? **c** a number divisible by 5?



# Challenge exercise

**1** In a raffle, 50 tickets are sold. The tickets are numbered from 1 to 50. They are placed in a hat and one is drawn out at random to win a prize. Find the probability that the number on the winning ticket:

**a** is 43

**b** is not 28

**c** is even

**d** is less than or equal to 20

**e** is an even number less than 20

**f** contains the digit 7

**g** does not contain the digit 9

**h** contains the digit 2 at least once

**i** contains the digit 2 only once

**j** does not contain the digit 2

**2** Suppose that the raffle tickets in Question **1** are coloured: those numbered 1 to 25 are green, the tickets numbered 26 to 40 are yellow and the rest are blue. Find the probability that the ticket that wins the prize:

**a** is blue

**b** is not green

**c** is green or blue

**d** is both green and blue

**e** is even-numbered and blue

**f** is prime and yellow

**3** Thomas, Leslie, Anthony, Tracie and Kim play a game in which there are three equal prizes. No player can win more than one prize.

**a** List the 10 ways the prizes can be allocated.

**b** What is the probability that Leslie wins a prize?

**c** What is the probability that Leslie does not win a prize?

**4** Find the probability that a randomly chosen three-digit number is divisible by:

**a** three

**b** seven

**5** A bag initially contains 15 red balls and 8 blue balls. Some of the red balls are removed. A ball is then drawn out at random. The probability that it is a red ball is $\frac{1}{3}$. How many red balls were removed?

**6** Describe how you can use a coin to decide between three menu items with equal probability.

**7** The six faces of a die are labelled with the numbers $-3$, $-2$, $-1$, 0, 1, and 2. The die is rolled twice. What is the probability that the product of the two numbers is negative (less than zero)?


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CHAPTER

# 18

Measurement and Geometry

# Transformations and symmetry

Consider the triangle $T$ drawn below. Each of the triangles $T_1$, $T_2$, $T_3$ is obtained from the triangle $T$ by moving it in different ways.

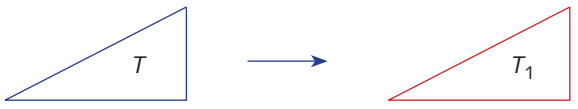



**Diagram 1**

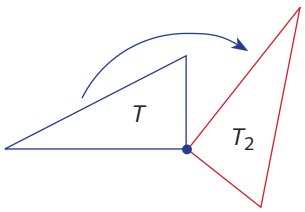



**Diagram 2**

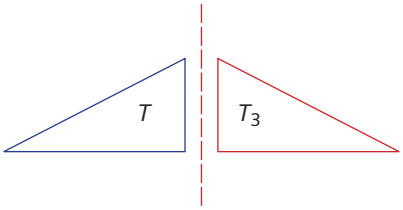



**Diagram 3**

When the figure is moved, we call this movement a **transformation**. The resulting figure is called the **image** of the original figure.

*continued over page*



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We will be dealing with only three types of transformation in this chapter:

1 translations (diagram 1)

2 rotations (diagram 2)

3 reflections (diagram 3).

Note that none of these transformations changes the size or shape of an object.

You will have seen these three transformations in many applications on your computer.

All of these transformations satisfy two important properties.

**Properties of translations, rotations and reflections**

When a figure is translated, rotated or reflected:

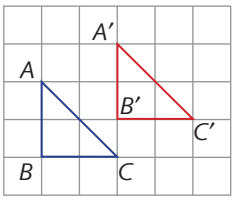
- intervals move to intervals of the same length
- angles move to angles of the same size.

Many of these exercises will require grid paper. The work on rotations will require some work on polar graph paper, also called circular graph paper. You can download samples of this from your Interactive Textbook. Carefully drawn diagrams are essential for understanding.

# 18A Translations

Shifting a figure in the plane *without turning it* is called a **translation**. To describe a translation, it is enough to say how far left or right, and how far up or down, the figure is moved.

In the diagram opposite, triangle $ABC$ has been translated to a new position on the page, and the new triangle has been labelled $A'B'C'$.




You can see that triangle $ABC$ has been shifted 2 units right and 1 unit up. Point $A$ has moved to $A'$, point $B$ to $B'$ and point $C$ to $C'$.

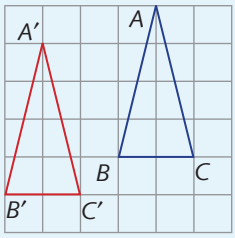
The image of a point is usually written with a dash attached. Thus the image of $A$ is written $A'$ (read as '$A$ prime' or '$A$ dash'). The image of $\Delta ABC$ is thus $\Delta A'B'C'$.

You can see that our translation has changed neither the side lengths nor the angles of the triangle, in agreement with the properties listed above.


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## Example 1

Describe how the triangle in the diagram opposite has been translated.



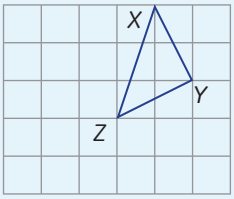

### Solution

The figure has been translated 3 units left and 1 unit down. (Note that 1 unit down followed by 3 units left is the same translation.)

Triangle $ABC$ is shown by a blue line while the image triangle $A'B'C'$ is shown with a red line, a convention we will stay with throughout the chapter.

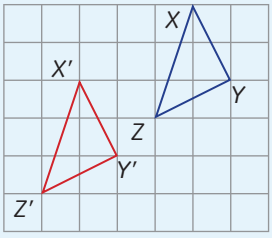
## Example 2

Translate $\Delta XYZ$ in the diagram opposite 2 units down and 3 units left.




### Solution

We only need to shift the vertices of this triangle. We then join them up to give the translated figure and label it $X'Y'Z'$.




Notice that in each of the examples above, *every* point of the triangle has moved. This is a special property of all translations, apart from the identity translation.

The translation that translates a figure 0 units right and 0 units up is called the **identity translation**. The identity translation leaves all points fixed.


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## Properties of translations

When a non-identity translation is applied:

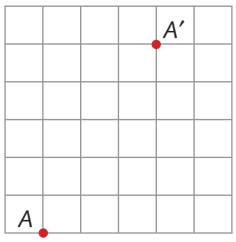
- every interval is translated to an interval of equal length
- every angle is translated to an angle of equal size
- every point moves; that is, there are no fixed points.

# Exercise 18A

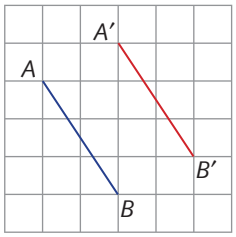
Example 1

**1** Describe the translation shown in each diagram below. You need not copy the diagrams.

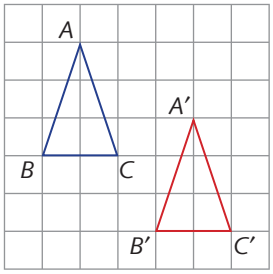
**a**



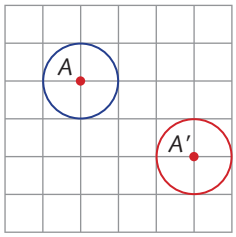

**b**



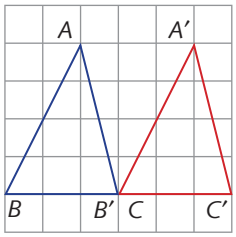

**c**



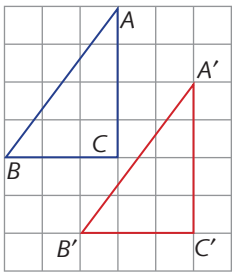

**d**




**e**



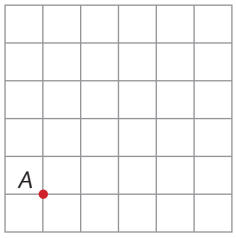

**f**



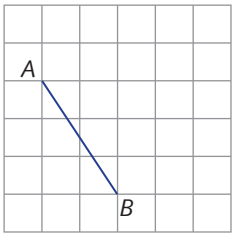

Example 2

**2** In each part below, a figure is shown and a translation described. Copy each figure and draw its image after the translation. Remember to label your images using dashes.

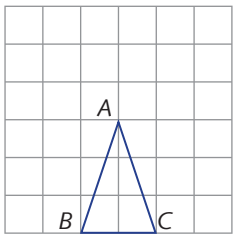
**a** 4 units up, 3 units right



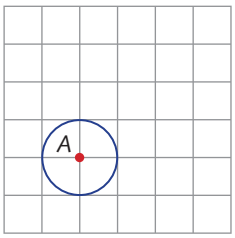

**b** 3 units right, 1 unit up




**c** 2 units up, 1 unit left



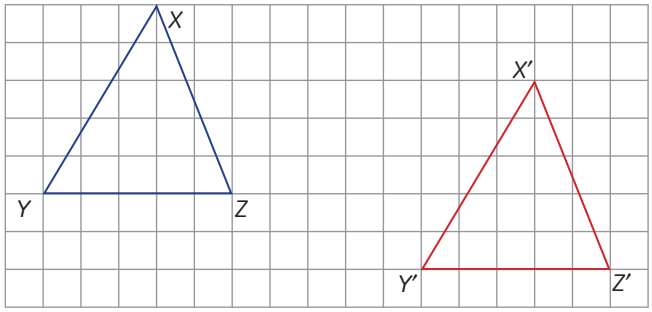

**d** 2 units right, 1 unit down






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**3** In the diagram below, $\Delta X'Y'Z'$ is the image under translation of $\Delta XYZ$. Copy the diagram onto your graph paper.




**a** Measure angles $XYZ$ and $X'Y'Z'$ with your protractor. What do you notice?

**b** Write down the angle in the image that is the same size as:

**i** $\angle XZY$

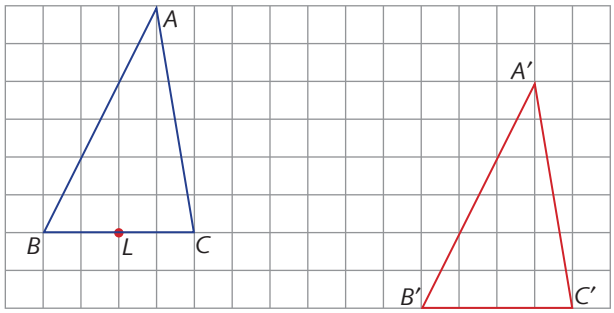
**ii** $\angle ZXY$

(Check with a protractor.)

**c** Accurately measure the lengths of the intervals $XY$ and $X'Y'$. Comment.

**d** Calculate the area, in square units, of $\Delta XYZ$ and $\Delta X'Y'Z'$. Comment.

**4** In the diagram below, $\Delta A'B'C'$ is the image under a translation of $\Delta ABC$. Copy the diagram onto your graph paper.




**a** Mark the image of $L$ under the same translation; label it $L'$.

**b** Describe the translation in words.

**c** Are the intervals $AB$ and $A'B'$ parallel?

**d** Write down the interval in the image that is parallel to:

**i** $AC$

**ii** $AL$

**e** Write down the interval in the image that is parallel to and the same length as:

**i** $BL$

**ii** $CA$

**f** Are there any points in the original figure that have not moved under the translation?


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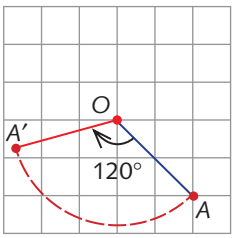
# 18B Rotations

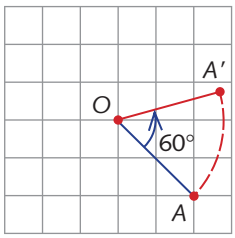
A **rotation** turns a figure about a fixed point, called the **centre of rotation**. The centre of rotation is usually labelled by the letter $O$.

A rotation is specified by:

- the centre of rotation $O$
- the angle of rotation
- the direction of rotation (clockwise or anticlockwise).

In the first diagram below, the point $A$ is rotated through 120° clockwise about $O$. In the second diagram, it is rotated through 60° anticlockwise about $O$.




The examples above demonstrate the following special properties of any rotation.

The **identity rotation** is the rotation that rotates a figure through 0° and leaves *every* point fixed.

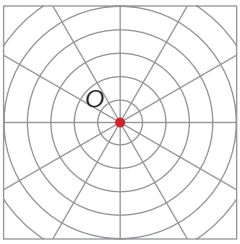
### Properties of rotations

For any non-identity rotation:

- every interval is rotated to an interval of equal length
- every angle is rotated to an angle of equal size
- there is only one fixed point – the **centre of rotation**
- the distance of a point from the centre of rotation does not change.

To help us when working with rotations, we may use special graph paper called **polar** or **circular graph paper**. This is available in your Interactive Textbook. We will use two types of such paper, examples of which can be seen in the next two diagrams. The first is marked with radial lines spaced at 30° apart, while in the second the lines are spaced at 45°. The centre of rotation $O$ is always the centre of the graph paper and is called the origin.

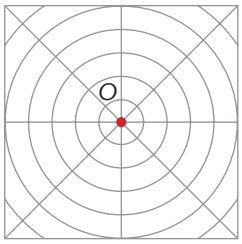
In this graph paper, lines radiate out from $O$ at 30° angles. The diagram is a bit like a clock face, with the lines pointing towards the hours.





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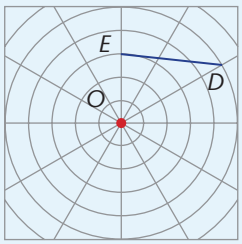
The second diagram is like a compass, with the lines 45° apart pointing to N, NE, E, SE, S, SW, W and NW.




The next example shows how the image of an interval under a rotation may be found by rotating its endpoints and joining their images.

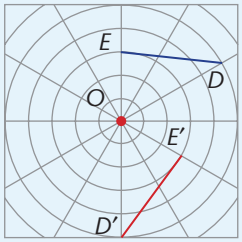
## Example 3

Rotate the interval $ED$ in the diagram by 120° in a clockwise direction about $O$.



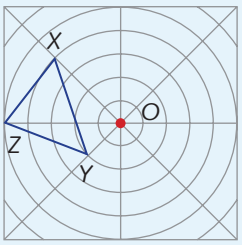

### Solution

To rotate the points $D$ and $E$ by $4 \times 30° = 120°$ clockwise, shift each point four 'hours' around the 'clock'. Each point stays the same distance from the centre of rotation; that is, it stays on the same circle. The points $D$ and $E$ are rotated and then joined up to form the image internal $D'E'$.



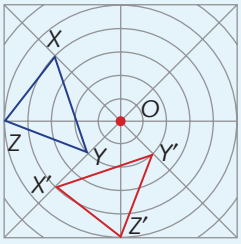

## Example 4

Rotate $\Delta XYZ$ in the diagram by 90° anticlockwise about $O$.




### Solution

A rotation of 90° anticlockwise moves every point though two divisions on the 'compass'. To move the triangle $XYZ$, we rotate the three corners and join them up to form the image triangle $X'Y'Z'$.



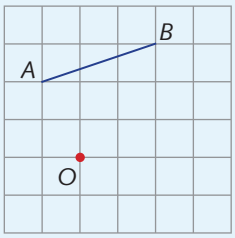

You can see that lengths of intervals do not change.


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In the following example, we will be concerned with rotating figures 90°, 180° or 270°, so a set square is ideal to construct these angles.

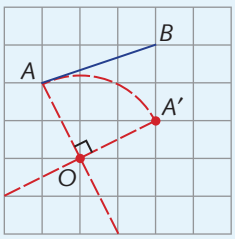
## Example 5

Rotate the line $AB$ by 90° clockwise about $O$.

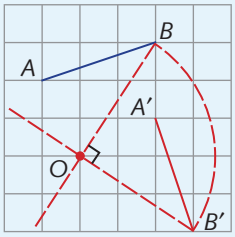



## Solution

We only need to rotate the endpoints $A$ and $B$. The image of the segment $AB$ is obtained by joining $A'$ and $B'$. This is explained in the following diagrams.




To rotate A we have drawn lines at 90°, starting with $OA$. A set square and a ruler can then help us rotate $A$ around $O$ to $A'$, keeping it always the same distance from $O$, since $A$ and $A'$ are an equal distance from $O$.




Repeat this process to rotate $B$ around $O$ to $B'$.


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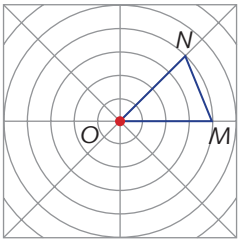
# Exercise 18B

Example 3, 4

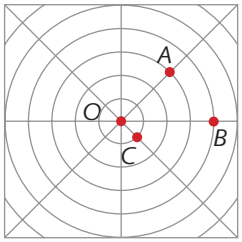
Use the appropriate type of polar graph paper in Question **1** and mark the points and figures carefully.

**1** Rotate each figure about the centre $O$ through the given angle. Remember to label your images with dashed letters.

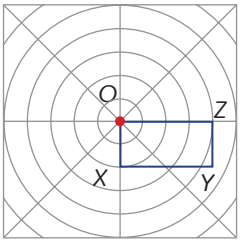
**a** Rotate 135° anticlockwise.



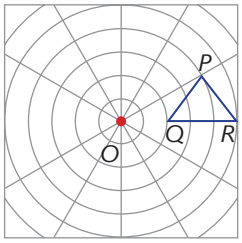

**b** Rotate 90° clockwise.



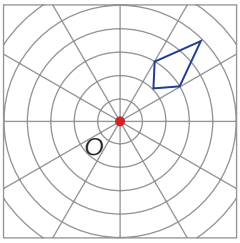

**c** Rotate 90° anticlockwise.



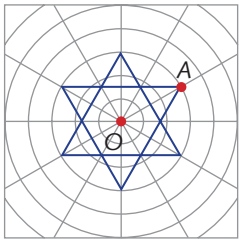

**d** Rotate 30° clockwise.




**e** Rotate 120° anticlockwise.



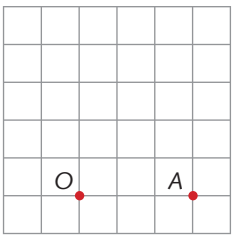

**f** Rotate 30° anticlockwise.



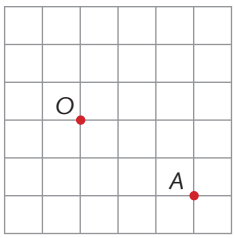

Example 5

**2** Rotate each figure through the given angle about the given centre $O$. Remember to label your images with dashed letters.

**a** Rotate 90° anticlockwise.



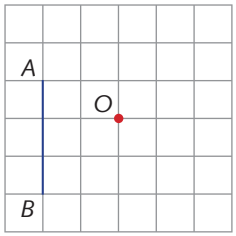

**b** Rotate 90° anticlockwise.



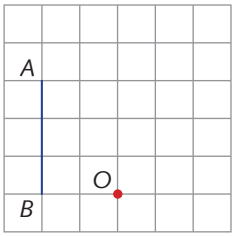


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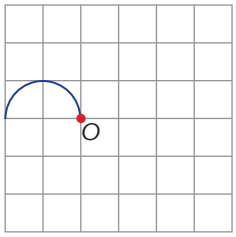
**c** Rotate 90° clockwise.



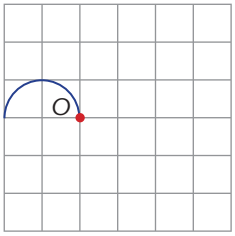

**d** Rotate 90° clockwise.



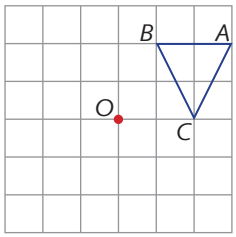

**e** Rotate 270° clockwise.



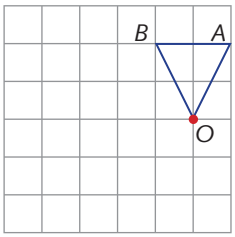

**f** Rotate 270° anticlockwise.



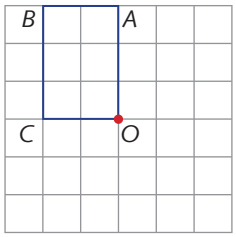

**g** Rotate 180° clockwise.



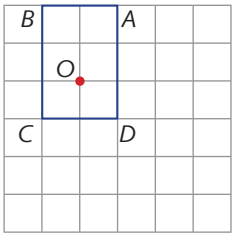

**h** Rotate 180° clockwise.



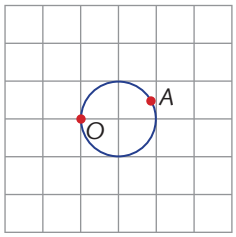

**i** Rotate 180° anticlockwise.



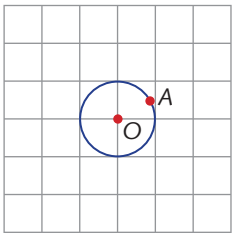

**j** Rotate 180° anticlockwise.




**k** Rotate 90° clockwise.

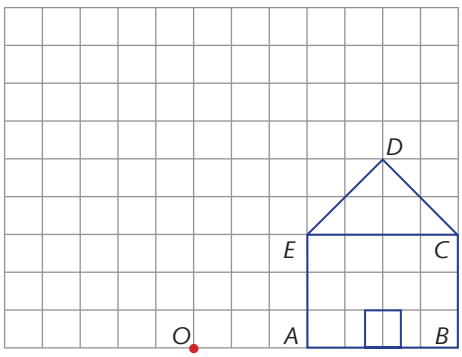



**l** Rotate 90° clockwise.




**3** Consider Question **2** above. Were any points fixed under the rotations, or did they all move?

**4** Consider the following diagram of a house.




**a** Draw the image of the house after a rotation of 90° anticlockwise about $O$. Label it $A'B'C'D'E'$.

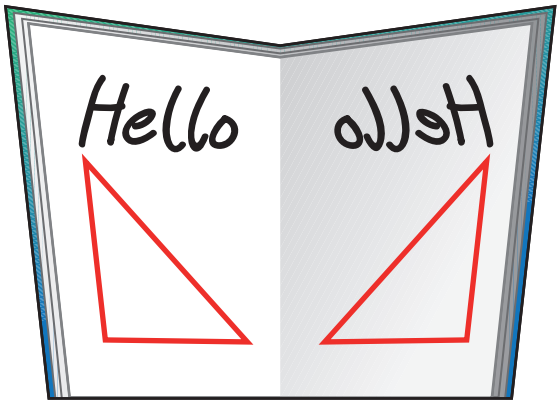
**b** $AE$ and $BC$ are parallel. What do you notice about $A'E'$ and $B'C'$?

**c** Find the area of $ABCE$ and $A'B'C'E'$ in square units. Comment.

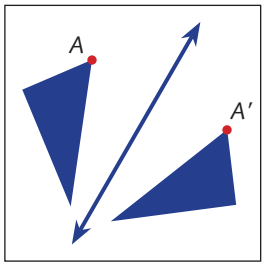

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# 18C Reflections

A reflection is a transformation that flips a figure about a line. This line is called the **axis of reflection**. A good way to understand this is to suppose that you have a book with clear plastic pages and a figure drawn, as in the picture below. If the page is turned, the triangle is flipped over. We say it has been reflected; in this case, the axis of reflection is the binding of the book.

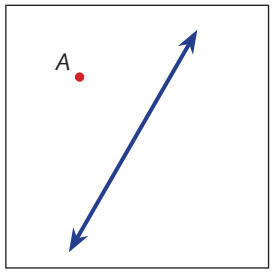


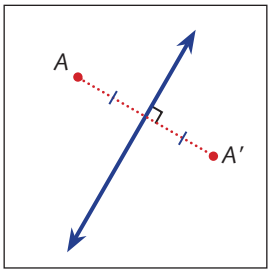

Our later examples will look more like the diagram opposite. The axis of reflection has also been drawn.




The image of each point can be determined in the following way.

To **reflect** $A$ in the axis of reflection, draw a line at right angles to the axis of reflection. Then use a ruler or compasses to mark $A'$ at the same distance as $A$, but on the other side. This is best done as a construction using a set square and compasses.

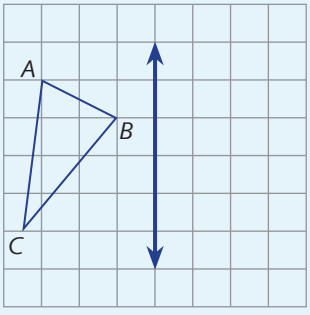



As was the case for rotation, we find the image of a triangle under a reflection by reflecting the vertices and joining the image vertices.

## Example 6

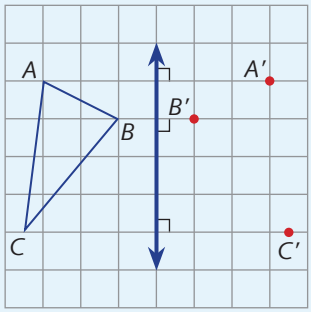
Reflect $\Delta ABC$ in the given axis of reflection shown.



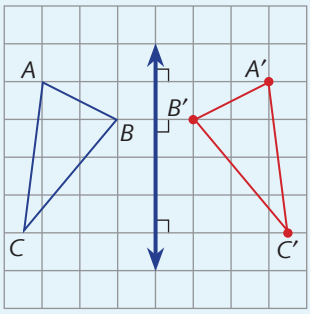


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**Solution**




Reflect each of the vertices.




Join them up to get the image triangle.

Here is a list of special properties of reflections. The second property can be seen in the previous example.

### Properties of reflections

When a reflection is applied:

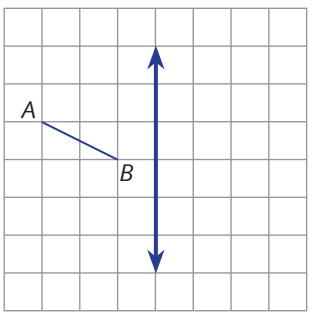
- all points on the axis of reflection are fixed points; the only fixed points are those on the axis of reflection
- if the points $A$, $B$, $C$, … are in a clockwise order, then the points $A'$, $B'$, $C'$, … will be in anticlockwise order, and vice versa
- if a reflection is applied twice, all figures are returned to their original position
- every interval is reflected to an interval of equal length
- every angle is reflected to an angle of equal size.

## Exercise 18C

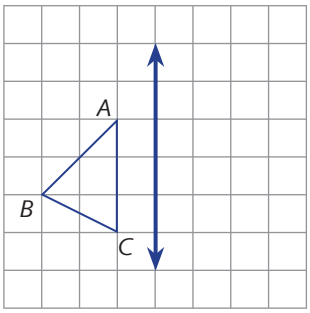
Example 6

**1** Reflect each figure in the given axis of reflection. (In each diagram, the axis of reflection is marked with an arrow at each end.) Remember to label your images in the proper way.

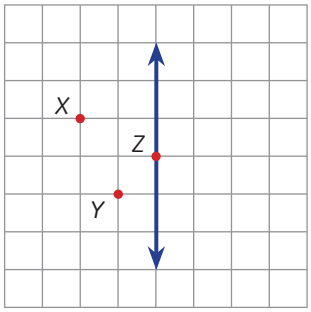
**a**




**b**



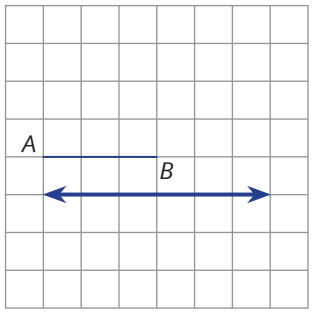

**c**



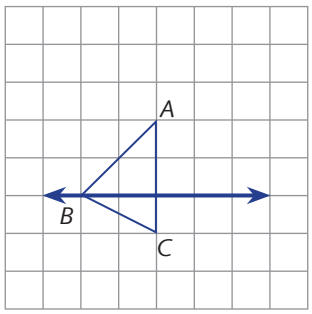



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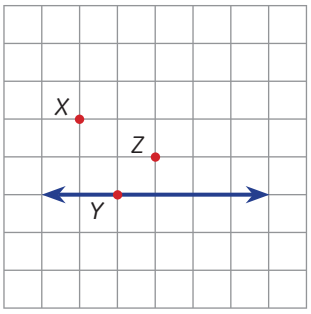
**d**



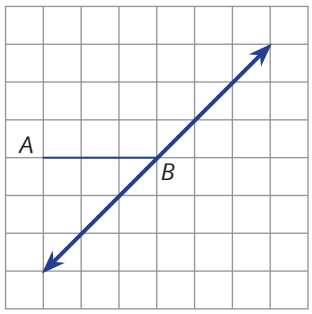

**e**



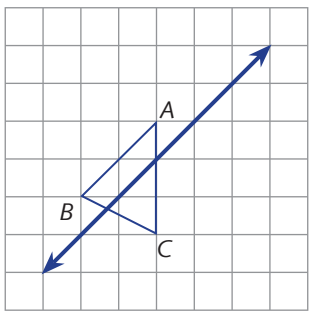

**f**



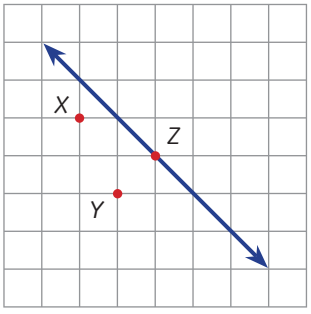

**g**



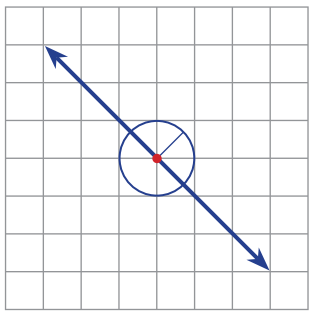

**h**



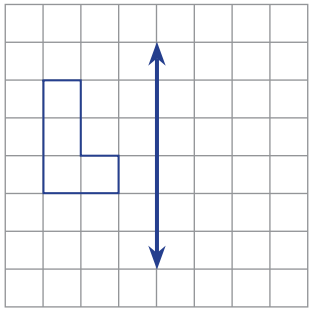

**i**



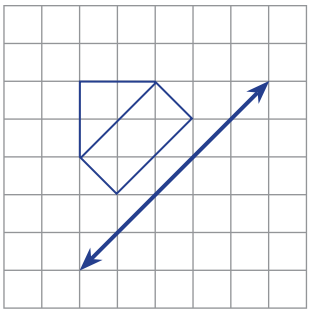

**j**



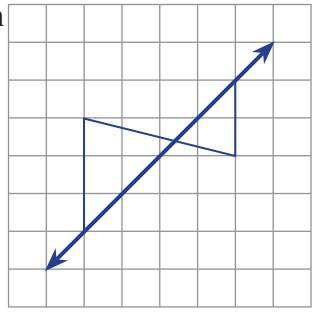
**k**



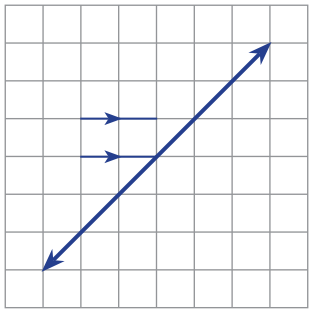
**l**



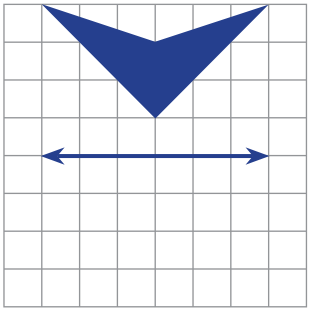
**m**



**n**

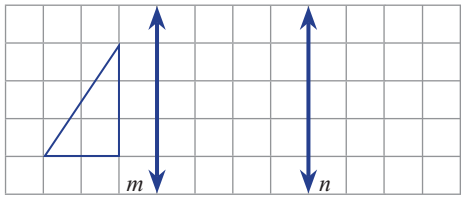


**o**

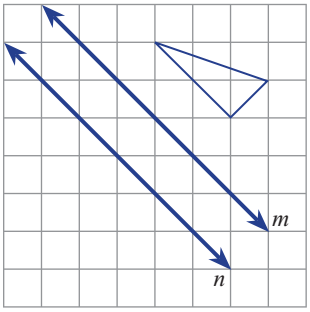


**2** This question involves *two* successive reflections. First reflect the triangle in the axis of reflection labelled $m$, and then in the axis of reflection labelled $n$.

**a**




**b**





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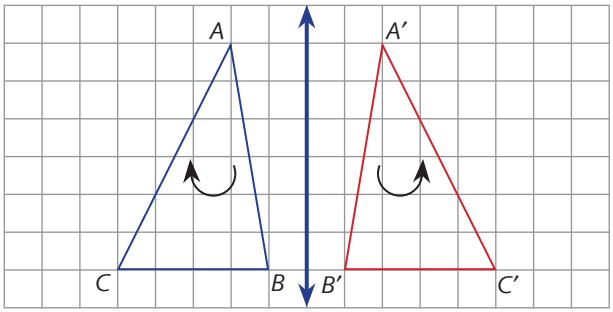
# 18D Combinations of transformations

Here is a summary of what we have discovered so far about translations, rotations and reflections:

- Intervals move to intervals of the same length.
- Angles move to angles of the same size.
- Pairs of parallel lines move to pairs of parallel lines.

| Transformation | Defined by | Fixed points |
|---|---|---|
| Translation | Vertical and horizontal shift | None |
| Rotation | Centre, angle and direction | The centre of rotation |
| Reflection | Axis of reflection | Points on the axis of reflection |

Reflection is the only one of the three transformations that reverses the order of vertices. Thus if a triangle is labelled $ABC$ in clockwise order, then the vertices of $\Delta A'B'C'$ will appear in anticlockwise order after reflection.

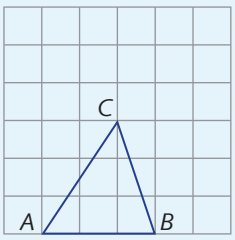



This is different from the case when a translation or rotation is applied to $\Delta ABC$ with vertices in clockwise order. The vertices of $\Delta A'B'C'$ will also be in clockwise order, as shown below.

We can combine two or more transformations by applying them one after another, as in the examples below.

## Example 7

Translate $\Delta ABC$ 2 units right and 1 unit up. Then translate the image 3 units left and 2 units up. Describe a single translation that has the same effect.

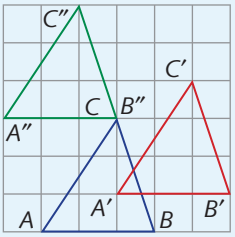




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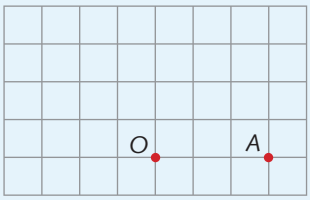
### Solution

The successive transformations are shown in the diagram opposite. The combined translation is the same as a translation 1 unit left and 3 units up.



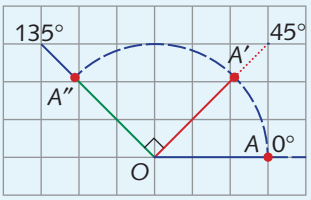

## Example 8

Rotate the point $A$ by 45° and then by a further 90°, both rotations anticlockwise about $O$. Describe a single rotation that has the same effect as the combination of these two rotations.



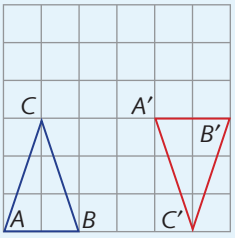

### Solution

The first rotation moves $A$ to $A'$ (see the diagram opposite). The second rotation then moves $A'$ to $A''$. This combined transformation is the same as a single rotation of 135° anticlockwise about $O$.




## Example 9

Describe two successive transformations that will have the combined effect of moving $\Delta ABC$ onto $\Delta A'B'C'$ as shown opposite.



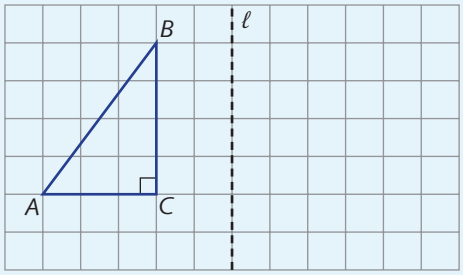

### Solution

There are many possible answers. One possible answer is:

- first translate $\Delta ABC$ so that $A$ moves to $A'$; that is, 4 units right and 3 units up
- then reflect the triangle in $A'B'$.


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## Example 10

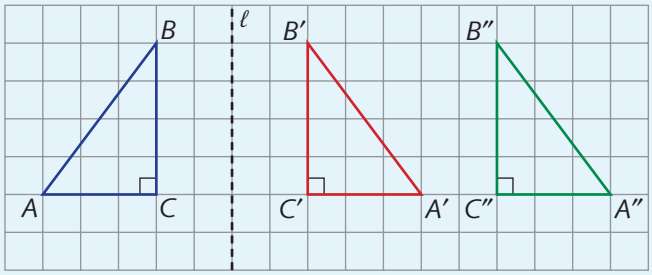



**a** Reflect triangle $ABC$ in the line $\ell$ and then translate the image 5 units to the right.

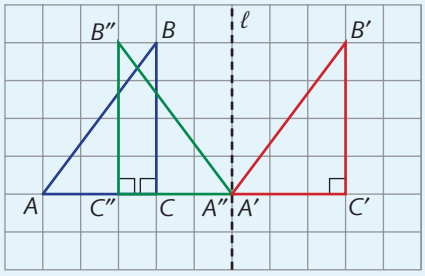
**b** Translate triangle $ABC$ 5 units to the right and then reflect the image in the line $\ell$.

### Solution

**a**




**b**




Note that in parts **a** and **b** a different result is obtained. The final image depends on the order in which the transformations are taken.

The following exercise involves no new transformations, but uses combinations of the three transformations we have learned about. If you get stuck on any step, look back at the earlier exercise dealing with the relevant kind of transformation.

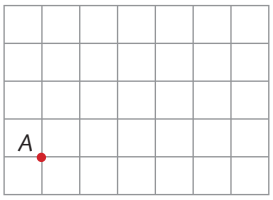

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# Exercise 18D

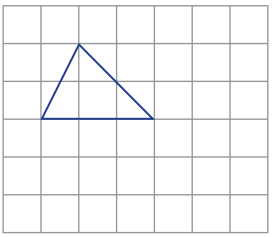
Example 7

**1** In each part perform the two transformations successively. Then find a single translation that has the same effect.

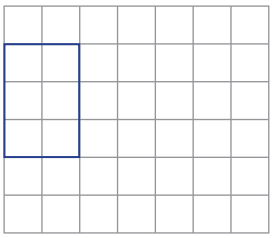
**a** Translate 1 right, 2 up, *then* 3 right, 1 up.



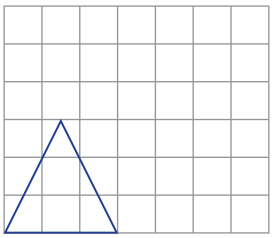

**b** Translate 3 down, *then* 2 right.



**c** Shift 3 right, 2 down, *then* 2 up.



**d** Shift 1 right, 3 up, *then* 2 down, 2 right.



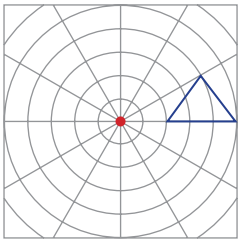
Example 8

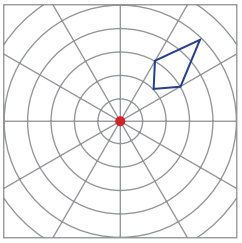
**2** Carry out the first rotation given for each figure, draw the image, and then carry out the second rotation. Find a single rotation that will have the same effect.

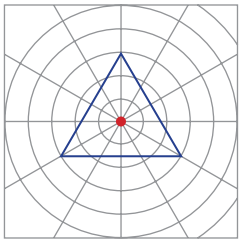
**a** Rotate 60° anticlockwise, then 30° anticlockwise.

**b** Rotate 30° anticlockwise, then 60° clockwise.

**c** Rotate 120° anticlockwise, then 90° anticlockwise.







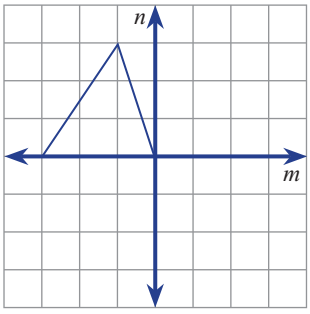
**3** **a** **i** Copy the first diagram below.

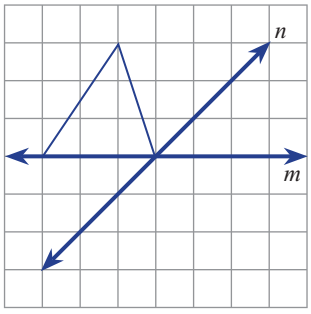
**ii** Reflect the triangle in $m$, then reflect the image in $n$.

**iii** Now reflect the original triangle in $n$, then reflect the image in $m$.

**iv** Are your answers to parts **ii** and **iii** the same?

**b** Repeat the steps **i** to **iv** for the second diagram.



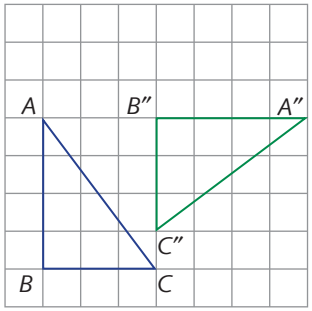


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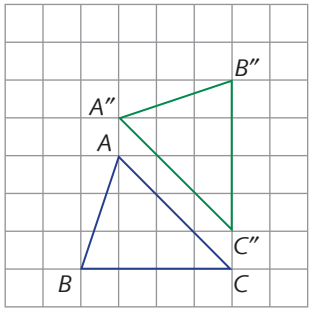
Example 9

**4** In each part, describe two successive transformations that will have the combined effect of moving $\Delta ABC$ onto $\Delta A''B''C''$.

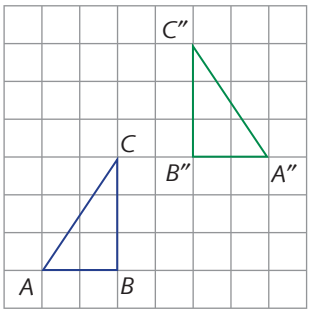
**a**



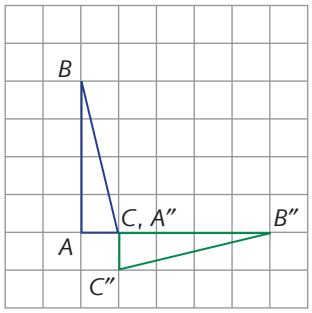

**b**




**c**




**d**



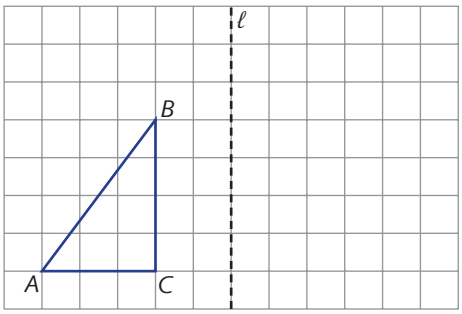

Example 10

**5** **a** Reflect triangle $ABC$ in the line $\ell$ and then translate the image 6 units to the right.

**b** Translate triangle $ABC$ 6 units to the right and then reflect the image in the line $\ell$.

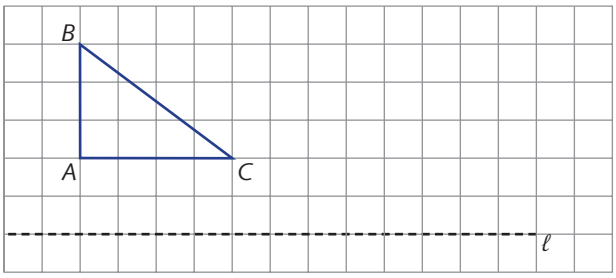
**c** Translate triangle $ABC$ 6 units up and the reflect the image in the line $\ell$.

**d** Reflect triangle $ABC$ in the line $\ell$ and then translate the image 6 units up.




**6** **a** Reflect triangle $ABC$ in the line $\ell$ and then translate the image 6 units to the right.

**b** Translate triangle $ABC$ to the right 6 units and then reflect in the line $\ell$.



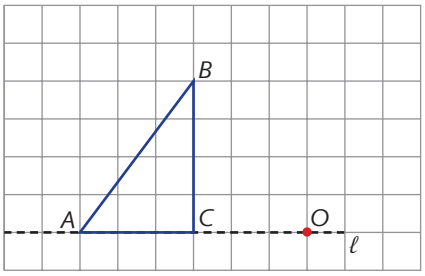




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**7** **a** Rotate triangle $ABC$ about point $O$ in an anticlockwise direction by 90° and then reflect the image in the line $\ell$.

**b** Reflect triangle $ABC$ in the line $\ell$ and then rotate the image about point $O$ in an anticlockwise direction by 90°.




# 18E Transformations in the Cartesian plane

In this section, transformations are described using coordinates.

## Example 11

Find the coordinates of the image of the point (1, 3) under:

**a** a translation of 1 unit to the right and 3 units up

**b** a translation of 3 units to the left and 4 units down

**c** a clockwise rotation of 90° about the origin

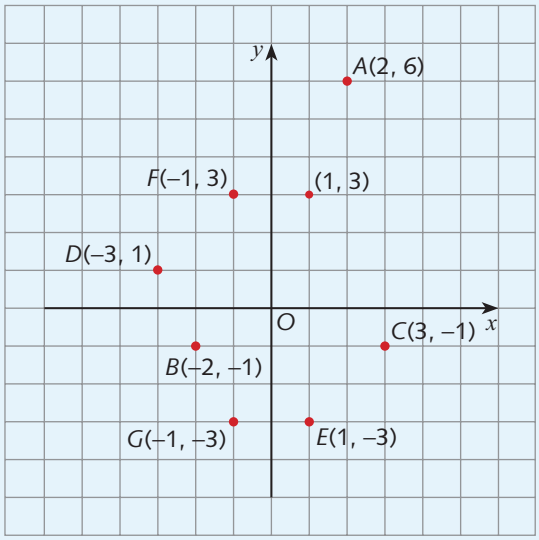
**d** an anticlockwise rotation of 90° about the origin

**e** a reflection in the $x$-axis

**f** a reflection in the $y$-axis

**g** a rotation of 180° about the origin

## Solution





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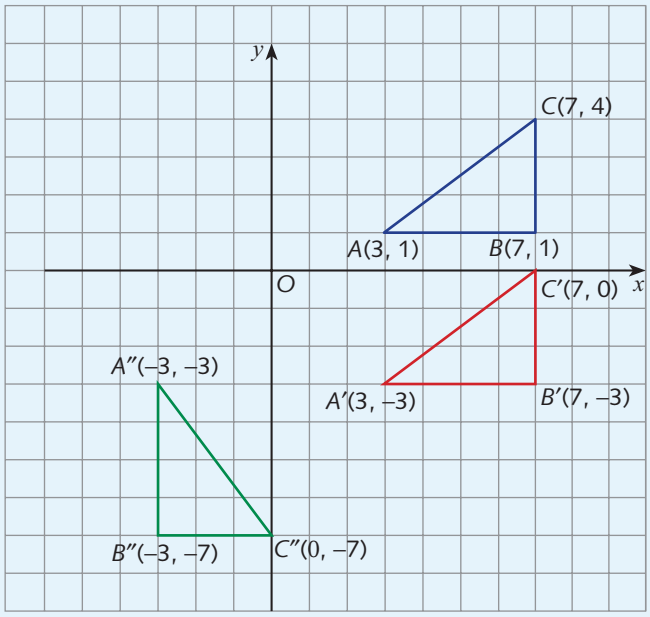
### Example 12

A triangle $ABC$ has vertices $A(3, 1)$, $B(7, 1)$, and $C(7, 4)$.

**a** Plot the points $A$, $B$ and $C$ and draw the triangle $ABC$.

**b** Find coordinates of the vertices of the image of the triangle under a translation of 4 units down followed by a clockwise rotation of 90° about the origin.

### Solution




## Exercise 18E

You will need graph paper to do these exercises. Each question requires you to draw a number plane with labelled $x$- and $y$-axes. A single number plane for each question is fine (for example, one number plane for question **1** parts **a–d**), if points are neatly labelled. The coordinates of the image point(s) should also be given.

### Translations

Example 11a, b

**1** Mark the following points on your graph paper and translate each one 2 units right and 3 units up. Be careful to label each point and its image. Also write down the coordinates of each image point.

**a** $A(0, 0)$ **b** $B(2, 1)$ **c** $C(-3, 1)$ **d** $D(-5, -5)$

**2** Given the following points and their images under a translation, plot the points and describe the translation.

**a** $A(0, 0)$ and $A'(2, 1)$ **b** $B(1, -2)$ and $B'(3, 3)$ **c** $C(0, 4)$ and $C'(0, -4)$


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## Rotations

Example 11c, d

**3** Mark the given points on a number plane and rotate them 90° anticlockwise about the origin $O$. Write down the coordinates of each image point.

**a** $A(2, -1)$ **b** $B(2, 1)$ **c** $C(-3, 1)$ **d** $D(0, 0)$

**4** Mark the given points on a number plane. Find their images under a rotation of 180° anticlockwise about the origin $O$. Draw each image point and write down its coordinates.

**a** $A(2, -1)$ **b** $B(2, 1)$ **c** $C(0, 0)$ **d** $D(0, 2)$

## Reflections

Example 11e, f

**5** Copy the given points onto a number plane. Find their images when they are reflected in the $x$-axis.

**a** $A(2, -1)$ **b** $B(-3, 3)$ **c** $C(-5, 0)$ **d** $D(0, 4)$

**6** Mark the given points on a number plane. Find their images under a reflection in the $y$-axis.

**a** **i** $A(3, -1)$ **ii** $B(-3, 3)$ **iii** $C(0, 4)$

**b** Join the points $A$, $B$ and $C$ in part a to form $\Delta ABC$. Find the image of $\Delta ABC$ under reflection in the $y$-axis.

## Combinations of transformations

Example 12

**7** Mark the given points on a number plane, translate them 4 units to the right and rotate them 90° anticlockwise about the origin $O$. Write down the coordinates of each image point.

**a** $A(2, -1)$ **b** $B(2, 1)$ **c** $C(-3, 1)$ **d** $D(-5, -5)$

**8** Mark the given points on a number plane. Find their images when they are first reflected in the $x$-axis and then in the $y$-axis.

**a** $A(2, -1)$ **b** $B(2, 2)$ **c** $C(-3, 3)$ **d** $D(-5, 0)$

**9** Mark the given points on a number plane. Find the coordinates of their images when they are first reflected in the $x$-axis and then translated 4 units down.

**a** $A(0, 0)$ **b** $B(2, 1)$ **c** $C(-3, 1)$ **d** $D(-5, -5)$

**10** A triangle $ABC$ has vertices $A(3, 1)$, $B(7, 1)$, and $C(7, 4)$.

**a** Plot the points $A$, $B$ and $C$ and draw the triangle $ABC$.

**b** Find the coordinates of the vertices of the image of the triangle under a translation of 4 units down followed by a reflection in the $y$-axis.

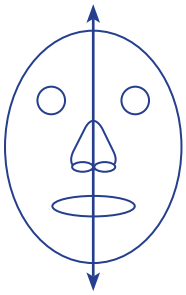

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# 18F Symmetry

**Symmetry** is an important idea in mathematics. We also see it in art and architecture. Here we are going to consider two types of symmetry, namely **rotational symmetry** and **reflection symmetry**.

## Reflection symmetry

The figure shown opposite has **reflection symmetry** because it falls back exactly on itself when it is reflected in the axis of reflection marked on the diagram. This line is called an **axis of symmetry** of the figure.



## Rotational symmetry

Think of rotating the figure shown opposite by 90° anticlockwise about $O$. The figure moves back on top of the original figure where $A$ has moved to where $B$ was, $B$ has moved to where $C$ was, $C$ has moved to where $D$ was, and $D$ has moved to where $A$ was. We say that the figure has **rotational symmetry**.

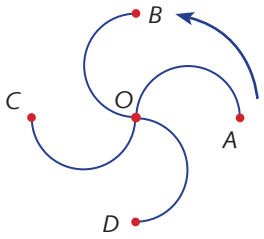
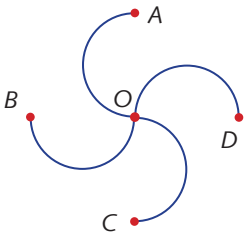
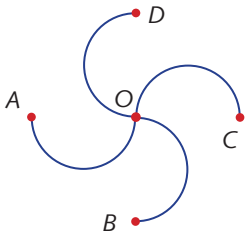



**Diagram 1**

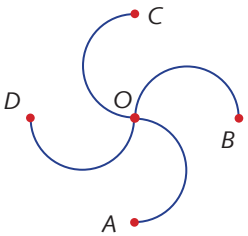
Next we look at the situation after repeated anticlockwise rotations of 90°. For example, the original point $A$ in diagram 1 moves to the point $A$ in diagram 2. It then moves to the point $A$ in diagram 3, and so on.



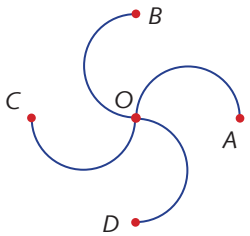

anticlockwise rotation of 90°
**Diagram 2**




2 anticlockwise rotations of 90°
**Diagram 3**




3 anticlockwise rotations of 90°
**Diagram 4**




4 anticlockwise rotations of 90°
**Diagram 5**

You can see that it takes four rotations of $\left(\frac{360}{4}\right)^\circ = 90^\circ$ anticlockwise for the point $A$ to return to its original position.

We say that our figure has **rotational symmetry of order 4**:

- 90° is the smallest angle of rotation that makes it fit exactly on top of itself, and
- when we repeat this rotation four times, we get the identity rotation of 360°.


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Can you see that this propeller  has rotational symmetry of order 3?

The smallest rotation that makes the propeller sit back on itself is 120°. When this is repeated three times, we get the identity rotation of 360°.

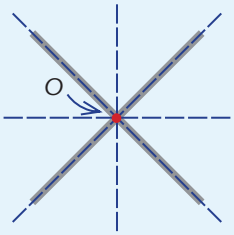
### Example 13

**a** Draw all axes of reflection symmetry for the figure shown opposite.
**b** Write down the order of its rotational symmetry.



### Solution

**a** The four axes of symmetry are shown in the solution diagram opposite. The axes of symmetry are shown as dotted lines.
**b** The order of rotational symmetry is 4, because the cross sits exactly on itself after rotations of 90°, 180°, 270° and 360° about the centre $O$.




These examples show that figures may have more than one axis of symmetry, and figures may have both rotational and reflection symmetry.

A figure can also have no rotational symmetry, for example, the number 5.

### Rotational symmetry

A figure has rotational symmetry of order $n$, $n > 1$, if $\frac{360°}{n}$ is the smallest angle of rotation that takes the figure so that it fits exactly on top of itself.

## Exercise 18F

Example 13

**1** In the table on the next page, the letters of the alphabet and several common symbols are drawn. Copy the table into your book.

**a** For each letter and symbol, mark any axes of symmetry.

**b** Does each figure have rotational symmetry? If so, mark the centre of the rotation on the diagram. State the order of rotational symmetry.

*Note*: The symmetry of some letters may vary, depending on how you draw them. Copy the letters accurately and comment on any issues of this type.


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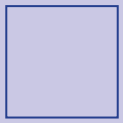
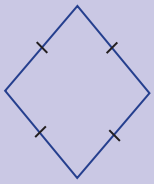
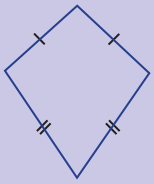
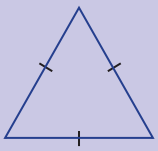
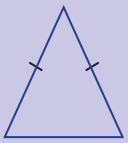
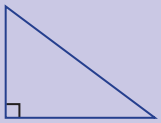
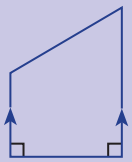
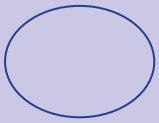
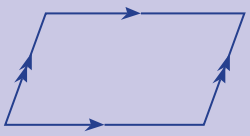
| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| **Order:** | | | | | | |
| | G | H | I | J | K | L |
| **Order:** | | | | | | |
| | M | N | O | P | Q | R |
| **Order:** | | | | | | |
| | S | T | U | V | W | X |
| **Order:** | | | | | | |
| | Y | Z | = | + | $ | % |
| **Order:** | | | | | | |

**2** In the following table, a number of common geometric figures are drawn. Copy the table into your book.

**a** For each figure, mark any axes of reflection symmetry.

**b** Does each figure have rotational symmetry? If so, mark the centre of rotation on the diagram and state the order of rotational symmetry.

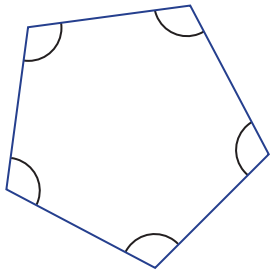
Assume that the rectangle is not a square, the rhombus is not a square, the kite is not a rhombus, the isosceles triangle is not equilateral, the trapezium has unequal vertical sides and the parallelogram is not a rhombus.

| | | | | |
|---|---|---|---|---|
| |  square |  rectangle |  rhombus |  kite |
| **Order:** | | | | |
| |  equilateral triangle |  isosceles triangle |  right triangle |  trapezium |
| **Order:** | | | | |
| |  circle |  ellipse |  parallelogram | |
| **Order:** | | | | |


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# 18G Regular polygons

A **regular polygon** is a plane figure in which every side is an interval of the same length and all internal angles are equal. A polygon is named after its number of angles; this is the same as the number of sides. For example, the figure on the right is called a **regular pentagon**, from the Greek words *pente*, meaning five, and *gonos*, meaning angled. The equal angles are marked.

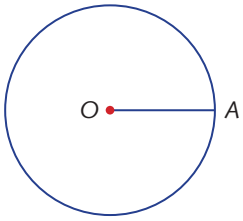


The names of some common regular polygons are given in the table below.

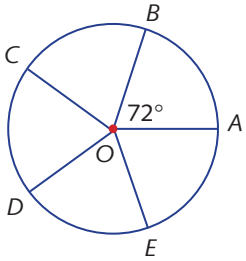
| Number of sides | Name |
|---|---|
| 3 | Equilateral triangle |
| 4 | Square |
| 5 | Regular pentagon |
| 6 | Regular hexagon |
| 7 | Regular heptagon |
| 8 | Regular octagon |
| 9 | Regular nonagon |
| 10 | Regular decagon |
| 12 | Regular dodecagon |

## Drawing a regular pentagon

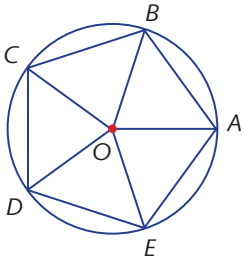
Use a protractor in the following to form angles of 72°.




Draw a circle of radius 5 cm. Mark a radius $OA$.



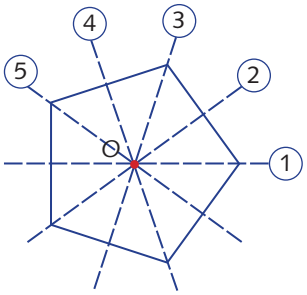

Draw rays $360° \div 5 = 72°$ apart, starting at $OA$. Let these rays intersect the circle at $B$, $C$, $D$, $E$.




Join $A$, $B$, $C$, $D$, $E$ to form the regular pentagon.


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## Symmetry of regular polygons




Regular polygons have a lot of symmetry. For example, the regular pentagon has five axes of reflection symmetry, as shown in the diagram opposite.

As we rotate the pentagon about the centre $O$, every 72° it will 'fall back on itself', looking identical to the original diagram even though the points have moved. The five rotations 72°, 144°, 216°, 288°, 360° about the centre $O$ are symmetries.

Thus the regular pentagon has rotational symmetry of order 5.

## Exercise 18G

**1** **a** Draw a regular pentagon, using a circle of radius 6 cm and the method above.

**b** Mark the axes of reflection symmetry.

**2** This question requires you to draw a regular nonagon (also called a 9-gon).

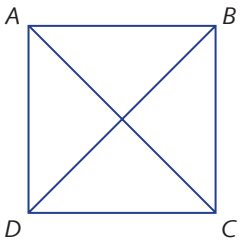
**a** Explain why the rays from the centre in your construction have to be 40° apart.

**b** Using a protractor, construct a regular nonagon in a circle of radius 5 cm.

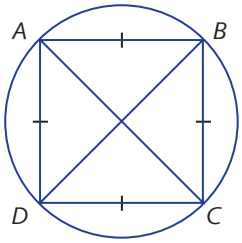
**c** Draw the axes of symmetry. Write down the order of rotational symmetry of a regular nonagon.

**3** **a** How many sides does a regular octagon have?

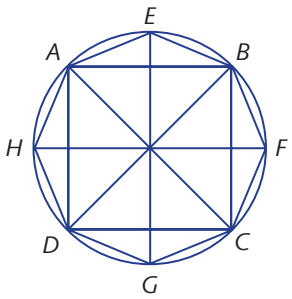
**b** Construct a regular octagon using only a straight edge and compasses by following the steps below.




Construct a square of side length 4 cm. Mark the diagonals of the square to locate its centre.




Place the point of your compasses on the centre of the square and draw a circle around the square, touching its four vertices. Mark the midpoints of the sides of the square.




Connect the opposite midpoints with a line that extends each end to meet the circle. Join the points $A$, $E$, $B$, $F$, $C$, $G$, $D$ and $H$ to form a regular octagon.

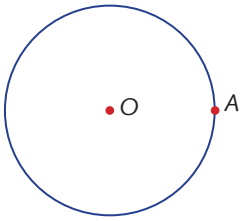
**c** What is the angle between adjacent rays from the centre in your construction?

**d** Draw the axes of symmetry and write down the order of rotational symmetry.

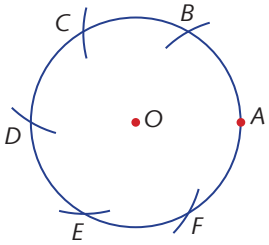
**e** Use a different colour to join every second vertex and so construct a regular 4-gon (a square!). Which of the axes of symmetry of the octagon are axes of symmetry of the square? What is the order of rotational symmetry of the square?


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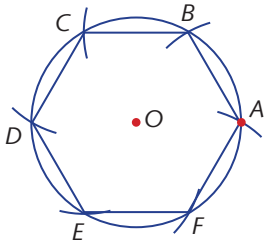
**4** **a** Follow the steps below to construct a regular hexagon without using a protractor.




Draw a circle of radius 5 cm. Leave your compasses set at 5 cm and mark a point $A$ on the circle.




Place the point of your compasses on $A$ and mark an arc intersecting the circle at $B$. Move the point of your compasses to $B$ and mark an arc intersecting the circle at $C$. Continue around the circle.

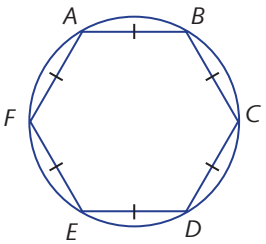



To check your accuracy, draw a final arc centred at $F$. It should pass through $A$. As before, join the points $A$, $B$, $C$, $D$, $E$ and $F$ to form a regular hexagon.

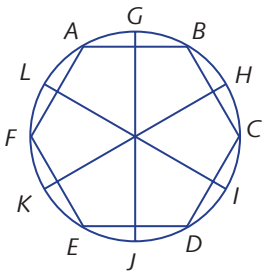
**b** Explain why this construction works.

**c** Draw the axes of reflection symmetry and write down the order of rotational symmetry.

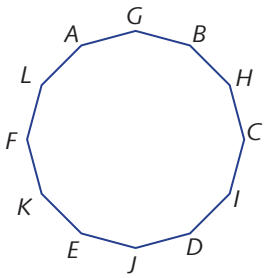
**5** **a** Construct a regular dodecagon in a circle of radius 5 cm, using the following steps.




Start by constructing a hexagon using the method in question **4**. Mark the midpoints of each side of the hexagon.



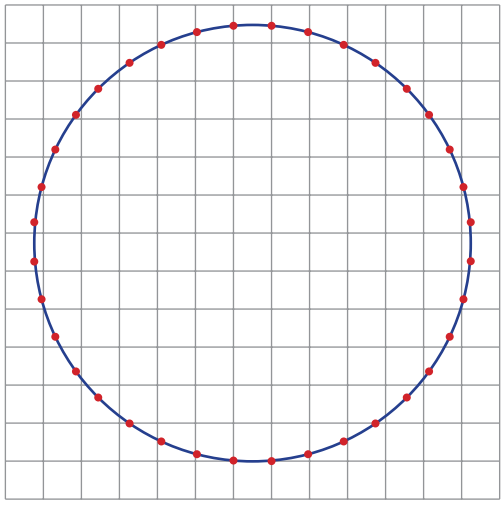

Connect the opposite midpoints with a line that extends each end to meet the circle.




Join the points $A$, $G$, $B$, $H$, $C$, $I$, $D$, $J$, $E$, $K$, $F$ and $L$ to form a regular dodecagon.

**b** Label the vertices 1, 2, 3, …, 12 clockwise (with 12 as the topmost vertex). What have you constructed?

**c** How many axes of symmetry does this polygon have, and what is its order of rotational symmetry?



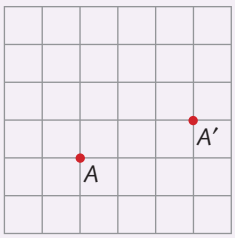
**6** What are the orders of the rotational symmetry of a regular 36-gon and a regular 37-gon? Do not attempt to draw them! A regular 36-gon is illustrated to the right, with straight lines between adjected points.


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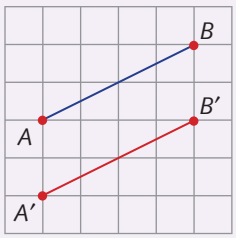
# Review exercise

**1** Describe the translation shown in each of these diagrams.

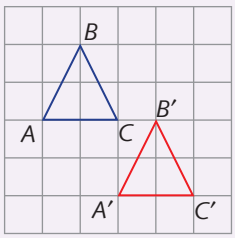
**a**



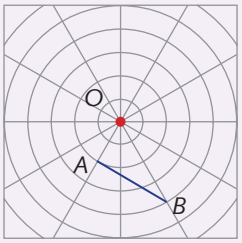

**b**



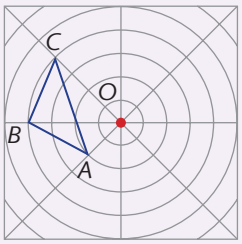

**c**



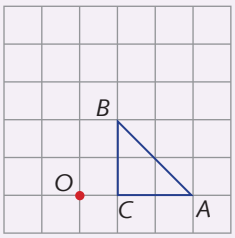

**2** Rotate the interval $AB$ by 60° in a clockwise direction about the point $O$.



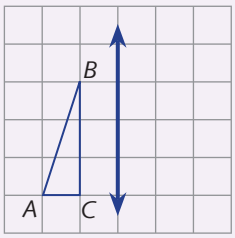

**3** Rotate the triangle $ABC$ by 90° clockwise about $O$.




**4** Rotate triangle $ABC$ in the axis of reflection.




**5** Reflect triangle $ABC$ in the axis of reflection.




**6** Plot the points given below on graph paper and translate them 3 units right and 1 unit up. Be careful to label each point and its image. Also write down the coordinates of each image point.

**a** $A(0, 0)$

**b** $B(2, 1)$

**c** $C(-3, 1)$

**7** Given a point and its image under translation in each part, plot the points and describe each translation.

**a** $A(0, 0)$ and $A'(3, 1)$

**b** $B(1, -3)$ and $B'(6, 2)$

**c** $C(0, 3)$ and $C'(0, -3)$


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**8** Plot the points given below on a number plane and rotate each one 90° anticlockwise about the origin $O$. Write down the coordinates of each image point.

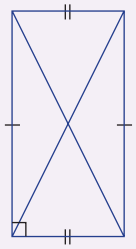
**a** $A(1, -2)$ **b** $B(1, 4)$ **c** $C(-2, 1)$

**9** Plot these points on a number plane. Find their images when reflected in the $x$-axis.

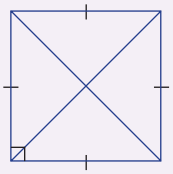
**a** $A(3, -2)$ **b** $B(3, 2)$ **c** $C(-2, 1)$

**10** State the order of rotational symmetry of each of the diagrams.

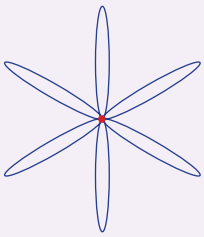
**a**



**b**



**c**

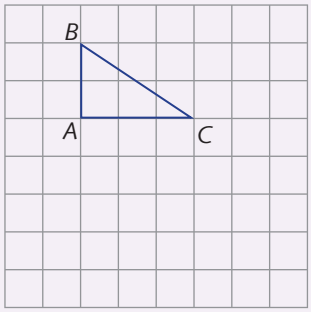


**d**

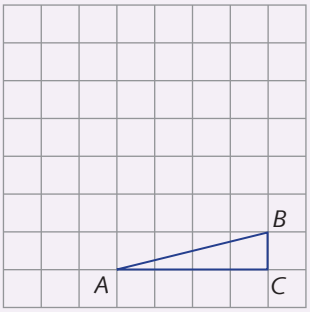


**11** Perform each pair of successive translations on the given figure. Then find a single translation that has the same effect.

**a** Translate 1 right, 2 down *then* 2 left, 1 down.

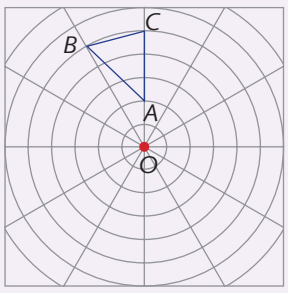



**b** Translate 2 left, 3 up *then* 1 left, 1 up.

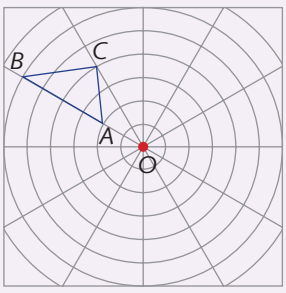



**12** Perform each pair of rotations on the given figure. Then find a single rotation that has the same effect. All rotations are about $O$.

**a** Rotation of 30° anticlockwise *then* 60° anticlockwise.




**b** Rotation of 30° anticlockwise *then* 60° clockwise.





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**13** Plot the given points on the number plane. Reflect them in the $x$-axis and then reflect the image in the $y$-axis. State the coordinates of the final image.

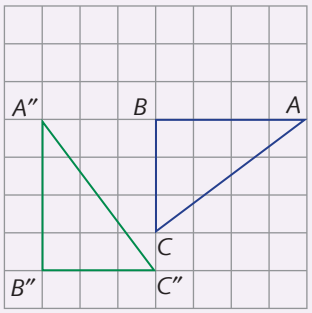
**a** $A(5, -1)$ **b** $B(2, -1)$ **c** $C(-2, -3)$

**14** Plot the following points on your graph paper. First translate them 1 unit up and 3 units to the right, and then translate the images 2 units up and 2 units to the left. State the coordinates of the final images.

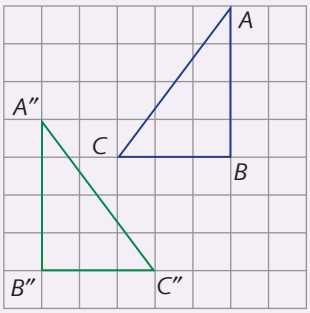
**a** $A(1, -2)$ **b** $B(-5, -3)$ **c** $C(-1, 4)$

**15** In each part, describe two successive transformations that will have the combined effect of mapping $\Delta ABC$ onto $\Delta A''B''C''$.

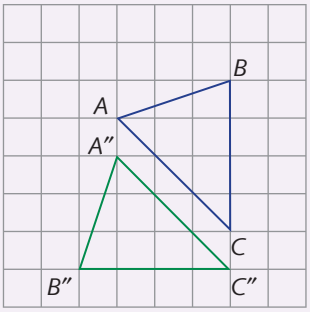
**a**



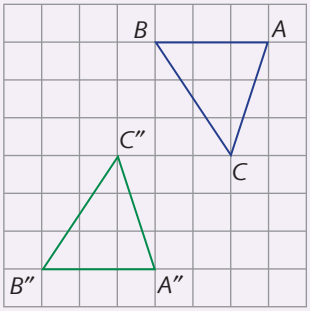

**b**




**c**




**d**



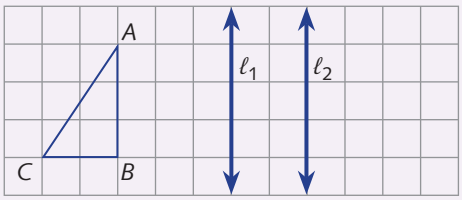


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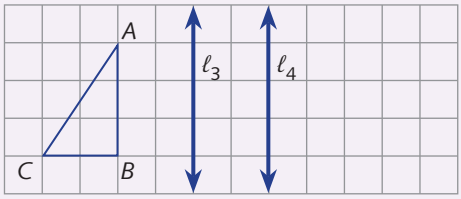
# Challenge exercise

Answer these questions on square grid paper.

**1** **a** Reflect triangle $ABC$ in line $\ell_1$ and then reflect the image in $\ell_2$. Label the first image $A'C'B'$ and the second $A''B''C''$.

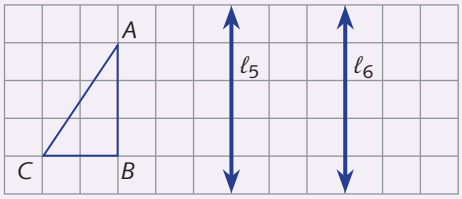



**b** This time, reflect triangle $ABC$ in line $\ell_3$ and then reflect the image in $\ell_4$. Label the first image $A'C'B'$ and the second $A''B''C''$ as before.




**c** Describe the translation that takes triangle $ABC$ to triangle $A''B''C''$ in both parts **a** and **b**. What do you notice?

**2** **a** Reflect triangle $ABC$ in line $\ell_5$ and then reflect the image in $\ell_6$. Label the first image $A'C'B'$ and the second $A''B''C''$.




**b** Describe the translation that takes triangle $ABC$ to triangle $A''B''C''$ in part **a**.

**c** Describe the translation that takes triangle $ABC$ to triangle $A''B''C''$ if the lines of reflection are parallel 4 units apart.

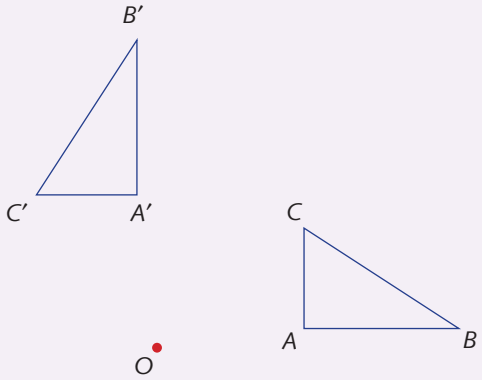
**d** Suppose that the lines are $a$ units apart. What is the translation?

**3** The triangle $ABC$ whose vertices have coordinates $A(0, 0)$, $B(2, 0)$, $C(2, 4)$ is translated to the triangle $A'B'C'$ with coordinates $A'(4, 0)$, $B'(6, 0)$, $C'(6, 4)$. Draw the two triangles on a number plane and draw in two axes of reflection, $\ell_1$ and $\ell_2$, such that when triangle $ABC$ is reflected in $\ell_1$ and its image is reflected in $\ell_2$, the final result is triangle $A'B'C'$. Is there only one way to do this? What is the distance between lines $\ell_1$ and $\ell_2$?


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**4** The triangle $ABC$ whose vertices have coordinates $A(0, 0)$, $B(2, 0)$, $C(2, 4)$ is translated to the triangle $A'B'C'$ with coordinates $A'(0, 4)$, $B'(2, 4)$, $C'(2, 8)$. Draw the two triangles on a number plane and draw in two axes of reflection, $\ell_1$ and $\ell_2$, such that when triangle $ABC$ is reflected in $\ell_1$ and its image is reflected in $\ell_2$, the final result is triangle $A'B'C'$. Is there only one way to do this? What is the distance between lines $\ell_1$ and $\ell_2$?

**5** Triangle $ABC$ has been rotated through 90° in an anticlockwise direction about $O$ to the triangle $A'B'C'$. Copy the diagram carefully into your book.

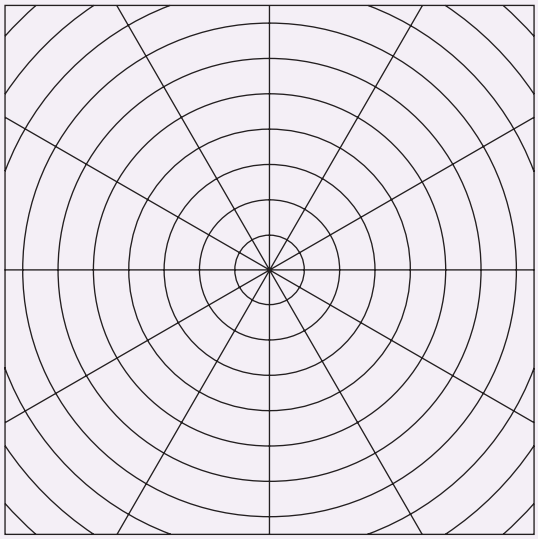



**a** Draw two lines $\ell_1$ and $\ell_2$ through $O$ such that, when triangle $ABC$ is reflected in $\ell_1$ and its image is reflected in $\ell_2$, the final result is triangle $A'B'C'$.

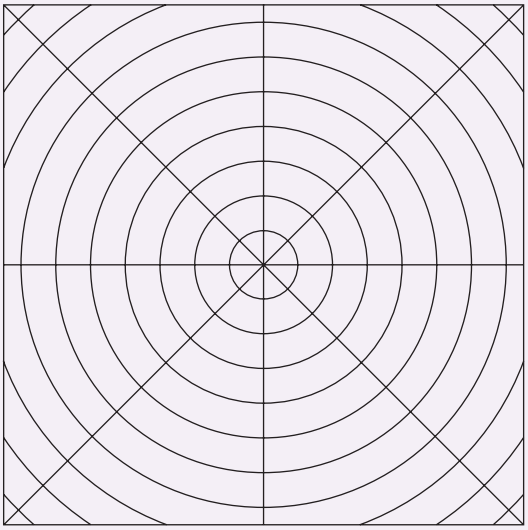
**b** Measure the acute angle between lines $\ell_1$ and $\ell_2$.

**c** Is there only one choice for the position of lines $\ell_1$ and $\ell_2$?

**Circular graph paper (30°)**



**Circular graph paper (45°)**



Circular graph paper is available to download from your Interactive Textbook.


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CHAPTER

# 19

Number and Algebra

# Graphs and tables

In this chapter, we discuss various ways of presenting data to make them easier to understand and analyse. Most people find it much easier to understand data if they are presented in tabular or pictorial form. Examples of tables and diagrams are found in newspapers, in advertisements, on television and on the internet. Many of these tables and diagrams present fair and accurate pictures of the situations they describe, but some diagrams are misleading and can give wrong impressions.

The purpose of a table or diagram is to:

- fix in the reader's mind the data it represents
- suggest relationships that may exist among the data
- create a quick, lasting and accurate impression of the significant facts.

Later in this chapter, we will look at four methods of pictorial presentation, namely the pictogram, the column graph or bar chart, the pie chart and the line graph.

In this chapter a calculator may be used where appropriate. Suitable computer software can also be used.



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# 19A Reading tables

Tables are constructed to record and present information. For example, the following table shows the numbers of voters who voted in an election. The table gives the numbers of male and female voters who voted for each candidate in a particular town. The information has been arranged so that it is easy to see at a glance a summary of the important election results.

| | Democrat | Liberal | Progressive |
|---|---|---|---|
| **Men** | 1256 | 4578 | 168 |
| **Women** | 1567 | 3987 | 42 |

From the table, it is easy to read off and compare the information. The information involves five **categories**. The categories are Men and Women, and the parties (Democrat, Liberal and Progressive) represented by each of the three candidates. The table presents the votes for the different candidates by gender. For example, we can see that most men voted Liberal.

### Example 1

The following table describes internet use for a group of 17 480 people over a particular 5-day period.

| | 0–17 years | 18–24 years | 25–54 years | 55+ years | Total |
|---|---|---|---|---|---|
| **Used the internet** | 1600 | 2570 | 2510 | 1320 | 8000 |
| **Did not use the intenet** | 450 | 850 | 8000 | 180 | 9480 |
| **Total** | 2050 | 3420 | 10 510 | 1500 | 17 480 |

**a** What percentage of people in the 55-and-over age group did not use the internet?

**b** What percentage of people who use the internet are aged 0–17?

### Solution

**a** $\left(\frac{180}{1500} \times \frac{100}{1}\right)\% = 12\%$ of people over 55 did not use the internet.

**b** $\left(\frac{1600}{8000} \times \frac{100}{1}\right)\% = 20\%$ of people who use the internet are aged 0–17.

## Exercise 19A

Example 1

**1** Data on the ages of male inhabitants of the Maldives in 2001 are presented in the table on page 475. The total number of males is 226 634.


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| Age group (years) | Number of males |
|---|---|
| 6–9 | 30 813 |
| 10–14 | 41 089 |
| 15–19 | 33 266 |
| 20–24 | 23 514 |
| 25–29 | 20 090 |
| 30–34 | 18 161 |
| 35–39 | 15 699 |
| 40–44 | 12 402 |
| 45–49 | 7461 |
| 50–54 | 5967 |
| 55–59 | 5996 |
| 60–64 | 6312 |
| 65–69 | 4611 |
| > 69 | 1253 |

**a** How many male inhabitants of the Maldives are aged between 10 and 14?

**b** What percentage of males are aged under 25? Give your answer to the nearest per cent.

**c** What percentage of males are aged between 15 and 24? Give your answer to the nearest per cent.

**d** What is wrong with the given data?

**2** Data has been collected on the nationalities of some of the visitors to certain island resorts. The table below gives the available data. In the table, (A) indicates that the country is in Asia, (E) indicates that the country is in Europe and (O) indicates that the country is neither in Asia nor in Europe.

| Nationality of visitor | Number of visitors (tens of thousands per year) |
|---|---|
| Australia (O) | 0.5 |
| Britain (E) | 6 |
| France (E) | 3.5 |
| Belgium (E) | 14 |
| Japan (A) | 23 |
| Singapore (A) | 5 |
| USA (O) | 1 |
| Italy (E) | 45 |
| Germany (E) | 23 |
| Korea (A) | 5 |
| Russia (E) | 12 |
| New Zealand (O) | 0.25 |


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Use the table on the previous page to answer the following questions.

**a** What is the total number of visitors from Japan?

**b** What is the total number of visitors from Asia?

**c** What percentage of the total number of visitors came from Europe? Give your answer to the nearest per cent.

**3** A small country has a number of island resorts. The following information was collected for five of these resorts.

| Name of resort | Capacity (maximum number of guests who can stay at one time) | Number of staff involved in recreation for guests |
|---|---|---|
| Belle de Mer | 115 | 5 |
| Dream Island | 243 | 20 |
| Hibiscus Garden | 146 | 13 |
| Thalassa | 76 | 5 |
| Tropic Isle | 400 | 22 |

**a** What is the maximum number of guests who can stay at the Hibiscus Garden resort?

**b** How many staff are involved in recreation for guests at Tropic Isle?

**c** What is the total capacity of the five resorts in the table?

**d** What is the ratio of staff to guests at the Hibiscus Garden?

**4** The table below gives the results of a tennis tournament between Rishi, Bryce, Gerald and John.

| | Sets won | | | |
|---|---|---|---|---|
| **Match** | **Rishi** | **Bryce** | **Gerald** | **John** |
| Rishi vs Bryce | 0 | 3 | | |
| Rishi vs Gerald | 3 | | 2 | |
| Rishi vs John | 3 | | | 2 |
| Bryce vs Gerald | | 3 | 1 | |
| Bryce vs John | | 3 | | 0 |
| Gerald vs John | | | 3 | 2 |

Complete the following summary table using the information given above.

| | Matches | | Sets | |
|---|---|---|---|---|
| | **Won** | **Lost** | **Won** | **Lost** |
| **Rishi** | | | | |
| **Bryce** | | | | |
| **Gerald** | | | | |
| **John** | | | | |

**a** Who won the most matches?

**b** Who played the most sets?

**c** What percentage of the matches Gerald played did he win?


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# 19B The pictogram

A **pictogram** uses a symbol to represent a block of data within a data set, and is the simplest way of representing data in a diagrammatic form. A pictogram has visual impact and is easily understood.

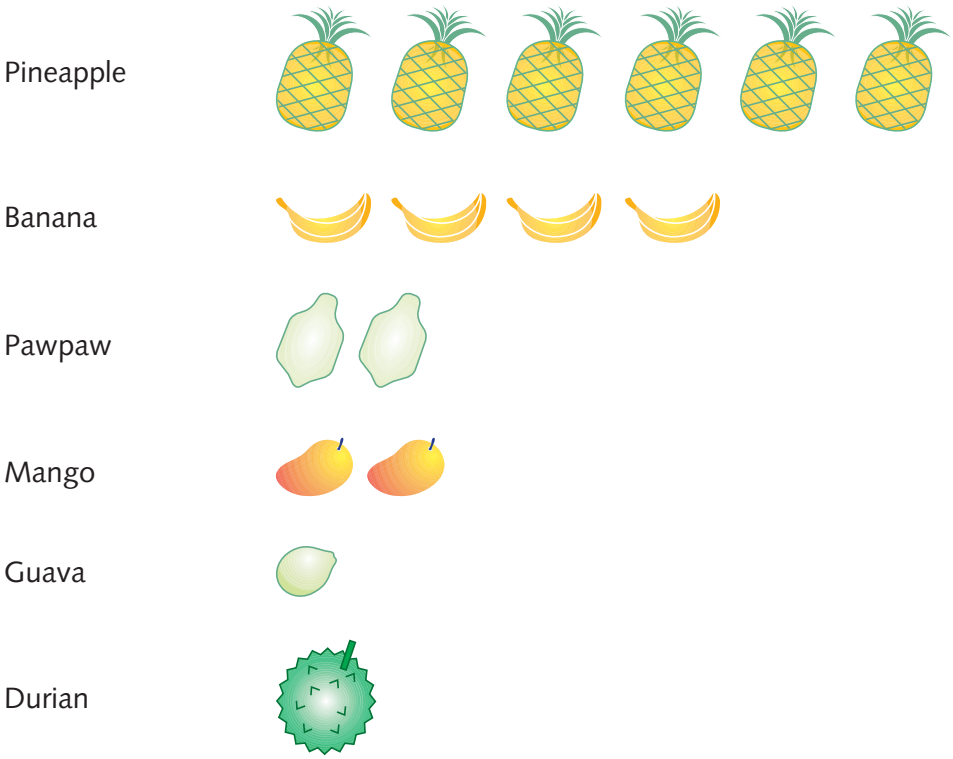
Consider the table below, which gives the areas under cultivation, in hectares, for different fruits on a tropical island.

| Fruit | Area under cultivation (hectares) |
|---|---|
| Pineapple | 30 |
| Banana | 20 |
| Pawpaw | 10 |
| Mango | 10 |
| Guava | 5 |
| Durian | 5 |

A symbol, such as FRUIT, can be used for all types of fruit to indicate the size of each area under cultivation.

Alternatively, you can use a different symbol for each block of data, as shown below.

**Fruits: area under cultivation**

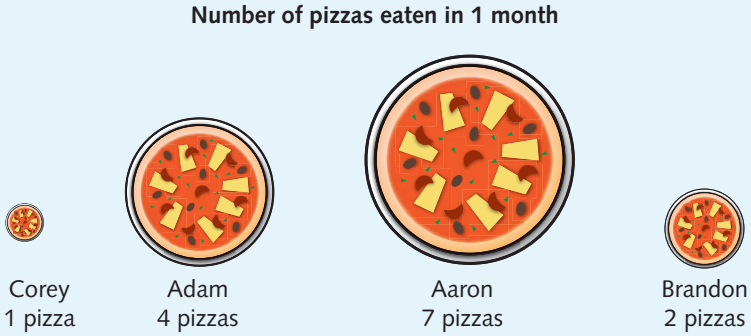



Each of the symbols represents 5 hectares.


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## Example 2

The pictogram below shows the numbers of pizzas consumed by four different people in a month.



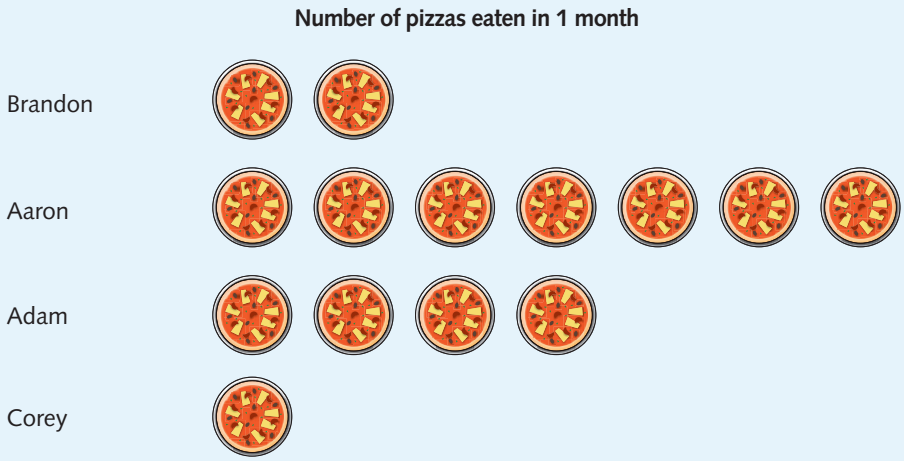

**a** Write in general terms your comments on the pictogram.

**b** Draw a better representation using a pictogram.

## Solution

**a** The pictogram gives a poor and distorted representation, as it is not easy to see how the size of each picture represents the number of pizzas eaten.

**b** It would be more informative to represent the data with different numbers of pizzas of the same size.





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### Example 3

The table below shows the average daily sales of ice-cream cones for a kiosk in each season over a 12-month period.

| Season | Summer | Autumn | Winter | Spring |
|---|---|---|---|---|
| Ice-cream sales | 90 | 35 | 5 | 55 |

Use a pictogram to represent these data.

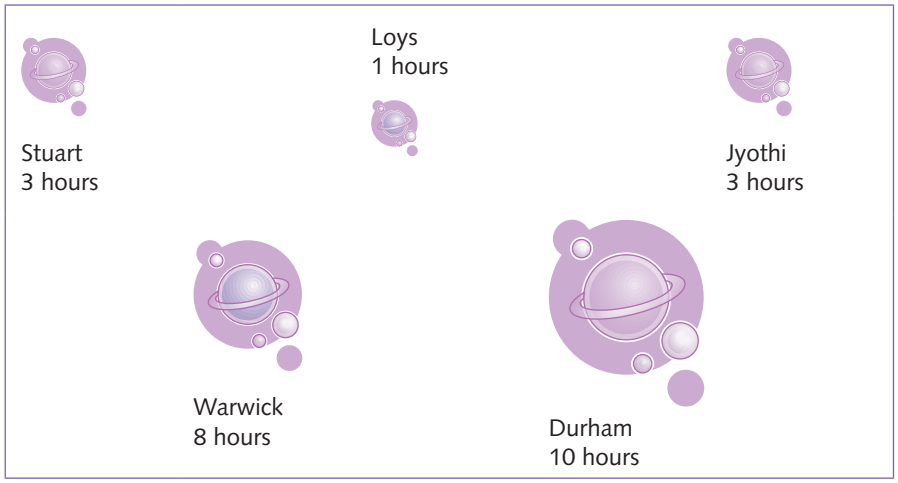
### Solution




## Exercise 19B

Example 2

**1** The pictogram below shows the amounts of time spent star-gazing by a group of budding young astronomers.




**a** Write in general terms your comments on the pictogram.

**b** Draw a better representation using a pictogram.



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**2** The pictogram below shows the numbers of trees grown on each of five neighbouring tree farms.




**a** How many trees are grown on Santa's Tree Farm?

**b** How many trees are grown in total?

**c** What percentage of the trees is grown at the Pines Tree Farm?

**d** What percentage, to the nearest per cent, of the trees is grown at the North Pole Tree Farm?

Example 2

**3** The table below shows the sizes of basketball singlets worn by the members of a basketball club.

| **Size** | Extra small | Small | Medium | Large | Extra large |
|---|---|---|---|---|---|
| **Number of embers** | 5 | 15 | 25 | 40 | 10 |

Draw a pictogram to display the data.

**4** The table below shows the numbers of large cats at an open range zoo. Draw a pictogram to display the data.

| **Type of cat** | Tiger | Lion | Jaguar | Cheetah | Puma |
|---|---|---|---|---|---|
| **Number of cats** | 70 | 100 | 40 | 50 | 30 |


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# 19C Column graphs

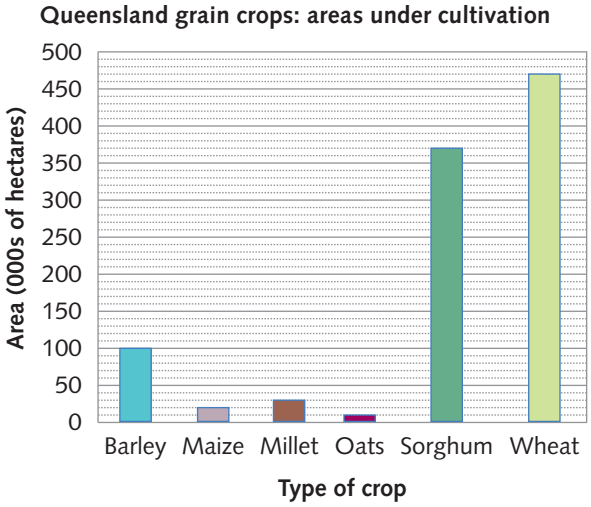
A **column graph** or **bar chart** uses bars of different lengths to compare different quantities. A column graph is easy to draw and easy to read. It is reasonably accurate and shows the relative sizes at a glance.

Consider the table below, which gives the areas under cultivation, in thousands of hectares, for different grain crops in a region of Queensland.

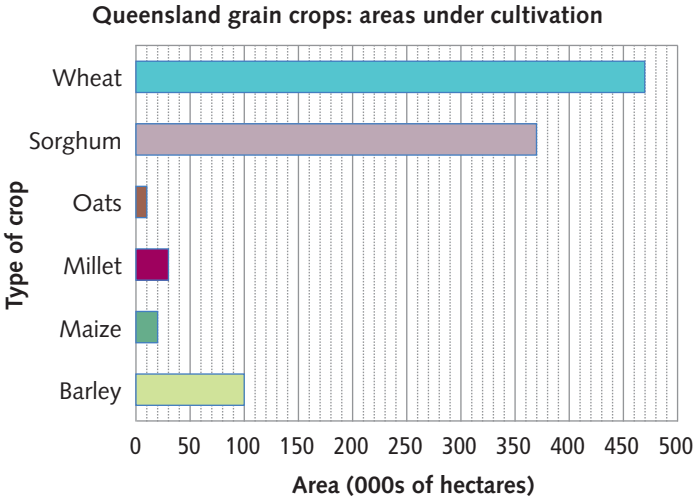
| Grain crop | Area under cultivation (in thousands of hectares) |
|---|---|
| Barley | 100 |
| Maize | 20 |
| Millet | 30 |
| Oats | 10 |
| Sorghum | 370 |
| Wheat | 470 |

A column graph makes it easy to compare the areas under cultivation for the different grain crops grown. The length of a column represents the area under cultivation for a particular grain crop using the scale on the left.

A spreadsheet package can be used to construct column graphs.




Note that all the columns have the same width. A column graph should have a title, and both axes should be clearly labelled. Column graphs may also be drawn horizontally rather than vertically.

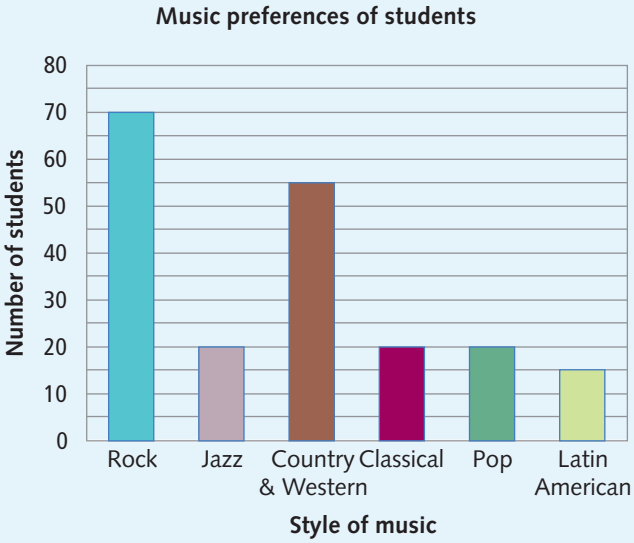




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## Example 4

A survey was conducted in which students were asked to pick their favourite style of music. They were given the categories Rock, Jazz, Country and Western, Classical, Pop and Latin American from which to choose. The responses are displayed in the column graph below.




Use the column graph to find:

**a** the number of students who chose Classical as their favourite style of music

**b** the least popular style of music, according to the survey

**c** the total number of students surveyed

**d** the percentage of students surveyed who chose Country and Western as their preferred option

**e** the percentage of students surveyed who did not choose Rock as their preferred option

## Solution

**a** Number of students who chose Classical = 20.

**b** The least popular style of music, according to the survey, is Latin American.

**c** Total number of students surveyed $= 70 + 20 + 55 + 20 + 20 + 15$
$= 200$

**d** 55 of the 200 students surveyed chose Country and Western.

Percentage who chose Country and Western $= \left(\frac{55}{200} \times \frac{100}{1}\right)\%$
$= 27\frac{1}{2}\%$

So $27\frac{1}{2}\%$ of students surveyed preferred Country and Western.

**e** $200 - 70 = 130$ students did not choose Rock.

Percentage who did not choose Rock $= \left(\frac{130}{200} \times \frac{100}{1}\right)\% = 65\%$

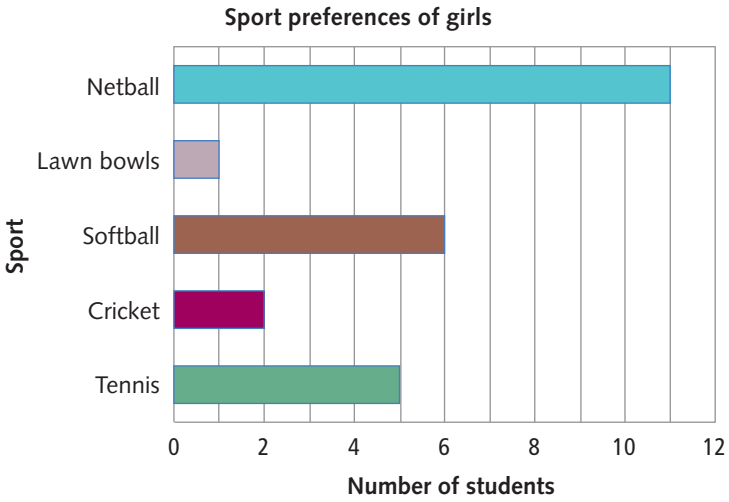
So 65% of students surveyed preferred a music style other than Rock.


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# Exercise 19C

Example 4

**1** The girls in a physical education class were asked which sport they liked best from a choice of netball, lawn bowls, softball, cricket or tennis. The responses are displayed in the column graph.




Use the column graph to find:

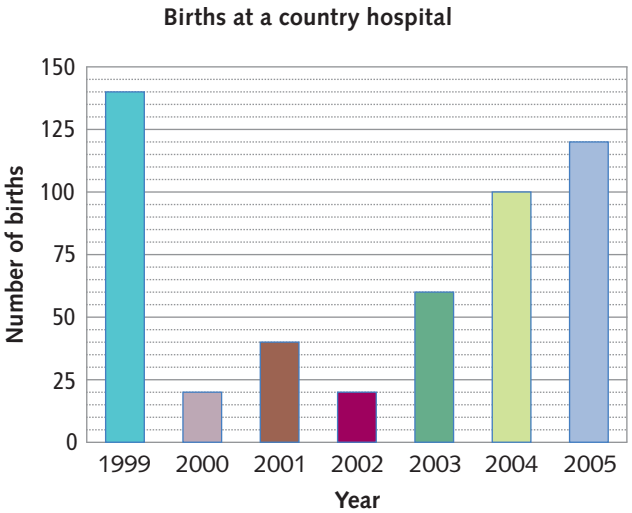
**a** how many girls chose tennis as their favourite sport

**b** the most popular sport

**c** the number of girls in the class

**d** the percentage of girls in the class who chose cricket as their favourite sport

**2** Use the column graph below, showing the number of births at a country hospital, to answer the following questions.




**a** How many babies were born at the hospital in 2003?

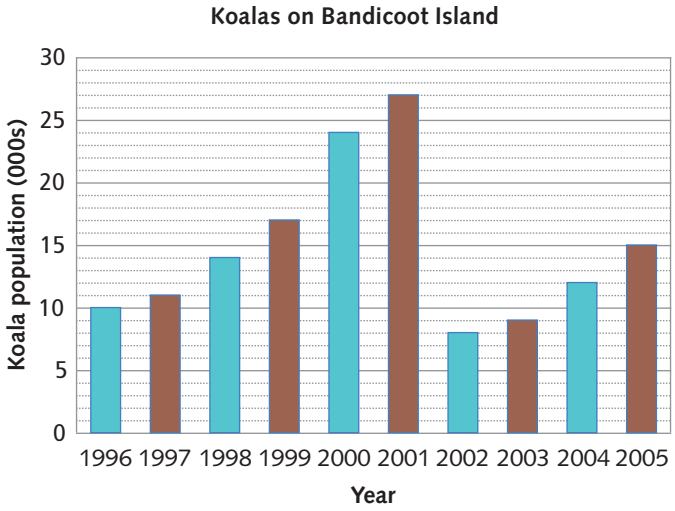
**b** In which year was the number of births the lowest?

**c** How many babies were born during the period 1990–2005?

**d** What percentage of the babies born during the period 1990–2005 were born in 1999?


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**3** The column graph below shows the number of koalas on Bandicoot Island for the years 1996 to 2005. The population is recorded on 1 January of each year. A large bushfire occurred in July of a particular year.




Use this graph to find:

**a** in which year the population of koalas was:

**i** the highest **ii** the lowest

**b** in which year you think the bushfire occurred

It is considered that the island's vegetation is able to sustain a population of 20 000 koalas.

**c** For how many years was the population above 20 000?

**d** How far above 20 000 was the highest population?

**e** How far below 20 000 was the lowest population?

**f** In which year would you expect the population to next reach 20 000?

**4** The table below shows the numbers of animals on a hobby farm.

| Type of animal | Chicken | Pig | Sheep | Horse | Goose | Dog | Cow |
|---|---|---|---|---|---|---|---|
| Number of animals | 6 | 3 | 25 | 2 | 4 | 4 | 6 |

**a** Use a column graph to display the data.

**b** What percentage of the animals on the farm are two-legged?

**c** What percentage of the animals on the farm are not sheep?

**5** Nick asked all of the boys in Year 10 at his school what time they usually woke up on Saturday mornings. The responses are shown in the table below.

| Time (a.m.) | 6:00 | 7:00 | 8:00 | 9:00 | 10:00 | 11:00 |
|---|---|---|---|---|---|---|
| Number of students | 4 | 8 | 8 | 10 | 13 | 17 |

**a** Use a column graph to display the data.

**b** What percentage of boys woke up before 8:30 a.m.?

**c** Give a conclusion that Nick could draw from these numbers.


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# 19D Divided bar charts and pie charts

## Divided bar charts

A **divided bar chart** (segmented bar chart) is a rectangle that has been divided into smaller sections by vertical lines. From the graph, you can compare the parts easily.

A vegetable farm grows potatoes, carrots and onions. The table below shows the number of hectares of the farm used for each crop.

| Crop | Potatoes | Carrots | Onions |
|---|---|---|---|
| Area (hectares) | 13 | 5 | 2 |
| Percentage (%) | 65 | 25 | 10 |

In this case, the divided bar chart is drawn as a rectangle of length 10 cm, representing 20 hectares in total. The length of the section for potatoes is therefore 6.5 cm, the length for carrots is 2.5 cm and the length for onions is 1 cm. These lengths correspond to percentages of 65% (potatoes), 25% (carrots) and 10% (onions).




### Example 5

Draw a divided bar chart to represent the following data about sales of different kinds of fruit from a stall at a market.

| Fruit | Oranges | Apples | Grapes | Bananas | Total |
|---|---|---|---|---|---|
| Value ($) | 360 | 80 | 160 | 200 | 800 |

### Solution

We can calculate the segment lengths, using an overall length of 10 cm for the divided bar chart, as follows.

Oranges: $\frac{360}{800} \times 10 \text{ cm} = 4.5 \text{ cm}$

Apples: $\frac{80}{800} \times 10 \text{ cm} = 1 \text{ cm}$

Grapes: $\frac{160}{800} \times 10 \text{ cm} = 2 \text{ cm}$

Bananas: $\frac{200}{800} \times 10 \text{ cm} = 2.5 \text{ cm}$

The segment lengths, and corresponding percentages, are shown in the next table.

| Fruit | Oranges | Apples | Grapes | Bananas |
|---|---|---|---|---|
| Segment length (cm) | 4.5 | 1.0 | 2.0 | 2.5 |
| Percentage | 45% | 10% | 20% | 25% |



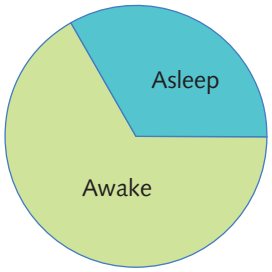


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## Pie charts

A **pie chart** (sector graph) is a circle that is divided into sectors by radii. The sectors are usually shaded in different colours. A pie chart is more complicated than a bar chart to construct, but gives an impression of the proportion of the whole represented by each category of data.

Consider the following simple example with only two categories of data.




Alex is awake for 16 hours and asleep for 8 hours. There are only two sectors required. Alex is asleep for $\frac{1}{3}$ of the day. We draw a sector with angle $\frac{1}{3} \times 360° = 120°$ to represent the part of the day Alex is asleep. The other sector represents the part of the day when he is awake. The angle for this sector is $\frac{2}{3} \times 360° = 240°$.

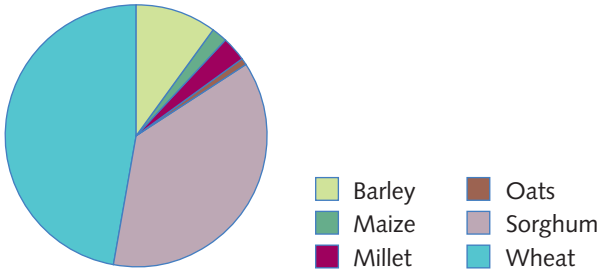
For the Queensland grain crop data on page 481, the size of each **sector** of the pie graph is determined by the area under cultivation for the given grain crop.

| Grain crop | Area under cultivation (thousands of hectares) | Angle of sector |
|---|---|---|
| Barley | 100 | $\frac{100}{1000} \times 360° = 36°$ |
| Maize | 20 | $\frac{20}{1000} \times 360° = 7.2°$ |
| Millet | 30 | $\frac{30}{1000} \times 360° = 10.8°$ |
| Oats | 10 | $\frac{10}{1000} \times 360° = 3.6°$ |
| Sorghum | 370 | $\frac{370}{1000} \times 360° = 133.2°$ |
| Wheat | 470 | $\frac{470}{1000} \times 360° = 169.2°$ |
| **Totals** | **1000** | **360°** |

A protractor can be used to draw the sectors, as in the pie graph at the top of the next page. You will not need to use the protractor to draw the last sector representing wheat but, as a check of your level of accuracy when using a protractor, measure the angle and check that it is close to 169°.


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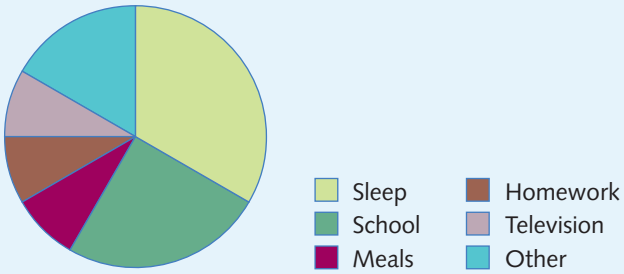
Queensland grain crops: areas under cultivation




## Example 6

The pie chart below represents the hours spent on various activities by a teenage boy on a typical 24-hour weekday.

Use of hours in a day, by activity




**a** Use your protractor to measure the angle of the sector for each activity.

**b** How many hours were spent at school?

**c** How many hours were spent not sleeping?

**d** If $\frac{3}{4}$ of the hours allocated to 'Other' are spent surfing, what would be the angle of the sector that would represent surfing on the same pie chart?

## Solution

**a** It is important to check that the angles add to 360° (although a sum close to 360° is an acceptable level of accuracy).

| Activity | Measured angle at centre of sector |
|---|---|
| Sleep | 120° |
| School | 90° |
| Meals | 30° |
| Homework | 30° |
| Television | 30° |
| Other | 60° |
| **Total** | **360°** |


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**b** The angle of the sector for 'School' is 90°.

$$\text{Fraction of day spent at school} = \frac{90}{360}$$
$$= \frac{1}{4}$$

Hence, $\frac{1}{4}$ of the day or 6 hours is spent at school.

**c** The angle of the sector for 'Sleep' is 120°.

$$\text{Fraction of day spent at school} = \frac{120}{360}$$
$$= \frac{1}{3}$$

Hence, $\frac{1}{3}$ of the day or 8 hours is spent sleeping.

Hence, $24 - 8 = 16$ hours of the day are spent not sleeping.

**d** For 'Other', the angle of the sector is 60°.

$$\text{Angle of the sector for 'Surfing'} = \frac{3}{4} \times 60°$$
$$= 45°$$

Hence, the angle of the sector for 'Surfing' on the same pie chart would be 45°.

## Example 7

A shop sells 90 beach balls in a week. The beach balls are produced in five different colours: red, blue, yellow, white and black. The table shows the number of balls sold.

| Colour of beach ball | Number sold |
|---|---|
| Red | 25 |
| Black | 5 |
| White | 5 |
| Yellow | 40 |
| Blue | 15 |

**a** Represent the data on a pie chart.

**b** What percentage of the balls sold were blue?

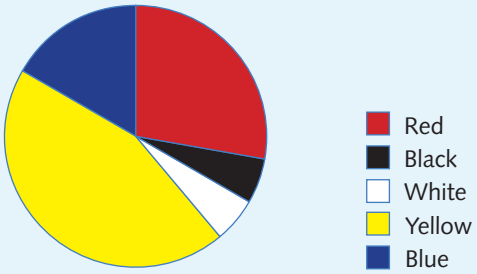

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**Solution**

**a** The angle of each sector must be calculated in order to draw the pie chart.

| Colour of beach ball | Angle at centre of sector |
|---|---|
| Red | $\frac{25}{90} \times 360° = 100°$ |
| Black | $\frac{5}{90} \times 360° = 20°$ |
| White | $\frac{5}{90} \times 360° = 20°$ |
| Yellow | $\frac{40}{90} \times 360° = 160°$ |
| Blue | $\frac{15}{90} \times 360° = 60°$ |

**Beach ball sales, by colour**




**b** 15 of the 90 beach balls sold were blue.

$$\text{Percentage of balls sold that were blue} = \frac{15}{90} \times 100\,\%$$
$$= 16\tfrac{2}{3}\%$$

## Exercise 19D

Example 5

**1** Draw a divided bar chart to represent the expenditure on the different types of advertising of a sporting event, as shown in the table below. The total expenditure was $100 000.

| Type of advertising | Amount spent |
|---|---|
| Newspapers | $15 000 |
| Radio | $10 000 |
| Television | $65 000 |
| Billboards | $10 000 |

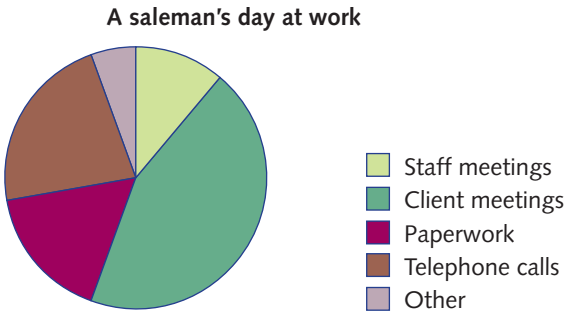

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**2** Draw a divided bar chart to represent the different types of sales by a car dealer over a 12-month period.

| Type of car | Number sold |
|---|---|
| Sedan | 540 |
| People mover | 120 |
| Station wagon | 200 |
| Four-wheel drive | 140 |

Example 6

**3** A salesman arrives at work at 8 a.m. and leaves at 5 p.m. The pie chart below shows how he spends his day.




**a** Use your protractor to measure the angle of each sector.

**b** How long does the salesman spend on telephone calls?

**c** What fraction of the salesman's work day is spent in meetings with staff?

**d** What percentage of the salesman's work day is spent meeting with clients? Give your answer to the nearest per cent.

**e** If $\frac{1}{4}$ of the time allocated to paperwork is spent claiming expenses, what would be the angle of the sector for the time spent claiming expenses?

Example 7

**4** 140 university students were asked their principal mode of travel to university that day. The table shows the responses they gave.

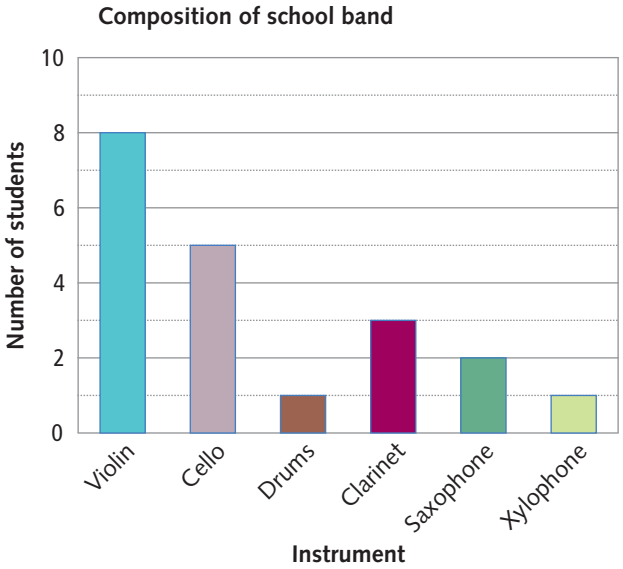
| Mode of transport | Number of students |
|---|---|
| Train | 56 |
| Tram | 35 |
| Bus | 28 |
| Car | 14 |
| Walk | 7 |

**a** Represent the data on a pie chart.

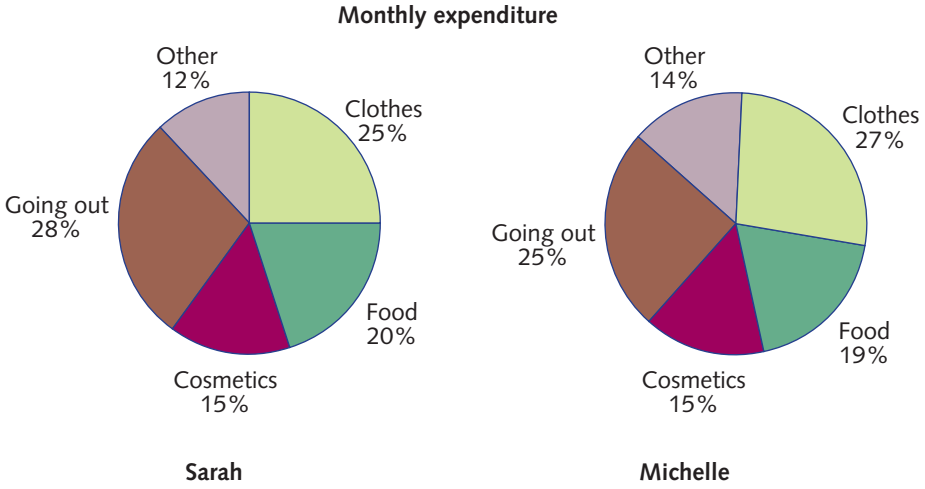
**b** What percentage of the students travelled by bus?


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**5** Display the data shown in the following column graph using a pie chart.




**6** The two pie charts shown below represent how two cousins, Michelle and Sarah, spent their monthly incomes from their part-time jobs.




**a** Suppose that in a particular month each girl earns $400. How much money does each of the girls spend on clothes?

**b** In the next month money Michelle earns $400 but Sarah earns $440. How much money does each of the girls spend on clothes?

**c** Comment on your answers to parts **a** and **b**.

**7** A charity uses its funds to provide food, shelter and employment training to homeless people. A pie chart is drawn to represent the different kinds of expenditure. The angles of the sectors for food and shelter are 109° and 125°, respectively.

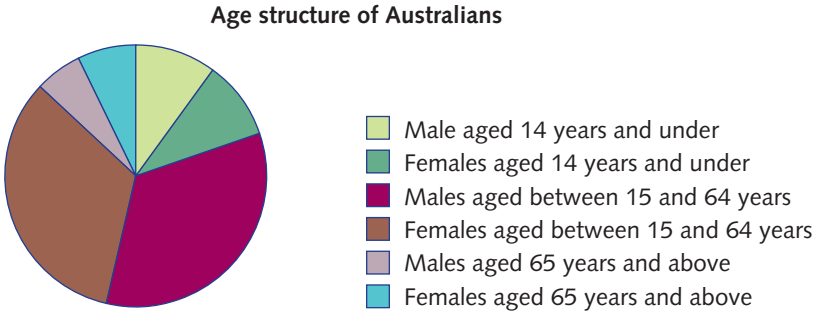
**a** Which category receives the smallest part of the funds?

**b** What is the angle of the sector used to represent employment training?

**c** Find the percentage of the charity's funds that is allocated to shelter. Give your answer to the nearest per cent.


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**8** The pie chart shows how 21 000 000 Australians are divided into various groups according to age and gender. The angles of the sectors for 'Females aged 65 years and above' and 'Males aged 65 years and above' are about 26° and 21°, respectively. Give your answers to the nearest 10 000.




(Source: www.cia.gov/library/publications/the-world-factbook/geos/as.html, accessed 11 April 2011)

**a** If $\frac{2}{3}$ of the 'Males aged 65 years and above' are under 70, how many males are 70 years of age or older?

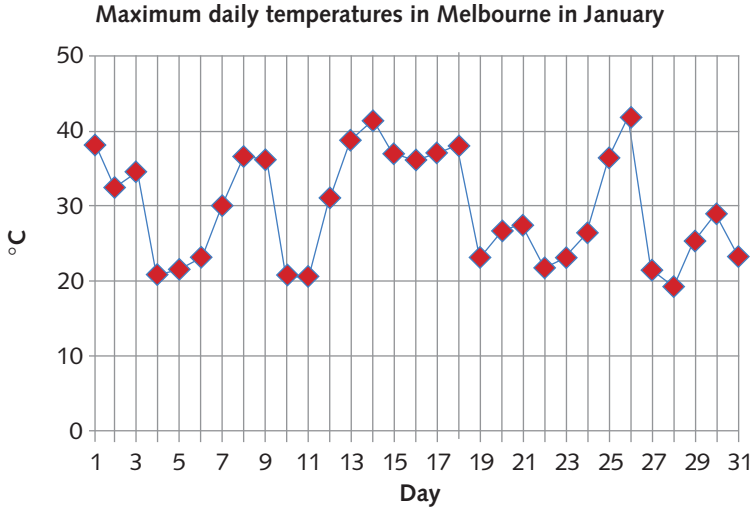
**b** If 67 % of Australians are aged between 15 and 64 years, how many Australians are 14 years of age or younger?

# 19E Line graphs

In this section and the next, **line graphs** are constructed. We do this by plotting points.

A line graph gives a general idea of trends, peaks, troughs and fluctuations.

Consider the maximum daily temperature in Melbourne, recorded over the month of January in one year. The data can be displayed on a line graph.





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Each dot represents the maximum temperature on a particular day. The lines between the dots show how the temperature increases or decreases as the month progresses.

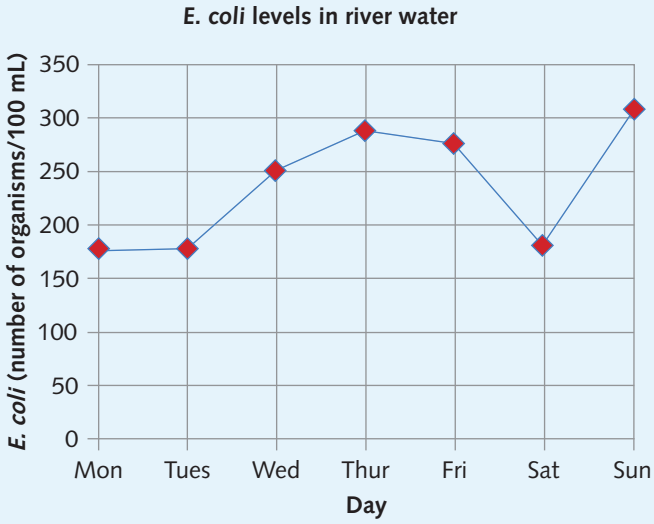
### Example 8

*Escherichia coli* is a common type of bacteria that can cause disease in humans. The table below shows average readings of the *E. coli* levels in part of a river. Readings that are above 200 organisms per 100 mL and below 1000 organisms per 100 mL are considered 'Fair', which means this part of the river is generally suitable for recreational activities such as rowing, canoeing and kayaking, but not for swimming.

| Day | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday | Sunday |
|---|---|---|---|---|---|---|---|
| ***E. coli* (number of organisms per 100 mL)** | 176 | 178 | 251 | 288 | 276 | 181 | 308 |

Use a line graph to display the data.

### Solution




## Exercise 19E

Example 8

**1** The table below shows the number of visitors per day to a national park in a particular week.

| Day | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday | Sunday |
|---|---|---|---|---|---|---|---|
| **Number of visitors** | 300 | 280 | 280 | 310 | 225 | 195 | 180 |

Use a line graph to display the data.


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**2** The table below is a record of Rohani's weight each month during her first year of life.

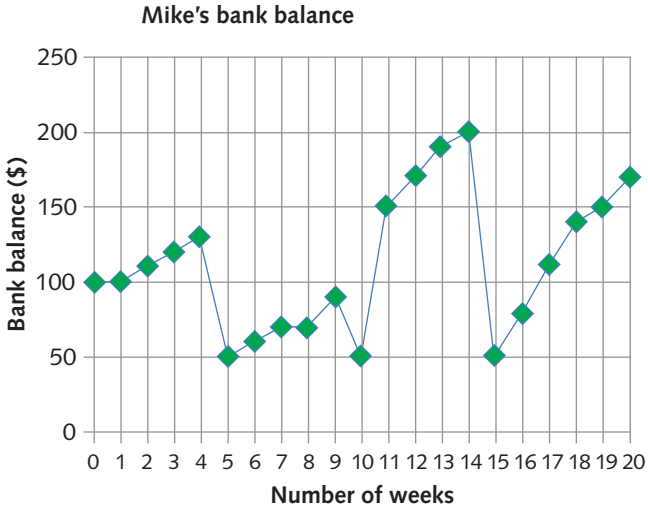
| Month | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Weight (kg) | 3.7 | 4.5 | 5.3 | 6.0 | 6.6 | 7.2 | 7.8 | 8.3 | 8.7 | 9.2 | 9.6 | 9.9 | 10.2 |

**a** How much weight did Rohani gain in her first month?

**b** How much weight did Rohani gain in her first year?

**c** Construct a line graph to display the data.

**3** The following line graph shows Mike's bank balance over a period of 20 weeks.



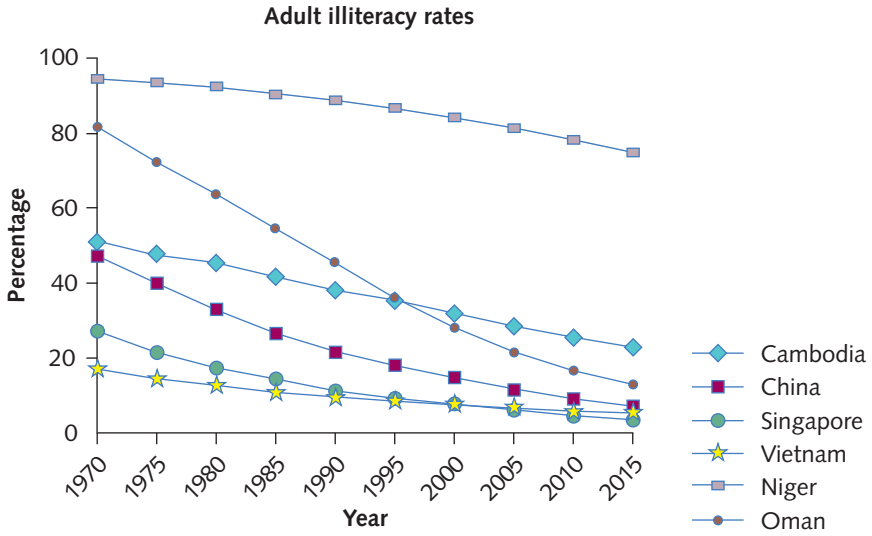

**a** How much money did Mike start with?

**b** How many times did Mike add to his savings?

**c** How many times did Mike withdraw some of his savings?

**d** At the end of the 20 weeks, did Mike have more or less money than he started with?

**4** The following line graphs represent the estimated and projected illiteracy rates of selected countries from 1970 to 2015.




(Source: www.uis.unesco.org/en/stats/statistics/UIS_Literacy_Country2002.xls, accessed 8 April 2006)


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**a** In which country is the illiteracy rate projected to be the lowest in the year 2015?

**b** In which country is the illiteracy rate projected to be the highest in the year 2015?

**c** Estimate the year when the illiteracy rate for Oman fell below that of Cambodia.

**d** In which country has the illiteracy rate fallen the most?

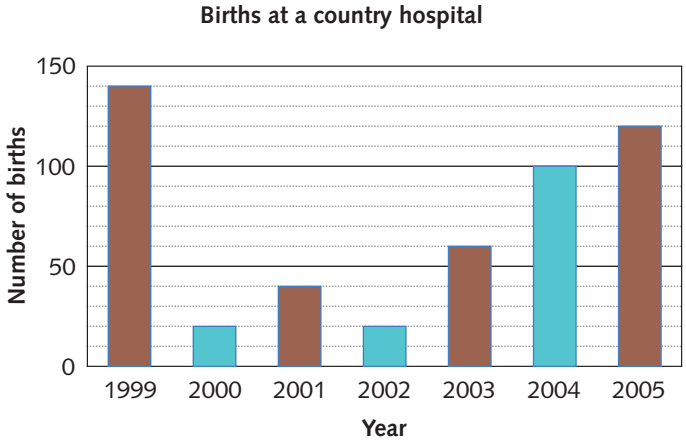
**e** In which country has the illiteracy rate fallen the least?

**5** The table below is a record of the maximum daily temperature in a certain city over the course of a week.

| Day | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday | Sunday |
|---|---|---|---|---|---|---|---|
| Maximum temperature (°C) | 18 | 14 | 17 | 20 | 12 | 13 | 15 |

Construct a line graph to display the data.

**6** Consider the graph below, showing the number of births at a country hospital.




**a** In which year were there 100 births at the hospital?

**b** Display the data shown in the column graph using a line graph.

**c** Which representation of the data do you consider to be the better one? The line graph or the column graph? Why?

# 19F Applications of the line graph

Line graphs are useful not only for representing data, as in the previous section, but also for showing the relationship between two quantities, such as age and height. Two particular applications of line graphs are discussed in this section: the conversion graph and the travel graph.

## The conversion graph

A **conversion graph** can be used for the conversion of measurements from one unit to another. Examples include converting one currency to another (for example, euros to Australian dollars), and changing heights from inches into centimetres, weights from pounds into kilograms, and areas from acres into hectares. In general, only approximate values can be found.


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## Example 9

The conversion graph below is based on the conversion factor on a particular day of A$1 = €0.60.

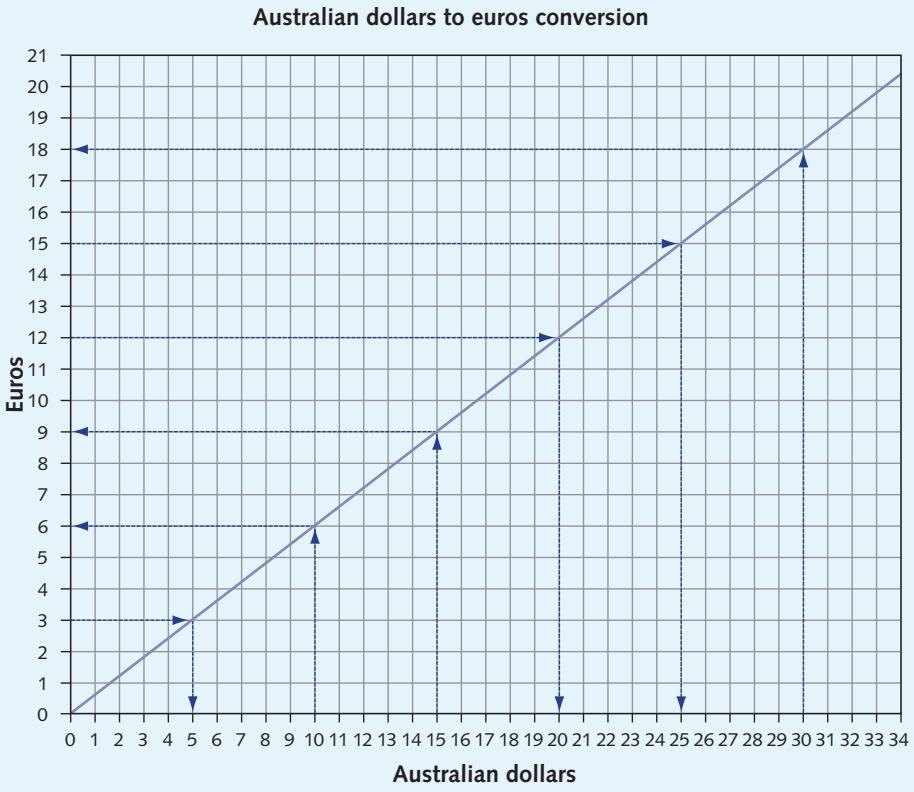
Use the conversion graph to:

**a** convert these amounts from Australian dollars to euros:

**i** A$10 **ii** A$15 **iii** A$30

**b** convert these amounts from euros to Australian dollars:

**i** €15 **ii** €3 **iii** €12




## Solution

**a** The approximate conversions from Australian dollars to euros are:

**i** A$10 ≈ €6 **ii** A$15 ≈ €9 **iii** A$30 ≈ €18

**b** The approximate conversions from euros to Australian dollars are:

**i** €15 ≈ A$25 **ii** €3 ≈ A$5 **iii** €12 ≈ A$20

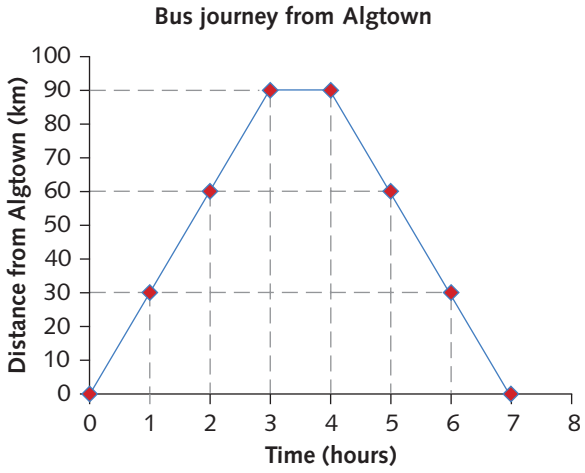
The pairs of arrows give the conversions.

## The travel graph

A **travel graph** is a graph that displays the distance travelled from a particular point at different times.

The graph shown on page 497 represents the journey of an old bus from Algtown to Gray Point. Gray Point is 90 km from Algtown. The bus travels for three hours at a constant speed to reach Gray Point, stays there for one hour and then returns to Algtown at the same constant speed.


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## Example 10

The graph below represents the journey of a bus from Trigtown to Picnic Point, then on to Tower Falls and finally back to Trigtown.

**a** How far is the bus from Trigtown after three hours?

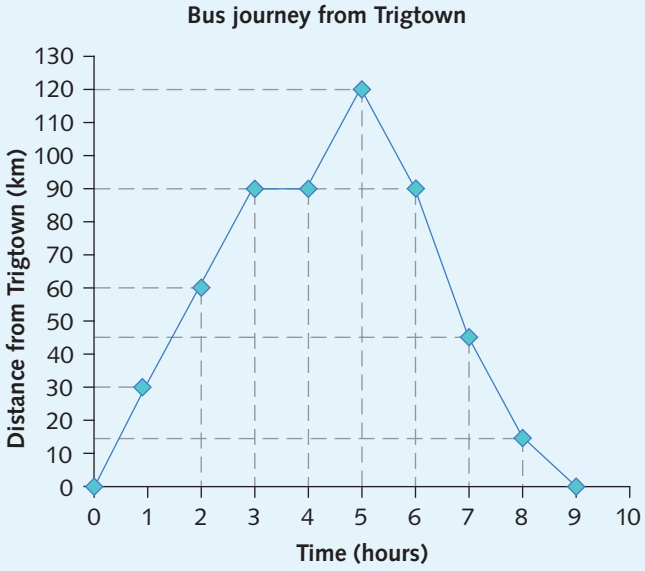
**b** What is the average speed of the bus for the first three hours?

**c** For how long does the bus stop at Picnic Point?

**d** For how long does the bus stop at Tower Falls, 120 km away from Trigtown?

**e** How far does the bus travel altogether?

**f** What is the average speed of the bus for the return journey from Tower Falls to Trigtown?




## Solution

**a** The bus is 90 km from Trigtown after three hours.

**b** Average speed $= \dfrac{\text{distance travelled}}{\text{time taken}}$

$= \dfrac{90}{3}$

$= 30$ km/h

The average speed of the bus for the first three hours is 30 km/h.

*(continued over page)*



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**c** After three hours of travelling, the bus stops at Picnic Point for one hour.

**d** The bus reaches Tower Falls after five hours. It turns around at Tower Falls and begins its return journey straight away.

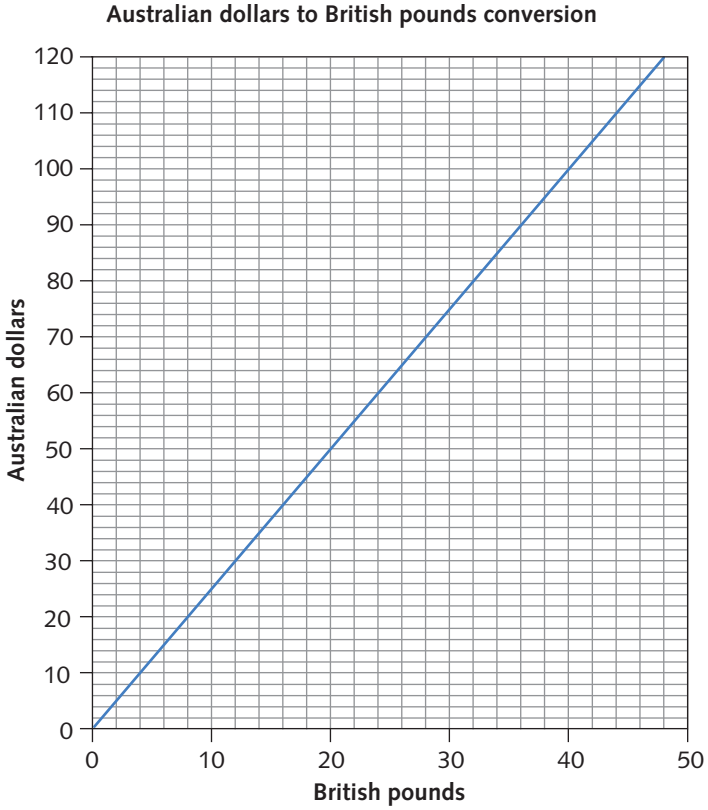
**e** The bus travels a total of 240 km.

**f** The average speed for the return journey is $\frac{120}{4} = 30$ km/h.

## Exercise 19F

Example 9

**1** The graph shown below was used to convert Australian dollars to British pounds and vice versa on a particular day. The graph is drawn for a conversion rate of A$1 = £0.40 or £1 = A$2.50.




**a** Use the conversion graph to convert the following amounts from Australian dollars to British pounds. Give approximate answers.

**i** A$10 **ii** A$20 **iii** A$40

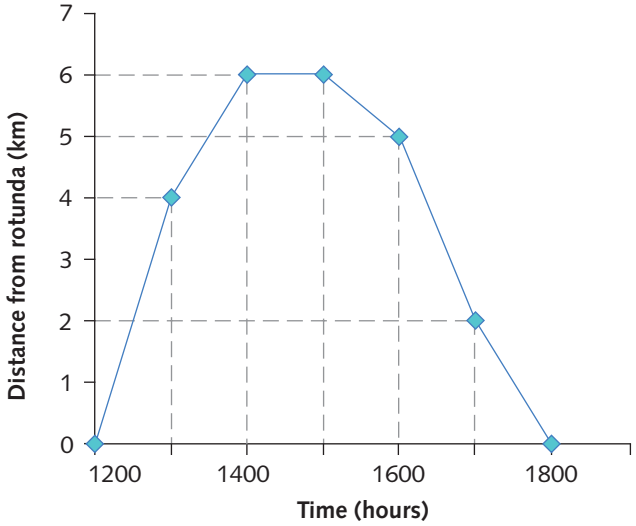
**b** Use the conversion graph to convert the following amounts from British pounds to Australian dollars. Give approximate answers.

**i** £10 **ii** £20 **iii** £32

Example 10

**2** The travel graph on the next page illustrates Sasha's journey during a walk along the Brisbane River. She leaves the rotunda in the Botanic Gardens at 1200 hours. The graph shows Sasha's distance from the rotunda for her walk.


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**a** How long did Sasha's walk take?

**b** How far is Sasha from the rotunda at the following times?

**i** 1300 hours **ii** 1400 hours **iii** 1700 hours

**c** When does she stop for a rest and for how long?

**d** What is the maximum distance from the rotunda that Sasha reaches?

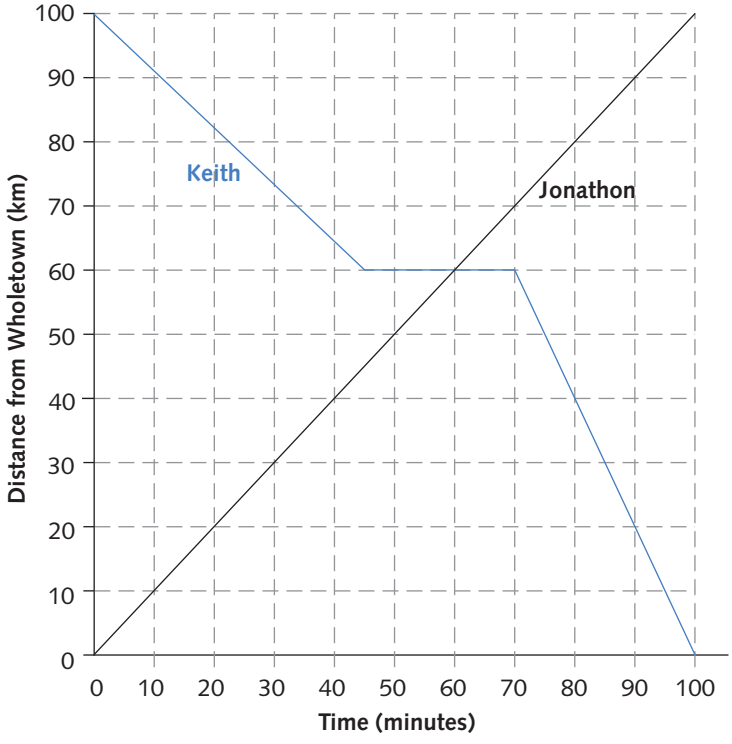
**e** What is the total distance walked by Sasha?

**f** What is Sasha's average speed for the 1-hour time interval from 1200 to 1300?

**g** What is Sasha's average speed for the 2-hour time interval from 1200 to 1400?

**h** What is Sasha's average speed for the 2-hour time interval from 1500 to 1700?

**3** Jonathon lives in Wholetown and Keith lives in Halftown. Wholetown is 100 km from Halftown. Jonathon leaves Wholetown at 1 p.m. and Keith leaves Halftown at the same time. They use the same road. The graphs show the journeys of Jonathon and Keith.





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**a** How far is Jonathon from Wholetown at 1:40 p.m.?

**b** How far is Keith from Wholetown at 1:40 p.m.?

**c** At what time does Jonathon pass Keith?

**d** For how long does Keith stop?

**e** What is Jonathon's average speed for the journey?

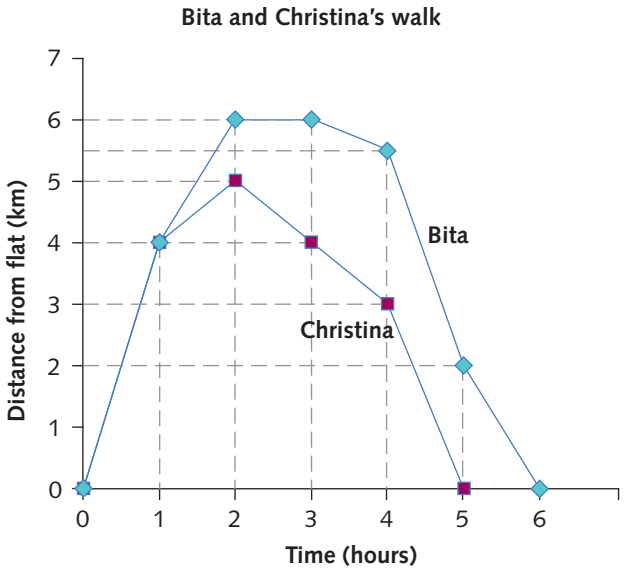
**f** What is Keith's average speed for:

**i** the first 45 minutes?

**ii** the last 30 minutes?

**iii** the whole journey (excluding his rest time)?

**4** Two friends start off together from their holiday flat to go for a long walk. They both walk along a pathway and return to the flat on the same pathway.




**a** For how long did the two friends walk together?

**b** What is the greatest distance along the pathway that Bita was from the flat?

**c** How long did Christina walk for?

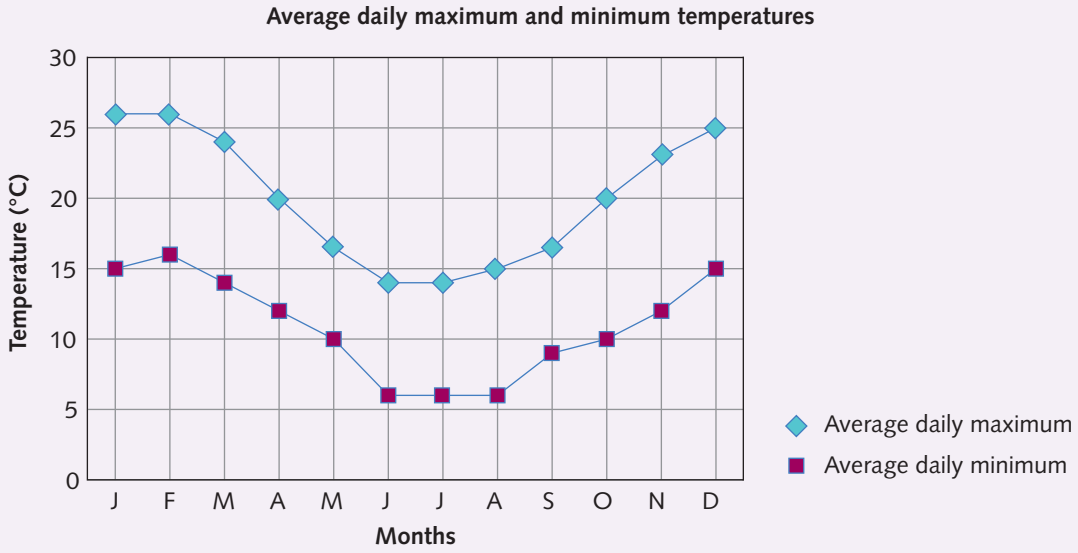
**d** What was the greatest distance apart of the two friends?

**e** What was Christina's average walking speed for the first 2 hours?


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# Review exercise

**1** The line graph below shows the average daily maximum and minimum temperatures for Melbourne.




**a** In which month(s) is the average daily maximum temperature the highest?

**b** In which month(s) is the average daily maximum temperature the lowest?

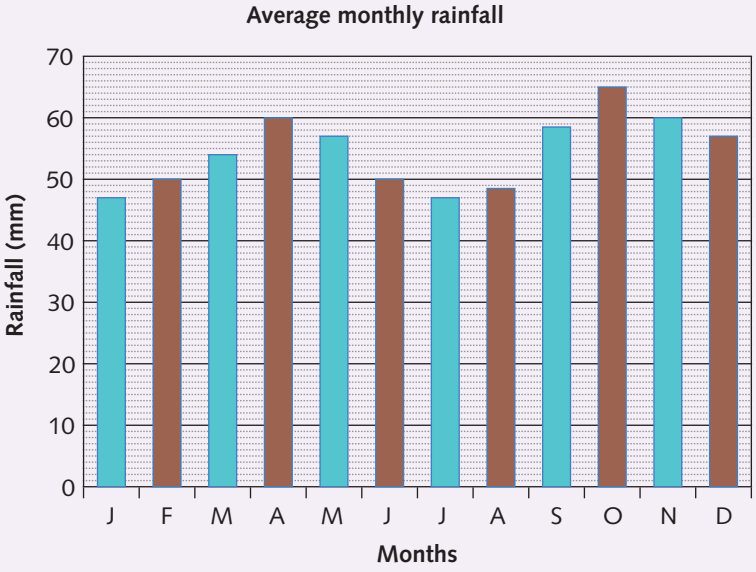
**c** In which month(s) is the average daily minimum temperature the highest?

**d** In which month(s) is the average daily minimum temperature the lowest?

**e** In which month(s) is the difference between average maximum and minimum temperature the greatest?

**f** In which month(s) is the difference between average maximum and minimum temperature the least?

**2** The column graph below shows the average monthly rainfall for Melbourne.





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**a** In which month(s) is the most rainfall recorded?

**b** In which month(s) is the least rainfall recorded?

**c** What is the total amount of rainfall recorded in the 12-month period?

**d** Which season is the wettest?

**e** Which season has the least rainfall?

**3** Five boys made paper gliders and were arguing about which would fly furthest, so they decided to have a competition. Adrian said his glider would come first and Jonathon's would come second. Lars said his would win, Khan's would come second and Rory's fifth. Khan thought his glider would come third, and that Jonathon's would be last. Jonathon said Rory's would come third and Adrian's fourth. Rory thought Lars's glider would be second-last.

**a** Use the above information to complete the table below.

**b** After the competition, it turned out that each boy had made at least one correct prediction. Determine the placings.

| | Prediction | | | | |
|---|---|---|---|---|---|
| **Boy** | **1st** | **2nd** | **3rd** | **4th** | **5th** |
| Adrian | Adrian | | | | |
| Jonathon | | | | | |
| Khan | | | | | |
| Lars | | | | | Rory |
| Rory | | | | | |




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**4** The table below shows the numbers of students enrolled in the various faculties of a university. There is a total of 2250 students.

| Faculty | Science | Medicine | Law | Arts | Commerce | Engineeering |
|---|---|---|---|---|---|---|
| Number of students | 225 | 125 | 54 | 815 | 620 | 411 |

**a i** How many of the students are not enrolled in Arts?

**ii** What percentage of students are studying Science?

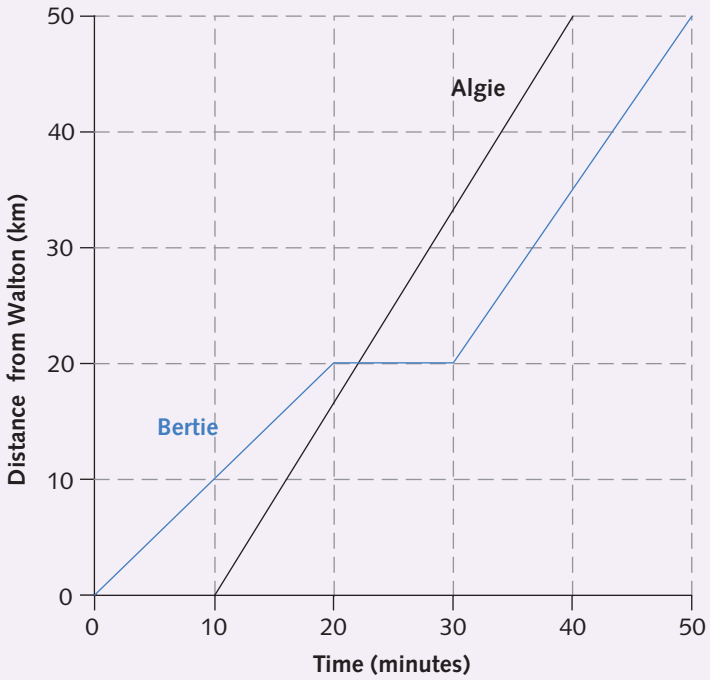
**b** Display the data using:

**i** a column graph

**ii** a divided bar chart

**iii** a pie chart

**5** The graph below shows the journey of two friends Algie and Bertie from the Post Office at Walton.




**a** What is the average speed in km/h of Bertie for:

**i** the first 20 minutes of his journey?

**ii** the last 20 minutes of his journey?

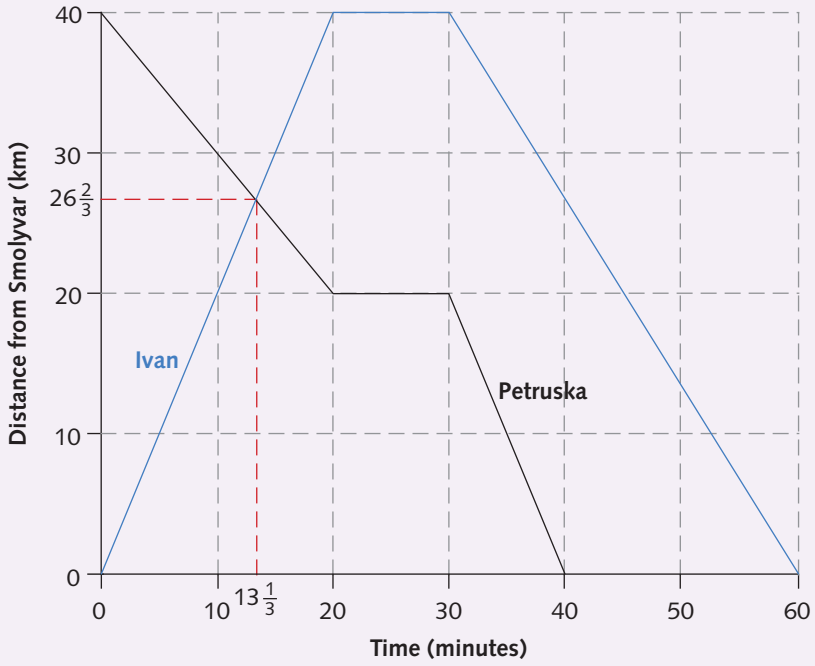
**b** When and where does Bertie stop and for how long?

**c** What is Algie's average speed in km/h?

**d** When and where does Algie pass Bertie?


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**6** The towns of Aral and Smolyvar are 40 km apart. The graphs show the journeys of Ivan and Petruska. Ivan starts from Smolyvar, travels to Aral and returns. Petruska travels from Aral to Smolyvar. They use the same route.




**a** What is the total distance travelled by Ivan?

**b** What is Petruska's average speed in km/h for the whole journey?

**c** How long does Ivan take to complete his journey?

**d** What is Ivan's average speed in km/h for the whole journey?

**e** How long does Ivan stay in Aral?

**f** When and where does Ivan pass Petruska?


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CHAPTER

# 20

Statistics and Probability

# Statistics

People deal with large amounts of information every day. When we read newspapers, watch television or open our mail, we may be looking at information that has been organised so that we can understand it easily. For example, we can tell how much water we use at home by looking at a water bill.

When we collect, organise, represent and analyse information, we are using **statistics**. The information collected is called **data**. A person who does this kind of work is called a **statistician**.

Analysing statistical data can help us understand more about a group of people, the habits of animals or changes in the weather over time. Statistical data help us make decisions and predict outcomes.

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# 20A Data and dot plots

If we wanted to discover whether green tree frogs in Queensland are surviving and are healthy, we might collect information about the numbers of frogs living in different areas, the weight of each frog and the length of their hind legs. The type of information collected would depend on the questions we wanted to answer.

The information we gather is called **data**. We may have many questions to answer, and so will need to gather data about different aspects of our population or sample. For example, here are some things that you might look at about a frog, if you were going to investigate the frogs in your area.

| | |
|---|---|
| **Gender** | Male, female |
| **Weight** | In grams |
| **Colour** | Green, brown, grey, gold |
| **Length of hind leg** | In millimetres |
| **Distance from water source** | In metres |
| **Age** | In months |

The data referred to in the table above are described using words, such as the name, gender and colour of the frogs, or numbers, such as the distance from water source, age, weight or hind-leg length of the frogs.

In previous years you may have learned about categorical data. **Categorical data** is where each observation falls into one of a number of distinct categories.

Some examples from above are:

- gender, where the categories are male and female
- the colour of the frog, where the categories are green, brown, grey and gold.

Count data can sometimes be considered as categorical data. A small number of different possible values (categories) is where this can be done. Examples of such count data include:

- the number of children in a family
- the number of people queuing at an ATM at a selected point of time
- the number of cars in a supermarket at a selected point of time
- the number of mobile phones owned by a family.

Measurement data are a type of numerical data. All measurement data need units of measurement and observations are recorded in the desired units of measurement.

Measurement data are recorded with a certain precision that depends on the measuring instrument, the choice of the investigator and practical restrictions.

Some examples of measurement data are:

- the weight in grams of a frog
- the length in millimetres of the hind leg of a frog
- the age in months of a frog.


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## Frequency tables

Categorical data are recorded with the number of observations that fall in each category. These are called the **frequencies** of the data – how often each category occurred. These frequencies are presented in a table and are graphed by a column graph in which each category has a column. The heights of the columns represent the frequency of the observations that fall in that category. Frequencies can be recorded in a **frequency table**.

In this section we use the same idea for **measurement** data.

When we collect data, we often use tally marks to organise the information we have collected. Tally marks help us keep track of values as they occur. We record how often a data value occurs by placing one tally mark against a value each time it occurs in our list. We write the tally marks next to one another until we get to four, as shown below.

| means 1 || means 2 ||| means 3 |||| means 4

To show a fifth tally mark, we cross through the four to show a group of five.

𝍸 means 5

The number of tally marks indicates how many times a value is repeated. The number of times a value is repeated is called the frequency of that value, and can be recorded in a frequency table.

### Example 1

The neck measurement of 22 people was taken. The measurements were taken to the nearest centimetre. Record the results in a frequency table.

39, 39, 37, 39, 39, 39, 35, 37, 35, 35, 39, 39, 39, 37, 40, 35, 35, 35, 41, 37, 39, 39

### Solution

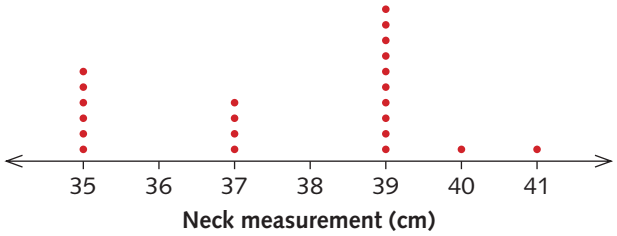
| Neck measurement (cm) | Tally | Frequency |
|---|---|---|
| 35 | \| 𝍸 | 6 |
| 36 | | 0 |
| 37 | |||| | 4 |
| 38 | | 0 |
| 39 | 𝍸 𝍸 | 10 |
| 40 | \| | 1 |
| 41 | \| | 1 |

## Dot plots

A **dot plot** plots numerical data, with a dot on the graph for each value in the set of data. If a value occurs three times, there are three dots in a line above that value. Hence, the heights of the columns of dots give the frequencies of the data values. For numerical data, a dot plot is a plot of the 'raw' data with no grouping of values. So if a set of numerical values is not large and has many different values, a dot plot is not very informative, but a dot plot is usually an excellent way to have a first look at data.


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The data given in Example 1 can be illustrated by a dot plot.



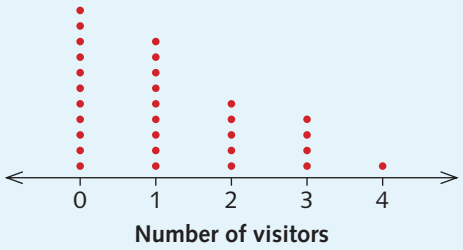

### Example 2

A group of 30 people in the building trade were asked how many times in the last week they had visited a particular hardware shop. Their responses were as follows:

0, 2, 2, 1, 0, 0, 3, 4, 1, 1, 0, 0, 0, 3, 1, 1, 1, 2, 3, 0, 0, 1, 1, 2, 2, 1, 3, 0, 0, 0

Draw a dot plot for the data.

### Solution




## Exercise 20A

Example 1, 2

**1** The local vet weighed each dog brought into the practice for one week. The tallies of the dogs with each of the different weights are shown in the table below.

| Weight (correct to nearest kg) | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Tally | I | | III | I | IIII | 𝍸 𝍸 | III | III | 𝍸 | 𝍸 | | III |
| Frequency | | | | | | | | | | | | |

**a** Complete the table.

**b** How many dogs were brought into the vet during the week?

**c** Draw a dot plot for the data.


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**2** Brad's golf scores over the past 20 weeks are as follows:

82, 79, 84, 82, 81, 79, 78, 72, 79, 79, 77, 88, 79, 81, 79, 79, 84, 78, 72, 72

**a** Complete a frequency table for the data.

**b** What was Brad's highest golf score?

**c** Which golf score was most frequent?

**d** Draw a dot plot for the data.

**3** A zoologist counted organisms living in droplets of water. The data are as follows:

21, 23, 24, 23, 21, 25, 34, 32, 21, 23, 34, 23, 22, 24, 25, 25, 26, 21, 24, 21, 20, 23, 27

**a** Complete a frequency table for the data.

**b** Draw a dot plot for the data.

**4** **[Class activity]** Record the length of the handspan of each member of your class, correct to the nearest centimetre.

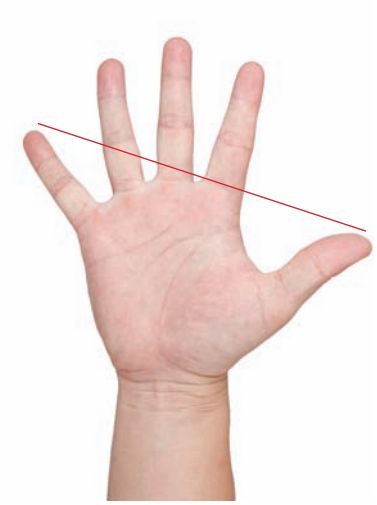
**a** Draw up a frequency table of the results.

**b** What is the shortest length?

**c** What is the longest length?

**d** What is the most commonly occurring length of handspan in your class?

**e** Draw a dot plot for the data.



# 20B Mode

One of the questions we often use statistics to answer is 'Which is the most popular?' The most popular (or common or favourite) value will be the most frequently occurring value in a data set. A value with the highest frequency is called the **mode**.

For example, in a survey of eye colour, the following results were obtained:

green, blue, blue, hazel, green, brown,

blue, green, brown, blue, brown, blue,

hazel, green, green, brown, brown, blue,

green, brown, green, green



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We can arrange the data into a frequency table.

| Eye colour | Tally | Frequency |
|---|---|---|
| Blue | 𝍸 I | 6 |
| Brown | 𝍸 I | 6 |
| Green | 𝍸 III | 8 |
| Hazel | II | 2 |

In the survey above, the eye colour with the highest frequency is green. Hence, the mode is green.

*Mode* is a French word for 'fashion'. It may help you to remember that the mode is the most fashionable (or popular) value.

## Example 3

The temperature at 3:30 p.m. on summer afternoon was recorded, to the nearest degree, at a number of secondary schools. The number of schools with each of the temperatures recorded are shown in the table below.

| Temperature (°C) | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Tally | III | 𝍸 III | 𝍸 𝍸 IIII | 𝍸 III | 𝍸 IIII | 𝍸 III | 𝍸 𝍸 III | 𝍸 𝍸 𝍸 III | 𝍸 𝍸 𝍸 𝍸 II | 𝍸 𝍸 | III | II |
| Frequency | | | | | | | | | | | | |

**a** Complete the table, filling in the frequencies of the temperatures.

**b** What is the mode?

**c** How many schools were involved?

### Solution

**a**

| Temperature (°C) | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Tally | III | 𝍸 III | 𝍸 𝍸 IIII | 𝍸 III | 𝍸 IIII | 𝍸 III | 𝍸 𝍸 III | 𝍸 𝍸 𝍸 III | 𝍸 𝍸 𝍸 𝍸 II | 𝍸 𝍸 | III | II |
| Frequency | 3 | 8 | 14 | 8 | 9 | 8 | 13 | 18 | 22 | 10 | 3 | 2 |

**b** The highest number of schools (22 schools) recorded a temperature of 39°C so the mode is 39°C.

**c** 118 schools recorded the temperature at 3:30 p.m. on that day.

Sometimes there is more than one mode. Data with two modes are called **bimodal** data. Data with more than two modes are called **multimodal** data.


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## Mode

The **mode** is the value (or values) with the highest frequency (or frequencies).

### Example 4

The ages of a sample of television newsreaders were recorded as follows.

41, 42, 48, 45, 51, 41, 38, 40, 48, 48, 52, 47, 46, 46,
46, 48, 42, 43, 51, 51, 52, 48, 46, 45, 42, 49, 48

**a** Complete a frequency table for the data.

**b** What is the mode? What does this mean in terms of the data?

**c** How many newsreaders are there in this sample?

**d** Draw a dot plot for the data.

### Solution

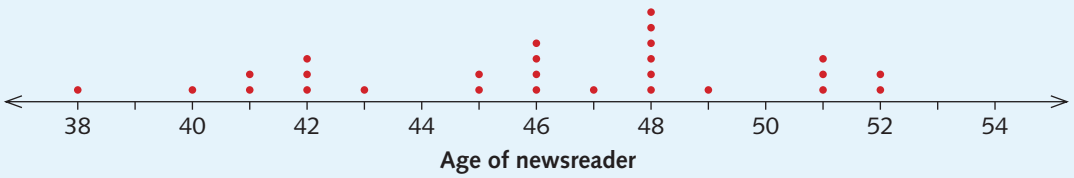
**a**

| Age (years) | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Tally | I | | I | II | III | I | | II | IIII | I | IIII | I | | III | II |
| Frequency | 1 | | 1 | 2 | 3 | 1 | | 2 | 4 | 1 | 6 | 1 | | 3 | 2 |

**b** The mode is 48. This means that more newsreaders are 48 years old than any other age.

**c** There are 27 newsreaders in the sample.

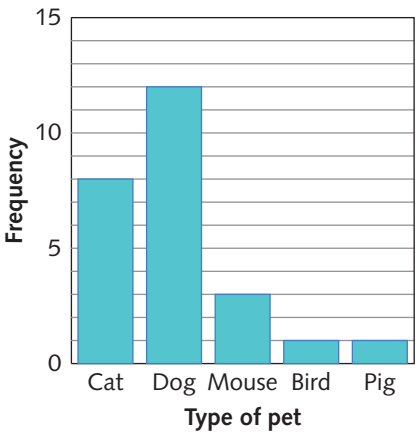
**d**




## Exercise 20B

Example 3

**1** A bar chart for the pets owned by a sample of kindergarten children is shown on the right.




**a** Draw a frequency table for the data.

**b** What is the mode?



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Example 4

**2** Chairs in each classroom of a school were counted. The result was:
23, 26, 23, 24, 28, 25, 26, 28, 29, 26, 14, 28, 25, 24, 28, 26,
24, 29, 24, 25, 28, 23, 33, 21, 25, 27, 23, 28, 28, 24, 25, 23.

**a** Draw a frequency table for the data.

**b** What is the mode?

**c** Draw a dot plot for the data.

**3** The following 21 values are the measurements (in centimetres) of lengths of wire.

371, 371, 371, 321, 321, 379, 379,
367, 311, 311, 311, 405, 405, 353,
353, 321, 367, 367, 387, 361, 371

Draw a dot plot.

# 20C Stem-and-leaf plots

We can arrange the data into a **stem-and-leaf plot**. For example, represented as follows.

| | Stem | Leaves |
|---|---|---|
| 71 is represented by | 7 | 1 |
| 68 is represented by | 6 | 8 |
| 54, 57 and 59 are represented by | 5 | 479 |

A number of shoppers were asked to record their ages as they left a department store. The ordered data are as shown below.

35, 36, 36, 36, 36, 36, 36, 38, 40, 40, 40, 40, 40, 40, 40, 41, 41, 42, 42, 44, 44, 44,
44, 45, 45, 46, 46, 46, 47, 47, 47, 47, 47, 48, 48, 48, 48, 48, 49, 49, 49, 49, 50, 50,
50, 50, 51, 51, 51, 52, 52, 53, 54, 54

The stem-and-leaf plot of the ages of shoppers is shown below. We can see that all the data in the group aged 50 to 54 have 5 as their first digit, so 5 becomes the stem for this group. The values are placed in order from smallest to largest. We can see each entry for this data set.

| Stem | Leaves |
|---|---|
| 3 | 56666668 |
| 4 | 00000001122444455666777778888899 99 |
| 5 | 000011122344 |

The stem-and-leaf plot makes it easy to see that most of the shoppers were aged between 40 and 49 years.


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## Stem-and-leaf plots

- **Stem-and-leaf plots** represent each value in a data set in the form of a leaf and a stem.
- The **leaf** is usually the last digit of the value, and the **stem** is the first digit of a two-digit number, or the first two digits of a three-digit number, and so on.

  For example, for the number 47, 4 is the stem and 7 is the leaf.

  For the number 251, 25 is the stem and 1 is the leaf.

### Example 5

Steve kept a record of the number of emails he received at work each day during the month of July. His data are shown below.

124, 112, 123, 145, 123, 109, 114, 94, 98, 144, 112, 101, 127, 147, 149, 142, 127, 122, 110, 140, 107, 112, 118, 114, 149, 105, 120, 122, 123, 92, 91

**a** Draw a stem-and-leaf plot of the data.

**b** What was the mode for these data?

### Solution

**a**

| Stem | Leaves |
|---|---|
| 9 | 1248 |
| 10 | 1579 |
| 11 | 0222448 |
| 12 | 022333477 |
| 13 | |
| 14 | 0245799 |

**b** There are two modes for these data. The modes are 112 and 123. Steve received 112 emails per day on three of the days in July and 123 emails per day on another three days in July.

## Exercise 20C

Example 5

**1** A survey of the prices of jeans was conducted at a shopping centre and the following prices were observed.

$78, $45, $68, $56, $39, $87, $85, $78, $99, $73, $49, $89, $87, $98, $99, $49, $67, $78

**a** Draw a stem-and-leaf plot for the data.

**b** What were the lowest and highest prices for jeans recorded?

**c** What is the mode?



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**2** James and Talis drew the stem-and-leaf plot shown below for the numbers of people travelling in buses from their suburb to the central train station over the course of a day.

| Stem | Leaves |
|---|---|
| 0 | 38 |
| 1 | 559 |
| 2 | 0223 |
| 3 | 02235888 |
| 4 | 0138 |
| 5 | 233 |

**a** How many buses did they see?

**b** What is the mode for this data set?

**3** Marianthi keeps a record of the number of text messages she sent from her mobile phone each day. Her records for one month one shown below.

103, 97, 102, 110, 101, 86, 98, 78, 79, 113, 99, 81, 100, 120, 114, 118,
101, 103, 92, 110, 86, 97, 94, 99, 112, 88, 102, 101, 103, 76

**a** Draw a stem-and-leaf plot of the data.

**b** What was the mode for these data?

# 20D Median, mean and range

## Median

The **median** is the 'middle value' when all values are arranged in order of size.

Here are some numbers in order of size:

2, 2, 3, 3, 3, 4, 5, 11, 13, 18, 18, 19, 21

This data set has an odd number of values. The middle value is 5, since it has the same number of values on either side of it. Hence, the median of this data set is 5.

Here are some more numbers:

1, 3, 4, 4, 5, 6, 8, 11, 13, 13, 19, 21

This data set has an even number of values. The middle values are 6 and 8. We take the average of 6 and 8 to calculate the median.

$$\text{Median} = \frac{6+8}{2}$$
$$= 7$$

Hence, the median of this data set is 7 even though it does not occur in the data set.


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### Median

- When the data set has an **odd** number of values, the **median** is the **middle value**.
- When the number of values is **even**, the median is the **average of the two middle values**.

#### Example 6

Students measured their heights to the nearest centimetre, and recorded the results shown below.

164, 168, 167, 158, 164, 154, 170, 175, 164, 168

Calculate the median.

**Solution**

Arrange the data in order.

$$\underset{\downarrow}{}$$

154, 158, 164, 164, 164, 167, 168, 168, 170, 175

The median lies between 164 and 167, so we need to take the average of these two values.

$$\text{Median} = \frac{164 + 167}{2}$$
$$= 165.5$$

## Mean

You may have already heard of the **mean** and know that it is commonly called the average. To calculate the mean, we find the *sum of the values* and divide this by the *number of values*.

### Mean

$$\text{Mean} = \frac{\text{sum of values}}{\text{number of values}}$$

#### Example 7

Students measured their heights to the nearest centimetre, and recorded the results shown below.

164, 168, 167, 158, 164, 154, 170, 175, 164, 168

Calculate the mean.


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### Solution

The mean is found by dividing the sum of the values by the number of values in the data set.

$$\text{Mean} = \frac{164 + 168 + 167 + 158 + 164 + 154 + 170 + 175 + 164 + 168}{10}$$
$$= \frac{1652}{10}$$
$$= 165.2\,\text{cm}$$

The difference between the mean and median can make quite an impact. Consider the following examples of the mean and median of house prices. You might think that you could not afford to buy a house in this suburb, based on the mean. This is because the mean is affected by the two extremely high prices. The median gives a clearer picture of what the 'average' house in this suburb might cost.

### Example 8

Listed below are some house prices achieved at auction last weekend.

\$320 000, \$299 000, \$308 000, \$335 000, \$1 005 000, \$325 000, \$985 000

**a** Calculate the average house price.

**b** Calculate the mean, excluding the two extremely high prices.

**c** Calculate the median house price.

### Solution

**a** $$\text{Mean} = \frac{\text{sum of values}}{\text{number of values}}$$
$$= \frac{320\,000 + 299\,000 + 308\,000 + 335\,000 + 1\,005\,000 + 325\,000 + 985\,000}{7}$$
$$= \frac{3\,577\,000}{7}$$
$$= \$511\,000$$

**b** $$\text{Mean} = \frac{320\,000 + 299\,000 + 308\,000 + 335\,000 + 325\,000}{5}$$
$$= \frac{1\,587\,000}{5}$$
$$= \$317\,400$$

**c** Arrange the values in order. The median is the middle value.

299 000, 308 000, 320 000, 325 000, 335 000, 985 000, 1 005 000

The median is \$325 000.


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## Using the frequency table to calculate the mean

The frequency table for a data set can be used to calculate the mean.

To calculate the mean, we:

- multiply each value by its frequency (this gives the total contribution of that value)
- add up all the products calculated in the previous step (this gives the sum of the values)
- divide the total by the number of values.

For example, the number of birds' nests in each tree in a park was recorded.

| Number of nests | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Frequency | 4 | 0 | 6 | 4 | 2 | 3 | 2 | 1 |

$$\begin{aligned}\text{Mean} &= \frac{\text{sum of values}}{\text{number of values}} \\ &= \frac{(1 \times 4) + (2 \times 0) + (3 \times 6) + (4 \times 4) + (5 \times 2) + (6 \times 3) + (7 \times 2) + (8 \times 1)}{22} \\ &= \frac{4 + 0 + 18 + 16 + 10 + 18 + 14 + 8}{22} \\ &= \frac{88}{22} \\ &= 4\end{aligned}$$

The mean of the number of nests in each tree is 4.

### Measures of central tendency

- The **median** is the 'middle value' when all values are arranged in order of size.
- The **mean** of a data set is found by taking the sum of the values and dividing the result by the number of values.

### Example 9

Barry the bricklayer laid the following numbers of bricks over seven days.

| Sunday | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
|---|---|---|---|---|---|---|
| 350 | 347 | 337 | 391 | 43 | 301 | 394 |

**a** Calculate the median number of bricks Barry laid each day for the week.

**b** Calculate the mean number of bricks he laid per day.

**c** Comment on the results. What number of bricks should Barry tell people he is capable of laying in one day?



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**Solution**

**a** Order the results below.

43, 301, 337, 347, 350, 391, 394

The median is the middle value.

Median = 347

**b** $\text{Mean} = \dfrac{\text{sum of values}}{\text{number of values}}$

$$= \frac{350 + 347 + 337 + 391 + 43 + 301 + 394}{7}$$

$$= \frac{2163}{7}$$

$$= 309$$

**c** The mean is affected by the very low number for Thursday. (Barry may have gone home.) Hence a number close to the median, say 350, would be a reasonable claim for the number of bricks he can lay in a day.

**Example 10**

Use the frequency table of Thea's netball scores below to calculate the mean of her scores.

| Points scored | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|
| Frequency | 2 | 2 | 7 | 10 | 1 | 3 |

**Solution**

$\text{Mean} = \dfrac{\text{sum of values}}{\text{number of values}}$

$$= \frac{(8 \times 2) + (9 \times 2) + (10 \times 7) + (11 \times 10) + (12 \times 1) + (13 \times 3)}{25}$$

$$= \frac{16 + 18 + 70 + 110 + 12 + 39}{25}$$

$$= \frac{265}{25}$$

$$= 10.6$$

This tells us that the mean of Thea's scores over the 25 games is 10.6 points. (The median score is 11.)

## Range

The word **range** is used in statistics in the same way as everyday speech as a verb, e.g. 'the values of the observations range from 8 to 13'. But as a noun it is used to give the distance between the smallest and the largest observations. In the example above, the range of the data is $13 - 8 = 5$.

The range of the data gives a summary of how spread out the observations are. Another person's scores might range from 5 to 14, so the range of the data for that person is 9.


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## Exercise 20D

Example 6, 7

**1** The winning margins (in metres) of the Onslow Cup winners over the last 12 years are shown below.

7, 6, 4, 7, 4, 4, 7, 8, 9, 8, 6, 2

**a** Calculate the median. **b** Calculate the mean. **c** Calculate the range.

**2** The weights of a group of students, in kilograms, are given below.

45, 44, 43, 44, 40, 39, 43, 39, 47, 44, 40, 41

**a** Find the mode.

**b** What is the median?

**c** Calculate the mean, correct to two decimal places.

**d** Calculate the range.

**3** The mean price of bags of potatoes at different supermarkets was $4.50. The sum of the data was $90.00. How many bags of potatoes were included in the survey?

Example 9

**4** Sue spent the following amounts on her lunch each day over the course of two working weeks.

$12, $4, $10, $42.50, $7.50, $10.50, $6, $8, $6.50, $7.50

**a** Calculate the median. **b** Calculate the mean.

**c** Compare the median and mean and comment on which is the better indicator of how much Sue usually spent on her daily lunch.

**d** Calculate the range.

Example 10

**5** Use the frequency table below to calculate the mean of the ages of Melbourne Cup winners from the last 12 years.

| Age of horse | Frequency |
|---|---|
| 4-year-old | 4 |
| 5-year-old | 4 |
| 6-year-old | 3 |
| 7-year-old | 1 |

**6** Twenty students took a maths test. Below are their results.

9, 1, 7, 8, 10, 6, 5, 2, 3, 7,
1, 7, 7, 7, 9, 7, 4, 2, 10, 8

**a** Prepare a frequency table of the scores.

**b** Using the frequency table, calculate the mean, median and mode.

**c** Calculate the range.

**7** Construct a stem-and-leaf plot for the data below, then highlight the median.

31, 21, 41, 54, 27, 53, 23, 41, 20, 33, 36, 38,
32, 30, 31, 42, 27, 38, 52, 36, 34, 53, 48

**8** Five positive whole numbers have a mean of 4, a median of 5 and a mode of 6. Find all five numbers.


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# Review exercise

**1** The heights of 42 students were measured, correct to the nearest centimetre. The results are shown below.

101, 156, 143, 98, 116, 99, 112, 121, 96, 145, 167, 171, 113, 108, 109,
139, 99, 99, 100, 103, 105, 115, 104, 102, 98, 99, 113, 112, 152,
144, 147, 138, 132, 155, 102, 122, 119, 142, 167, 101, 100, 108

**a** Construct a stem-and-leaf plot. **b** Find the median.

**c** Find the range.

**2** Over a 22-day period, the numbers of employee absences from work were recorded.

1, 2, 0, 0, 1, 2, 2, 2, 1, 0, 0, 0, 4, 0, 1, 1, 3, 0, 3, 3, 0, 1

Draw a dot plot representing this information.

**3** Find the mean, median and range of each of the following sets of data.

**a** 34, 45, 67, 89, 45, 56, 34, 21

**b** 11, 23, 24, 45, 61, 43, 21, 56, 78, 562

**4** The following table gives the salaries of 16 employees in a small company.

| Job | Number having job | Annual salary ($) |
|---|---|---|
| Manager | 1 | 300 000 |
| Deputy manager | 2 | 220 000 |
| Supervisor | 2 | 115 000 |
| Sales representative | 4 | 75 000 |
| Warehouse worker | 4 | 56 000 |
| Clerical worker | 3 | 52 000 |

**a** Find the median salary. **b** Find the mean salary.

**c** Find the range of the salaries.

**5** Find the mean and median of the data given in the stem-and-leaf plot.

| Stem | Leaves |
|---|---|
| 1 | 2 |
| 2 | 4 7 |
| 3 | 2 4 |
| 4 | 2 2 |
| 5 | 2 2 8 8 |
| 6 | 3 4 4 5 5 7 8 9 |


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**6** The number of hours lasted by 25 light bulbs of a particular brand was recorded and entered in stem-and-leaf plot as shown.

| Stem | Leaves |
|---|---|
| 3 | 86 90 |
| 4 | 20 21 26 |
| 5 | 72 |
| 6 | 30 43 46 71 82 90 |
| 7 | 22 24 26 30 63 64 77 82 |
| 8 | 12 40 64 |
| 9 | 12 33 |

(Here 7|24 means 724.)

**a** How many light bulbs lasted between 500 and 700 hours?

**b** What was the median time?

**7** The lasting times of another brand of light bulb is recorded here. There are 25 light bulbs involved in the study.

506 556 561 598 630 721 734 763 765 772 778
790 882 824 846 843 853 852 864 865 910 912
914 935 958

**a** Construct a stem-and-leaf plot.

**b** How many light bulbs lasted between 500 and 700 hours?

**c** What was the median time?

**d** What was the range?

**8** The mean of the numbers 11, 17, 18, 22, 38, $x$ is 24.

**a** What is the value of $x$?

**b** What is the median?

**9** The range of a set of data is 56. The minimum value is 22. The data in order from smallest to largest is 22, 45, 50, 52, 58, $x$, 67, 71, $y$.

**a** Find the maximum value.

**b** Find the median.

**c** The mean is 56.5. Find the value of $x$.


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CHAPTER

# 21

Measurement and Geometry

# Polyhedra and three-dimensional drawing

In the real world we have to deal with three-dimensional objects. When we represent these objects in drawings, we draw on paper and this allows us to work only in two dimensions. This is a significant challenge, as real objects can be viewed from many different directions and look quite different from each direction. If a viewer looks at a three-dimensional object from three different directions, an understanding of the shape of the object is achieved. A drawing on paper, on the other hand, can represent this object only as viewed from one particular direction. No matter how much we move to view the drawing from different directions, it does not change our understanding of the drawing.

The difficulty of drawing three-dimensional objects is exemplified by the fact that it is only about 500 years ago that painters first mastered the techniques of perspective that made their paintings in the real world look three dimensional.

Drawing simple three-dimensional shapes is an important skill for a student of mathematics. In this chapter we consider some basic techniques and apply them to some three-dimensional geometric shapes.

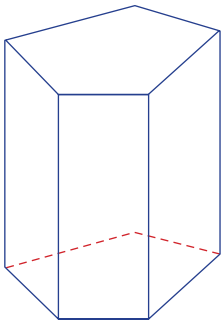


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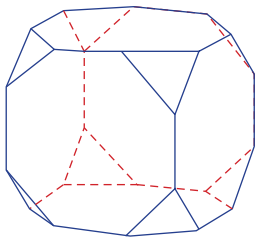
# 21A Polyhedra

A **polyhedron** is a solid where the outside surface is made up wholly of flat panels called **faces** with simple polygonal shapes and is not pierced by holes. We note that the plural of polyhedron is **polyhedra** or **polyhedrons**. A **polygonal shape** is a closed plane shape with a border made up of intervals. Triangles and quadrilaterals are examples of polygonal shapes. So, solids like spheres, tubes and living things are not polyhedra, but there are many manufactured solids such as boxes, houses and books that are very close to being ideal mathematical polyhedra.

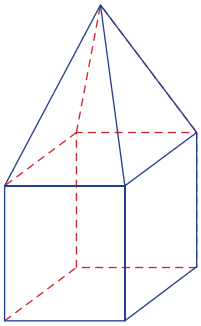
Here are three polyhedra. The two to the left are named and the third described.



Pentagonal prism



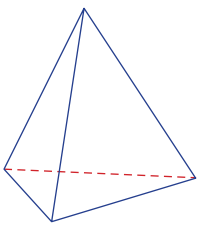
Truncated cube



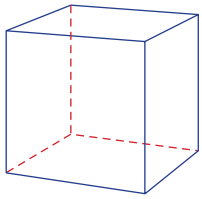
Cube with square pyramid on top of it

## Platonic solids

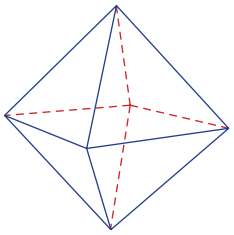
We recall from Chapter 18 that a regular polygon is a plane figure with sides that are intervals of the same length. A **regular polyhedron** is a three-dimensional solid with faces that are all identical regular polygons and with the same number of edges meeting at each vertex. There are only five such solids and they are shown below. They are also known as the **Platonic solids**. They have many interesting properties.



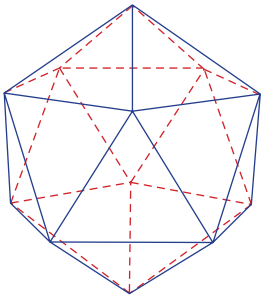
Tetrahedron



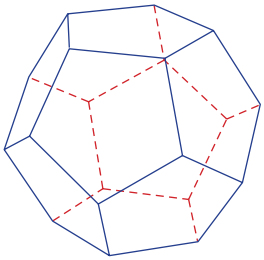
Cube



Octahedron



Icosahedron



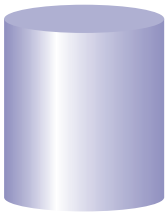
Dodecahedron


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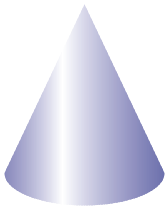
The following are not polyhedrons since their faces are not polygons.



Sphere



Cylinder



Cone

### Names for parts of a polyhedron

Each polygon forming the surface of a polyhedron is called a **face**.

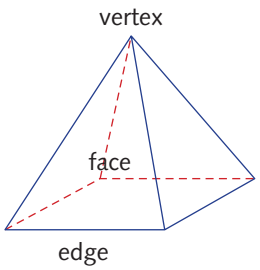
The pyramid shown opposite has five faces.

Two faces of a polyhedron meet along a line called an **edge**.

The pyramid shown opposite has eight edges.

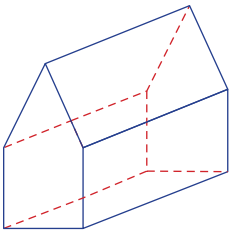
The point at which three or more edges meet is called a **vertex**.

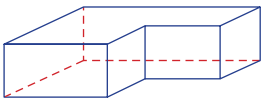
The pyramid shown opposite has five vertices.




## Prisms

A **prism** is a polyhedron that has two identical and parallel faces and all of its remaining faces are parallelograms. The following are examples of prisms.





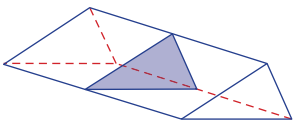
### Cross-section of a prism

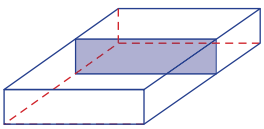
A **cross-section** of a prism is the polygon produced by intersecting the prism by a plane parallel to the base.

If the intersecting plane is parallel to the planes containing the identical faces, the cross-section will be identical to the faces at the ends and so, a prism has a **uniform cross-section**.

For example, for the prism shown to the left below, the cross-section is always a triangle that is indentical to the triangle that is the base. The figure is called a **right-triangular prism**.

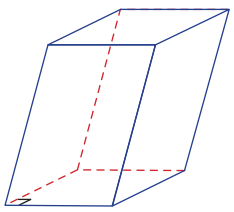
The figure to the right is called a **right-rectangular prism**.





### Oblique prisms

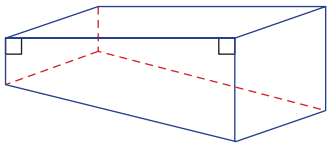
The figure shown to the right is an **oblique rectangular prism**.

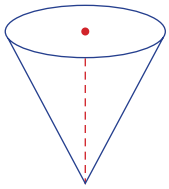


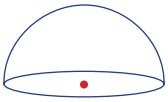

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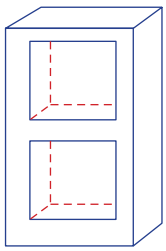
## Exercise 21A

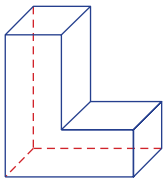
**1** Determine which of the following shapes are polyhedra.

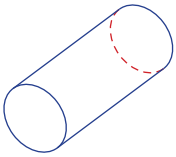
**a**


**b**


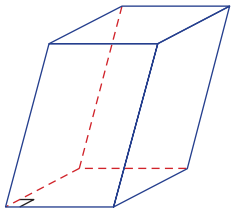
**c**


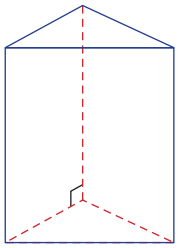
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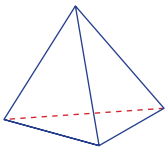
**e**


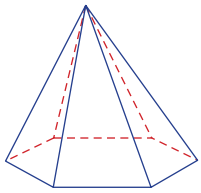
**f**


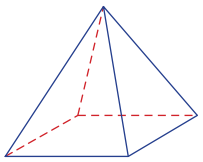
**2** Find the number of faces, edges and vertices of the following solids.

**a**


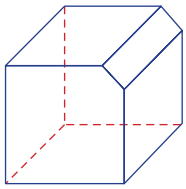
**b**


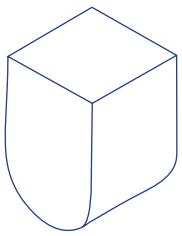
**c**


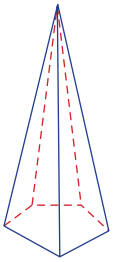
**d**


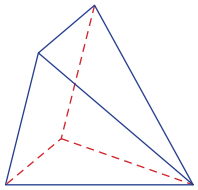
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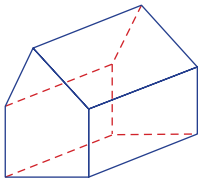
**3** Which of the following are prisms?

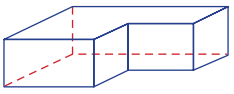
**a**


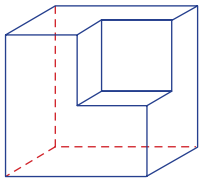
**b**


**c**


**d**


**e**


**f**


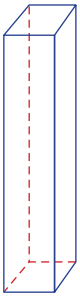
**g**



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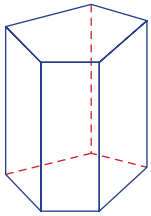
**4** **a** Complete the following table for prisms with the given cross-sections.



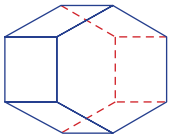
Triangular prism



Rectangular prism



Pentagonal prism



Hexagonal prism

| Type of cross-section | Number of faces ($F$) | Number of edges ($E$) | Number of vertices ($V$) |
|---|---|---|---|
| Triangular | | | |
| Rectangular | | | |
| Pentagonal | | | |
| Hexagonal | | | |

**b** For each of the above, evaluate $V - E + F$

**c** If the polygonal cross-section of a prism has $n$ sides, find in terms of $n$:

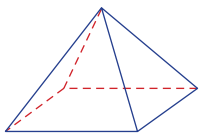
**i** the number of faces ($F$)

**ii** the number of edges ($E$)

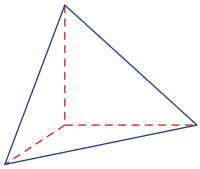
**iii** the number of vertices ($V$)

**d** Find $V - E + F$ using the results of parts **i**, **ii** and **iii**.

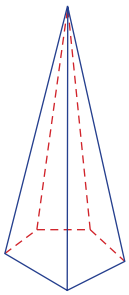
**5** If a pyramid has a triangular base it is called a **triangular pyramid**. If it has a pentagonal base it is called a pentagonal pyramid. Several pyramids are illustrated below.



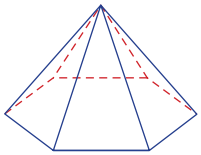
Triangular pyramid



Rectangular pyramid



Pentagonal pyramid



Hexagonal pyramid

**a** Complete the table.

| Type of pyramid | Number of faces ($F$) | Number of edges ($E$) | Number of vertices ($V$) |
|---|---|---|---|
| Triangular | | | |
| Rectangular | | | |
| Pentagonal | | | |
| Hexagonal | | | |

**b** For each of the above evaluate $V - E + F$.


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**c** If the polygonal base has $n$ sides, find in terms of $n$:

**i** the number of faces ($F$)

**ii** the number of edges ($E$)

**iii** the number of vertices ($V$)

**d** Find $V - E + F$ using the results of parts **i**, **ii** and **iii**.

**6** **a** Complete the following table. Diagrams of the platonic solids are on page 523. It is a good idea to have models of these solids when doing this question.

| Platonic solid | Number of faces ($F$) | Number of edges ($E$) | Number of vertices ($V$) |
|---|---|---|---|
| Cube | | | |
| Tetrahedron | | | |
| Icosahedron | | | |
| Octahedron | | | |
| Dodecahedron | | | |

**b** For each of the above evaluate $V - E + F$.

In Questions **4**, **5** and **6** you calculated $V - E + F$. You should have got 2 in each case.
The formula $V - E + F = 2$ is known as **Euler's formula**.

# 21B Drawing a solid

## Views or elevations

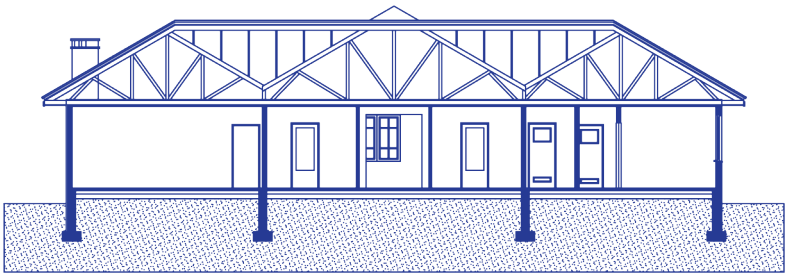
The most basic drawings of a solid are done according to the following rules.

The **front view** (or **front elevation**) outlines what is seen as you stand directly in front of the solid.
The **rear view** (or **rear elevation**) outlines what is seen as you stand directly behind the solid.

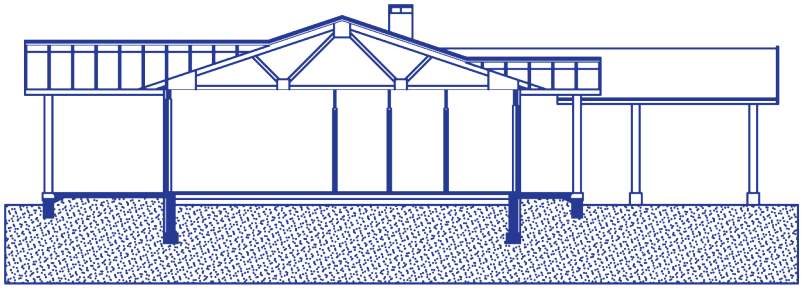
A **side view** (or **side elevation**) outlines what is seen as you look at the solid from a side. This line of view should be at right angles to the front view.

The **plan view** outlines what is seen when the solid is observed from directly above.

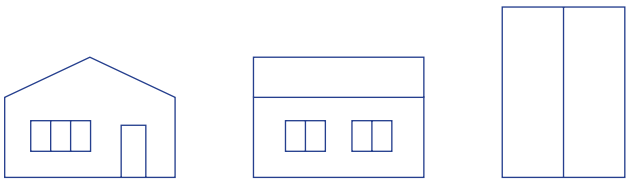
House plans are often presented in this way. The front and one side view of a house are shown here.




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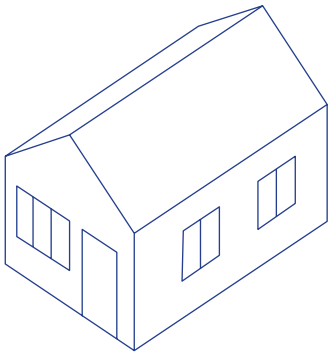


Three views of a house are shown below.



Front view  Side view  Plan view

Although none of these drawings by itself can give a good representation of the solid, we have the ability to use the three drawings to produce an image of the solid in our mind.



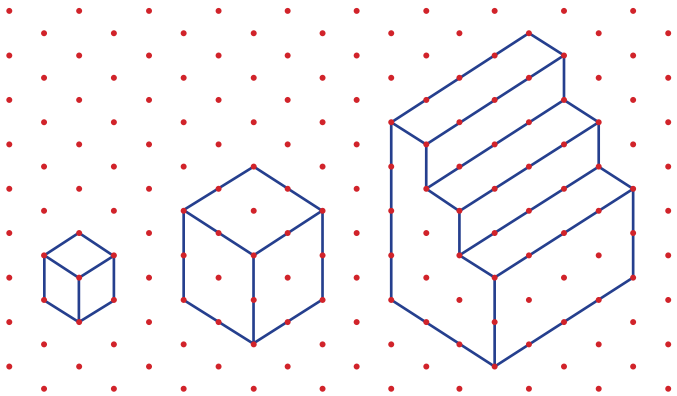
## Isometric drawing

An **isometric drawing** combines the three views into one. The object is drawn so that the front, side and top views of the solid are all visible to the person making the drawing. This drawing is called the isometric drawing of the solid. The isometric drawing for the house in the previous section is shown.

It should be noted that, from this view, the geometrical properties of the faces of the solid are no longer maintained. The right angles are not right angles when measured. However, parallel lines in the original solid remain parallel in the isometric drawing.

### Drawing on isometric paper

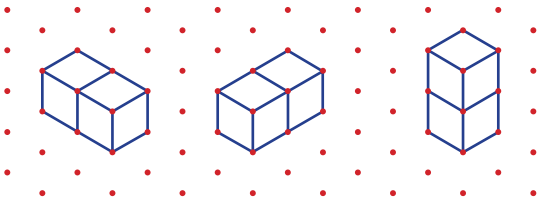
Isometric paper is a type of paper used for drawing solids from an isometric viewpoint. The angles formed by the lines on the paper represent right angles from the isometric viewpoint, so this paper is best suited for drawing shapes with horizontal bases and side faces that are upright and at right angles to each other. Isometric paper is available to download from your Interactive Textbook. The simplest solid with these properties is a cube. The diagrams below show two cubes of different sizes and a short set of steps drawn on isometric paper.





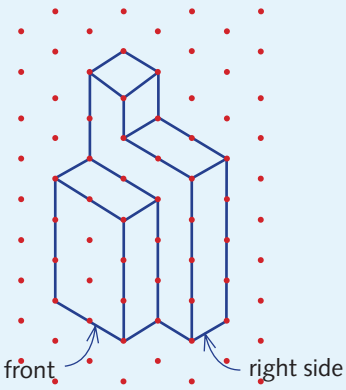

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On the right are the three different ways two cubes stuck together to form one prism may be drawn on isometric paper.

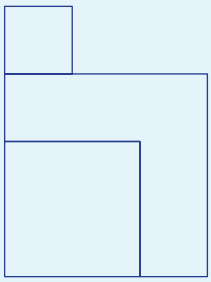


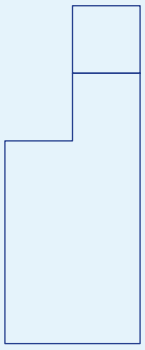
**Example 1**

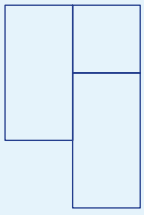
Draw the front view, right side view and plan view of the solid.




**Solution**


Front view


Right side view

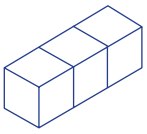

Plan view

## Exercise 21B

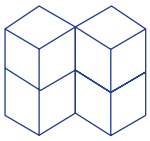
For this exercise it is useful to have a set of unit cube blocks.

**1** Draw each of the following on isometric paper.

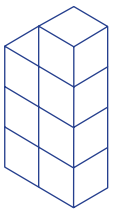
**a**



**b**

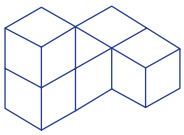


**c**

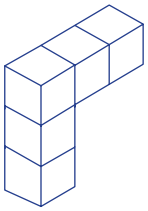



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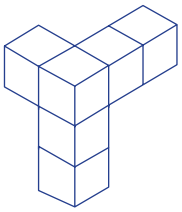
**d**



**e**



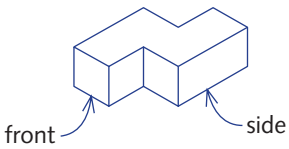
**f**



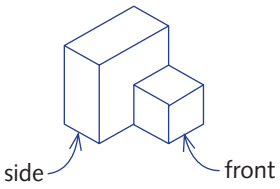
Example 1

**2** Draw the front, side and plan views of the following solids.

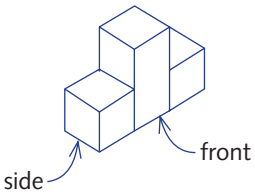
**a**



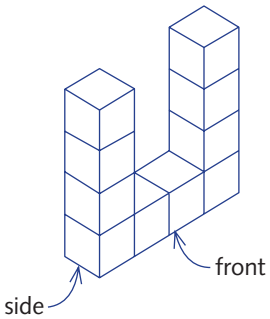

**b**



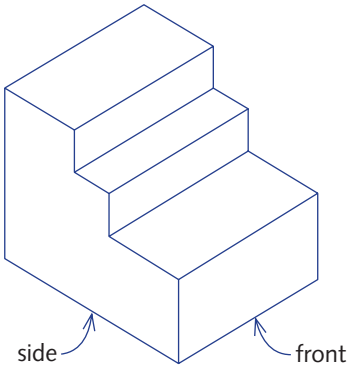

**c**



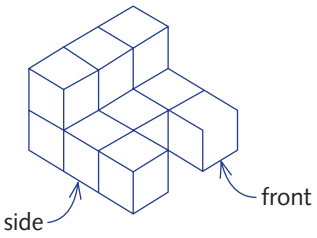

**d**




**e**




**f**

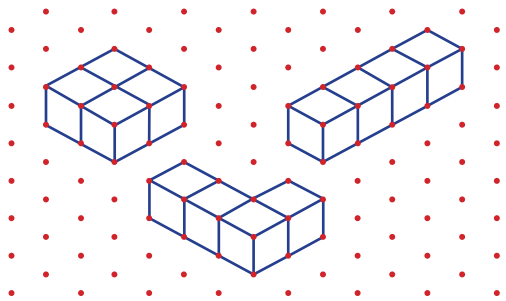



**3** **a** On isometric paper, draw three cubes, all on the same level and stuck together to form a prism, in as many ways as you can. You will find that there are only two such prisms but different isometric drawings of each of them.

**b** Is it possible to make an object that is not a prism if the three cubes are stuck together such that each cube has at least one face stuck to the face of another cube?

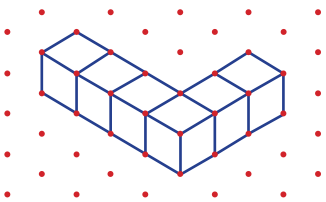
**4** **a** Draw four other objects that can be made from four cubes stuck together as described in Question **3b**. Only one version of each object needs to be drawn on isometric paper. Three of them have been drawn for you here.

**b** How many of these objects were not prisms?

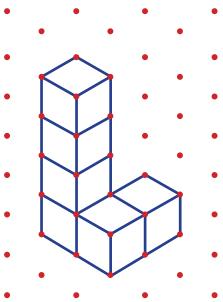


**5** Do two other isometric drawings of each of the following solids.

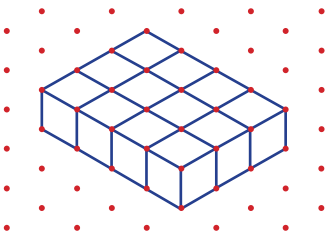
**a**



**b**



**c**

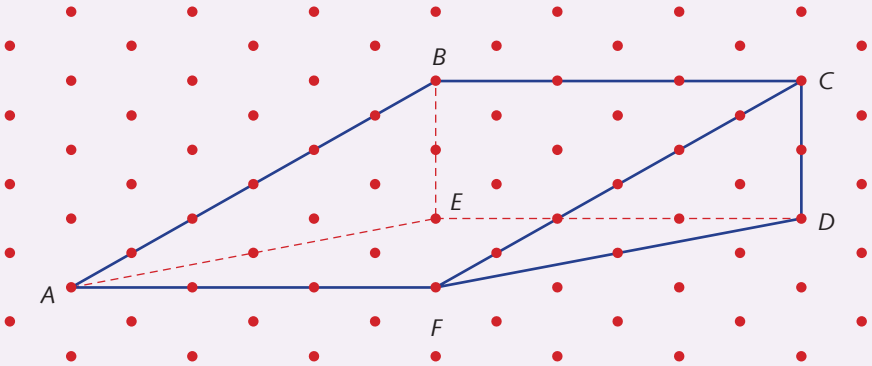




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# Review exercise

**1** A wedge-shaped solid is shown. It is drawn on isometric dot paper.




**a** What type of prism is this?

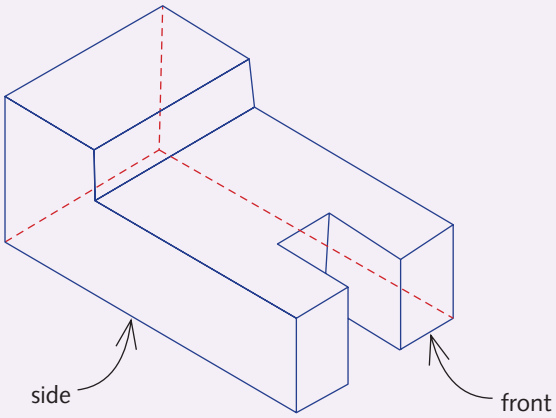
**b** Name the faces of this prism.

**c** Draw the plan view, back view and side view of this solid.

**d** Draw the prism again from a different view on isometric dot paper.

**e** Verify that $V - E + F = 2$.

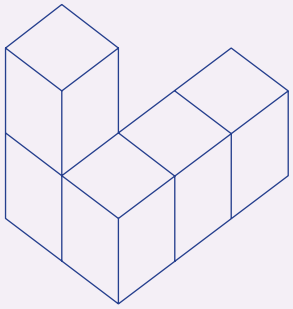
**2** **a** For the figure shown, draw the plan view, front view, back view and side view.



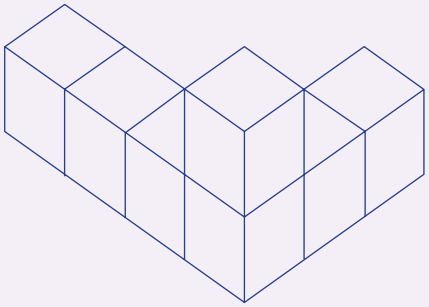

**b** Is the figure a prism?

**3** Draw two different isometric drawings of each of the following.

**a**



**b**




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# Challenge exercise

*In the following work with Euler's formula, $V - E + F = 2$.*

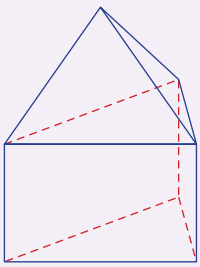
**1** Find a polyhedron with the given number of edges and vertices. First find the number of faces using Euler's formula.

**a** 8 edges, 5 vertices **b** 9 edges, 6 vertices **c** 16 edges, 8 vertices

**2** A tetrahedron has four faces and four vertices. Name a polyhedron with:

**a** five vertices and five faces **b** six vertices and six faces

**3** The diagram is of a solid called an **extended triangular pyramid**. It has seven faces and seven vertices.



**a** Draw an extended square pyramid.

**b** How many faces, edges and vertices does an extended square pyramid have?

**c** A pyramid has a polygonal base with n sides. How many vertices and faces does the associated extended pyramid have? How many edges does it have?

**d** Name two polyhedra that have nine faces and nine vertices.

**4** **a** Draw a prism with a cross-section that is a quadrilateral with two parallel sides of unequal length and the other two sides of equal length.

**b** Show with a drawing how two of the prisms of part **a** may be put together to form a hexagonal prism.

**c** Show with a drawing how a number of prisms with cross-section a regular hexagon may be put together to form a 'honeycomb'.

**5** **a** Draw two tetrahedrons joined at a base. Describe the polyhedron you get. Calculate $V - E + F$ for this polyhedron.

**b** A parallelepiped is a prism with all faces a parallelogram. Draw a parallelepiped.

**c** Show how a tetrahedron can be inscribed in a parallelepiped with each edge of the tetrahedron lying on a face of the parallelepiped.

**d** Find, by means of a rough sketch, what polyhedron has its vertices at the midpoints of the edges of a tetrahedron.

**6** Show with a drawing how a cube can be cut up into six identical square pyramids.


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CHAPTER

# 22

# Review and problem-solving

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# 22A Review

## Chapter 11: Integers

**1** List the integers less than 4 and greater than $-6$.

**2** The sequence $-60,\ -57,\ -54,\ \ldots$ is 'going up by threes'. Write down the next four terms.

**3** Simplify:

**a** $-(-5)$ **b** $-(-34)$ **c** $-(-(-7))$ **d** $-(-(-13))$

**4** Write the answers to these additions.

**a** $-3 + 11$ **b** $-5 + 7$ **c** $-5 + 12$ **d** $-29 + 18$

**e** $-35 + (-7)$ **f** $-12 + 18$ **g** $-3 + (-2)$ **h** $-6 + (-11)$

**5** Write the answers to these subtractions.

**a** $3 - 6$ **b** $6 - 18$ **c** $-3 - 11$ **d** $-15 - (-8)$

**e** $-8 - 26$ **f** $17 - (-3)$ **g** $-16 - 19$ **h** $-11 - (-22)$

**i** $-36 - (-32)$ **j** $-23 - (-9)$ **k** $-20 - (-11)$ **l** $20 - (-30)$

**6** The temperature in Montreal on a winter's day went from a minimum of $-22°C$ to a maximum of $-7°C$. By how much did the temperature rise?

**7** Write the answers to these multiplications.

**a** $5 \times (-7)$ **b** $6 \times (-3)$ **c** $10 \times (-11)$ **d** $12 \times (-7)$

**e** $10 \times (-16)$ **f** $-5 \times (-10)$ **g** $-11 \times 4$ **h** $-12 \times (-4)$

**8** Complete these divisions.

**a** $-18 \div (-3)$ **b** $-36 \div 4$ **c** $-35 \div (-7)$ **d** $-27 \div (-3)$

**9** Complete these divisions.

**a** $\dfrac{18}{-6}$ **b** $\dfrac{-63}{7}$ **c** $\dfrac{-84}{12}$

**10** Evaluate:

**a** $(-9)^2$ **b** $-8^2$ **c** $-7 - 3 + 6$

**d** $9 \times (-8) + 7$ **e** $-3 \times (-8) + 24 - 15$

**11** In an indoor cricket match, a team has made 30 runs and lost 8 wickets. What is the score of the team? (A run adds 1 to the score and a wicket subtracts 5.)

**12** Evaluate:

**a** $-6 \times (6 - 7)$ **b** $8 \times (12 - 20)$ **c** $-3 \times (25 + 15)$

**d** $-6 \times (-14 - 6)$ **e** $-12 \times (-6 + 40)$ **f** $-(-5)^2$

**g** $(2 - 11) \times (11 - 20)$ **h** $(10 - 3) \times (-3 + 10)$ **i** $(-5 - 10) \times (10 - 4)$


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## Chapter 12: Algebra and the Cartesian plane

**1** Evaluate each expression for $x = -4$.

| | | | |
|---|---|---|---|
| **a** $2x$ | **b** $-x$ | **c** $x + 2$ | **d** $x - 3$ |
| **e** $2x + 3$ | **f** $x^3$ | **g** $-x^3$ | **h** $(-x)^2$ |
| **i** $3 - x$ | **j** $3 - 2x$ | **k** $5 + 2x$ | **l** $2 - x^2$ |

**2** Substitute $m = -5$, $n = 6$ and $p = -60$ to evaluate:

| | | | |
|---|---|---|---|
| **a** $m + n$ | **b** $m + p$ | **c** $m - p$ | **d** $mp$ |
| **e** $np$ | **f** $\frac{p}{m}$ | **g** $mnp$ | **h** $\frac{p}{n}$ |

**3** Given that $m = -12$, $n = 4$ and $p = -3$, evaluate:

| | | |
|---|---|---|
| **a** $m + n$ | **b** $m + p$ | **c** $m - p$ |
| **d** $mp$ | **e** $np$ | **f** $\frac{p}{m}$ |

**4** Evaluate each expression for $x = -5$.

| | | |
|---|---|---|
| **a** $5x + 4$ | **b** $-5x + 4$ | **c** $5(x + 4)$ |
| **d** $-5(x + 4)$ | **e** $-5x^2$ | **f** $(-5x)^2$ |

**5** A piece of string is $x$ m in length. It is divided into five equal parts.

**a** Find the length of each part in terms of $x$.

**b** Find the length of each part for the following values of $x$.

**i** $x = 20$ **ii** $x = 42$ **iii** $x = 96$

**6** Angel has \$2000 in a bank account. She takes \$$x$ from the bank account every day.

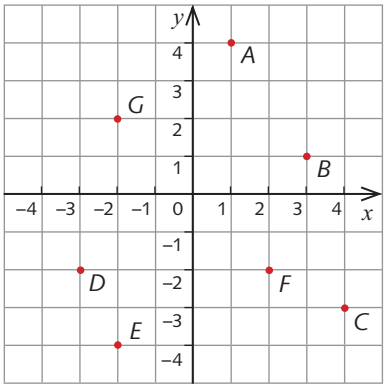
**a** How much money does she have in the account after:

**i** 1 day? **ii** 5 days?

**b** Find the value of her bank account after five days if:

**i** $x = 100$ **ii** $x = 200$ **iii** $x = 450$

**7** Write down the coordinates of each of the points marked on the Cartesian plane below.





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**8** Do each part of this question on graph paper.

**a** Plot the points $O(0, 0)$, $A(5, 0)$, $C(5, 5)$ and $D(0, 5)$, and join points to form $OA$, $AC$, $CD$ and $DO$. Describe the shape formed.

**b** Plot the points $A(-5, 0)$, $B(5, 0)$ and $C(0, 10)$, and join points to form $AB$, $BC$ and $CA$. Describe the shape formed.

**c** Plot the points $A(-3, 0)$, $B(3, 0)$, $C(3, 3)$ and $D(-3, 3)$, and join points to form $AB$, $BC$, $CD$ and $DA$. Describe the shape formed.

**d** Plot the points $O(0, 0)$ and $A(4, 4)$, and draw the line passing through them. Now plot the points $B(2, -3)$ and $C(5, 0)$ and draw the line passing through these points. Describe the relationship between the lines.

**9** For each given rule, complete the table, list the coordinates of the points, and plot the points on the Cartesian plane. (After you have completed the table, decide on the scale of your axes.)

**a** $y = 4x$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

**b** $y = -x$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

**c** $y = x - 4$

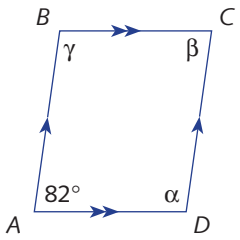
| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

Check the points are collinear.

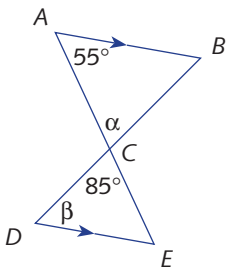
## Chapter 13: Triangles and constructions

**1** Find $\alpha$, $\beta$, $\gamma$ or $\theta$ in each diagram below, giving reasons.

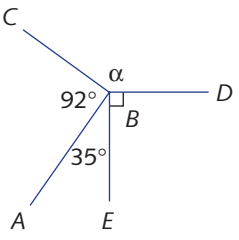
**a**



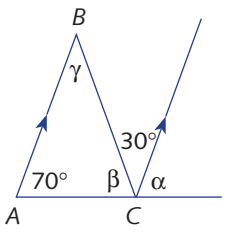

**b**



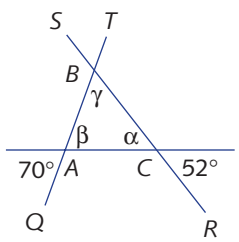

**c**



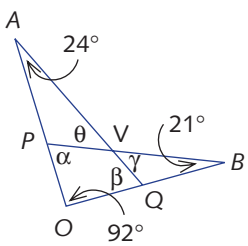

**d**



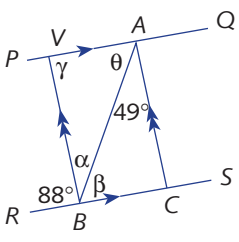

**e**



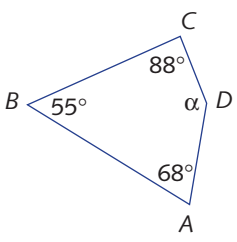

**f**




**g**



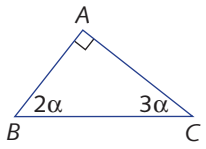

**h**



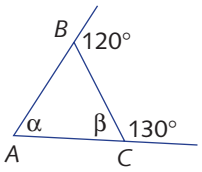


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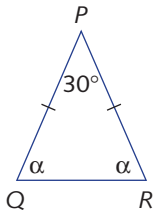
**i**



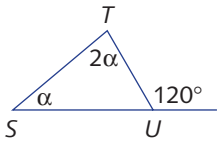

**j**



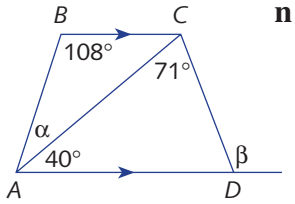

**k**



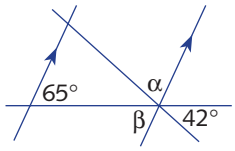

**l**



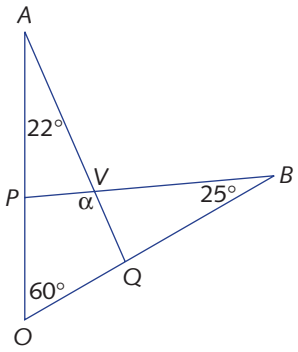

**m**



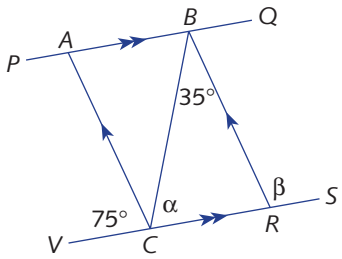

**n**



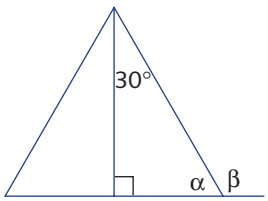

**o**




**p**




**q**




**2** Use a ruler and compasses to construct a triangle:

**a** with side lengths 5 cm, 8 cm and 10 cm

**b** with side lengths 6 cm, 6 cm and 4 cm

## Chapter 14: Negative fractions and decimals

**1** Arrange the numbers $-3\frac{1}{4}$, $2\frac{1}{2}$, $-1\frac{3}{4}$, $0$, $2$, $-2\frac{1}{2}$, $3\frac{1}{2}$ from smallest to largest.

**2** Calculate:

**a** $-\frac{3}{4}+4$ **b** $-1\frac{5}{6}+4$ **c** $-\frac{2}{5}+\frac{3}{5}$ **d** $-\frac{5}{8}+\frac{3}{8}$

**e** $-1\frac{1}{4}-1\frac{1}{4}$ **f** $-2+\frac{3}{5}$ **g** $-5+2\frac{2}{5}$ **h** $-11\frac{1}{2}-13\frac{1}{2}$

**3** Calculate:

**a** $\frac{3}{5}+\frac{3}{4}$ **b** $\frac{3}{5}-\frac{3}{4}$ **c** $\frac{1}{2}-\frac{7}{8}$ **d** $\frac{3}{11}-\frac{3}{10}$

**e** $\frac{2}{3}+\frac{7}{8}$ **f** $-\frac{3}{5}+\frac{7}{10}$ **g** $\frac{1}{2}-\frac{7}{20}$ **h** $\frac{3}{7}-\frac{5}{8}$

**4** Calculate:

**a** $\frac{2}{3}\times\left(-\frac{3}{4}\right)$ **b** $\frac{7}{8}\times\left(-\frac{16}{21}\right)$ **c** $-1\frac{1}{2}\times\left(-\frac{4}{3}\right)$ **d** $-\frac{3}{4}\times\left(-\frac{5}{8}\right)$

**e** $-\frac{5}{8}\times\left(-\frac{2}{3}\right)$ **f** $1\frac{1}{5}\times\left(-\frac{35}{48}\right)$ **g** $-\frac{2}{3}\div\left(-\frac{7}{12}\right)$ **h** $1\frac{1}{3}\div\left(-\frac{6}{11}\right)$


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**5** Evaluate each expression for $x = -\frac{3}{4}$.

**a** $3x$ **b** $-x$ **c** $x - 3$ **d** $4x$

**e** $(-x)^3$ **f** $-x^3$ **g** $3 - x$ **h** $4 - 4x$

**6** Given that $m = -2\frac{1}{4}$, $n = \frac{5}{8}$ and $p = -5$, evaluate:

**a** $m + n$ **b** $m + p$ **c** $mn$ **d** $np$

**e** $mnp$ **f** $2m + p$ **g** $4m$ **h** $-8n$

## Chapter 15: Percentages and ratios

**1** Write the following fractions as percentages.

**a** $\frac{9}{10}$ **b** $\frac{17}{20}$ **c** $\frac{6}{5}$ **d** $\frac{12}{25}$

**2** Write the following decimals as percentages.

**a** 0.78 **b** 0.095 **c** 0.97 **d** 1.35

**3** **a** Express 30 cents as a percentage of \$1.

**b** Express 1 kg as a percentage of 800 grams.

**c** Express 4 months as a percentage of 1 year.

**4** Arrange the following from smallest to largest.

**a** 0.39, $\frac{12}{32}$, $4\frac{1}{2}\%$

**b** 64%, 0.6, $\frac{2}{3}$

**5** Fill in the table with the equivalent fraction, decimal and percentage for each number.

| Fraction | Decimal | Percentage |
|---|---|---|
| | 0.375 | |
| $\frac{1}{3}$ | | |
| | | 75% |
| $2\frac{3}{4}$ | | |
| | | 300% |

**6** Write each percentage as a fraction.

**a** $62\frac{1}{2}\%$ **b** $87\frac{1}{2}\%$ **c** $1\frac{1}{4}\%$ **d** $5\frac{1}{4}\%$


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## Chapter 16: Solving equations

**1** Solve:

**a** $x + 3 = 5$ **b** $x - 2 = 9$ **c** $x - 4 = 8$

**d** $2x = 10$ **e** $3x = 7$ **f** $z - 10 = 12$

**g** $5x = 20$ **h** $\frac{x}{3} = 4$ **i** $z + 8 = 16$

**2** Solve:

**a** $2x + 3 = 7$ **b** $3x - 6 = 9$ **c** $2x - 4 = 8$

**d** $2x + 8 = 16$ **e** $3x - 5 = 7$ **f** $6z - 10 = 12$

**g** $2x - 11 = 13$ **h** $\frac{m}{3} + 6 = 10$ **i** $\frac{z}{5} - 4 = 6$

**3** Solve each equation for $x$.

**a** $x + 3 = 2$ **b** $2x + 4 = -2$ **c** $2 - x = 8$

**d** $\frac{x}{5} + 4 = 11$ **e** $2x - 5 = -6x + 7$ **f** $2(x - 2) = 11x$

**g** $2(2x - 1) = 10$ **h** $3(2x + 5) = 15$ **i** $5(3x - 1) = 20$

**4** In each case below, write an equation and solve it.

**a** Three is added to a number and the result is 6.

**b** Five is subtracted from a number and the result is 10.

**c** A number is multiplied by 7 and the result is 84.

**d** A number is divided by 4 and the result is 15.

**e** A number is multiplied by 4, and 3 is subtracted from the result. The result of this is 17.

**f** Six is added to a number and the result is multiplied by 3. The result of this is 28.

**5** A rectangle is five times as long as it is wide. The perimeter of the rectangle is 120 cm. Find the length and the width of the rectangle.

**6** The difference between two numbers is 18. The sum of the two numbers is 20. Find the two numbers.

**7** Solve:

**a** $x + 5 = -12$ **b** $x - 7 = -14$ **c** $x + 7 = 8$

**d** $5x = -12$ **e** $-6x = -24$ **f** $-3x = 9$

**g** $\frac{x}{4} = -20$ **h** $\frac{-3x}{2} = -6$ **i** $4x + 1 = -11$

**j** $5x - 3 = -18$ **k** $-2x + 10 = -12$ **l** $4x - 6 = -8$

**8** Solve each equation for $x$.

**a** $5x + 4 = 2x + 5$ **b** $6x + 7 = 2x + 10$ **c** $5x + 2 = x + 1$

**d** $4x - 2 = 5x + 1$ **e** $x + 14 = 3x + 10$ **f** $10x + 4 = 2x + 7$

**g** $3(x - 2) = x + 4$ **h** $5x - 4 = 2(x - 1)$ **i** $2(3x - 1) = 5(2 - x)$


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**9** If you add 13 to a number, you get the same result as when you subtract half the number from 4. What is the number?

## Chapter 17: Probability

**1** A jar contains 20 blue marbles and 30 red marbles. A marble is chosen at random. What is the probability that it is:

**a** red? **b** blue?

**2** A box contains 24 balls. There are eight red balls numbered 1 to 8, six blue balls numbered 9 to 14, and ten green balls numbered 15 to 24. One ball is taken out of the box. Find the probability of getting:

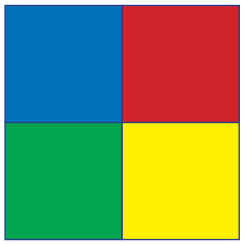
**a** a red ball **b** a green ball **c** an odd number

**d** a green ball with an odd number **e** a green or a red ball

**f** a number greater than 10 **g** a number divisible by 5

**h** a prime number **i** a ball that is not red

**3** A dart is thrown at the board shown opposite. There is an equal chance of hitting any point on the board. What is the probability of:



**a** hitting the blue square?

**b** not hitting the blue square?

**c** hitting one of the bottom two squares?

**4** The letter tiles that make up the word MATHEMATICS are put in a sack together with the letter tiles for the word FOOTBALL. There are a total of 19 tiles. A tile is drawn. What is the probability of obtaining:

**a** an M? **b** an A? **c** an F? **d** a vowel?

**e** a consonant? **f** a letter other than an M? **g** an M or an A?

**5** A school has 1800 students. The table below gives information about whether or not a student studies an Asian language.

| | Boys | Girls |
|---|---|---|
| **Studies an Asian language** | 500 | 600 |
| **Does not study an Asian language** | 400 | 300 |

A student is chosen at random. What is the probability that the student is:

**a** a boy?

**b** a boy who studies an Asian language?

**c** a girl who does not study an Asian language?


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## Chapter 18: Transformations and symmetry

**1** Copy the points listed below onto your graph paper and translate them 3 units right and 1 unit up. Be careful to label each point and its image. Also write down the coordinates of each image point.

**a** $A(0, 0)$ **b** $B(1, 2)$ **c** $C(-2, 1)$ **d** $D(-5, -5)$

**2** Given the following points and their images under a translation, plot each point and describe the translation.

**a** $A(0, 0)$ and $A'(1, 3)$ **b** $B(2, -2)$ and $B'(3, 4)$

**3** Plot the points listed below onto a number plane and rotate them 90° anticlockwise about $(0, 0)$. Write down the coordinates of each image point.

**a** $A(2, -1)$ **b** $B(2, -2)$ **c** $C(-3, 4)$ **d** $D(-4, -4)$

**4** Plot the points listed below onto a number plane. Find their images under a rotation of 180° anticlockwise about $(0, 0)$. Draw each image and write down its coordinates.

**a** $A(2, -1)$ **b** $B(2, -2)$ **c** $C(-3, 4)$ **d** $D(-4, -4)$

**5** Copy these points onto a number plane and find their images under reflection in the $x$-axis.

**a** $A(2, -1)$ **b** $B(-3, 3)$ **c** $C(-5, 0)$ **d** $D(0, 4)$

## Chapter 19: Graphs and tables

**1** In a school, 720 students were asked to name their favourite colour. The results are shown below. Copy and complete the table and draw the corresponding pie chart.

| Colour | Number of students | Fraction of students | Angle required for pie chart |
|---|---|---|---|
| Blue | 120 | $\frac{1}{6}$ | 60° |
| Green | 240 | | |
| Red | 200 | | |
| Yellow | 40 | | |
| Purple | 60 | | |
| Brown | 60 | | |

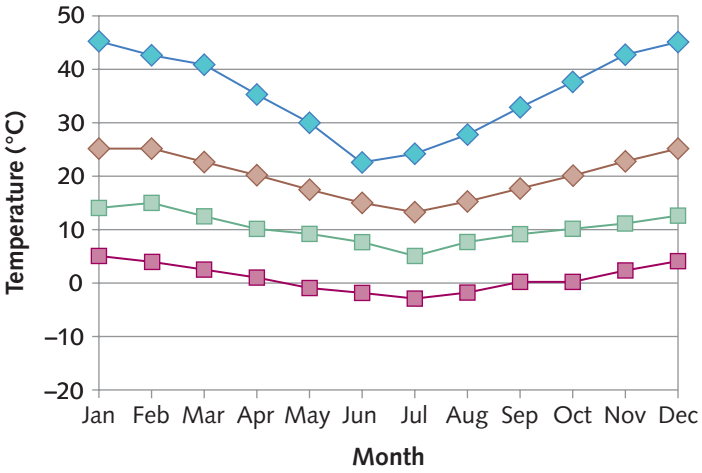
**2** A pie chart is drawn to show how a university student spends her money. She spends 20% of her money on food and 40% on rent. Calculate the angles of the sectors used to represent:

**a** the amount she pays in rent **b** the amount she pays for food

**3** The line graphs shown over page give information about the temperatures in a city in the south of Australia. The graphs shown are (from the bottom up):

- the lowest temperature for each month
- the average daily minimum temperature for each month
- the average daily maximum temperature for each month
- the highest temperature for each month.


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**a** Which month is the coldest month?

**b** Which two months are the hottest months?

**c** What is the average maximum temperature for May?

**d** What is the average minimum temperature for May?

**4** **a** Draw a pie chart to represent the amount of wheat, barley and rice produced in a particular region, given that the angles of the sectors representing wheat, barley and rice are 90°, 120° and 150°, respectively.

**b** If the total production of the three crops is 48 000 tonnes, calculate how many tonnes of each crop are produced.

**5** A sum of money is divided in the ratio 1:3:5. Draw a pie chart to illustrate the division. Indicate the angles of the sectors carefully.

**6** The table below shows the number of people visiting a museum each day of a particular week.

| Day | Mon | Tue | Wed | Thu | Fri | Sat | Sun |
|---|---|---|---|---|---|---|---|
| Number of people | 1250 | 2500 | 2800 | 2800 | 3200 | 6500 | 7000 |

Draw a line graph to represent the information in the table.

## Chapter 20: Statistics

**1** The numbers of wombats that crossed a particular road over a number of weeks are recorded in the table below.

| Week number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Number of wombats | 24 | 41 | 44 | 26 | 27 | 38 | 35 | 39 | 40 | 41 | 23 | 46 | 33 | 36 | 38 | 41 | 24 | 22 | 31 |

**a** Draw a stem-and-leaf plot of the data.

**b** Find the median of the data.

**c** Find the range of the data.


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**2** Twenty students took a maths test. Here are their results:

11, 12, 13, 15, 15, 20, 11, 9, 5, 15, 19, 19, 16, 8, 12, 12, 16, 16, 20, 10

**a** Find the mean. **b** Find the median. **c** Find the range.

**3** For the data shown in the stem-and-leaf plot:

| Stem | Leaves |
|---|---|
| 7 | 46788999 |
| 8 | 001122236666667889 |
| 9 | 01223356679 |
| 10 | 013 |

**a** find the median **b** find the range

**4** Students are asked to cut a piece of string of length 50 cm. The following observations were taken. Measurements are to the nearest centimetre.

48, 48, 48, 49, 49, 49, 49, 50, 50, 50, 50, 50, 51, 51, 51, 51, 52

Draw a dot plot for the data.

**5** The numbers 5, 12, 16, $x$, $y$, 24, 30 have mean 18. If $x$ and $y$ are positive integers and the numbers are arranged from smallest to largest with no two numbers the same, find the possible values of $x$ and $y$.

**6** For the data sets below find the median and range:

**a** 12, 14, 16, 18, 20, 22, 24 **b** 3, 7, 9, 11, 13, 17, 19, 29

**7** The waist measurements of 22 people measured to the nearest centimetres are:

85, 91, 81, 92, 92, 101, 76, 84, 74, 76, 80, 86, 82, 82, 96, 81, 76, 84, 88, 82, 96, 99

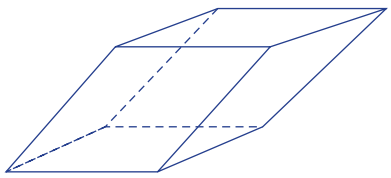
**a** Find the mean. **c** Find the range.

**b** Find the median. **d** Construct a stem-and-leaf plot.

## Chapter 21: Polyhedra and three-dimensional drawing

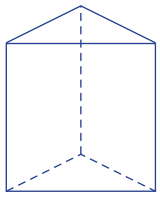
**1** **a** How many faces, edges and vertices does each solid have?

**i**



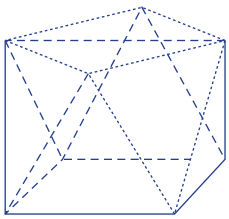
Parallelepiped

**ii**



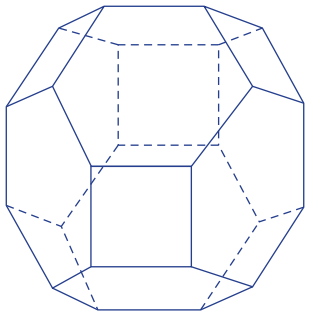
Triangular prism

**iii**



Solid consists of square at the top and bottom and equilateral triangles joining them

**iv**



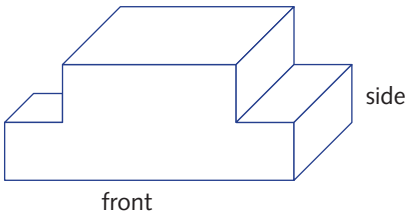
Truncated octahedron

**b** Calculate $V - E + F$ for each of these.

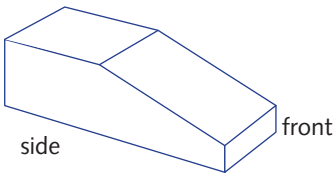

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**2** Draw the front view, side view and plan view.

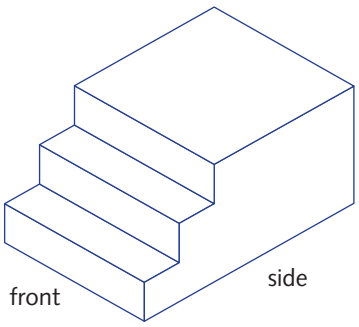
**a**



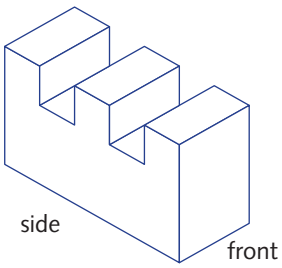

**b**




**c**



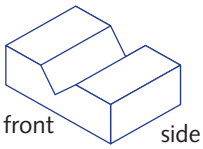

**d**




**3** **a** Draw the diagrams shown in **2a**, **c** and **d** using isometric paper.

**b** Draw two different drawings of a triangular prism on isometric paper.

**4** Draw a front, plan and side view for the diagram below.




# 22B Tessellations

A tiling pattern with no gaps or spaces between the tiles is called a **tessellation**. The tessellations we are going to look at are made up of tiles that are polygons and completely cover a plane. You will need a ruler, compasses and a protractor. Set squares will be useful.

A single tile is said to tessellate if the tessellation is made up of copies of the one tile.

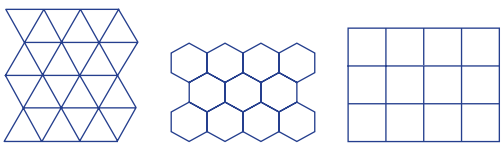
We will begin with tessellations that can be created using just **one polygon.** If we limit ourselves to regular polygons, there are only three possibilities:

**1** equilateral triangles will tessellate

**2** squares will tessellate

**3** regular hexagons will tessellate.

The resulting patterns are called the **regular** tessellations.



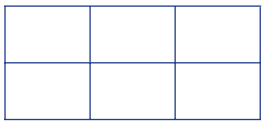

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## Activity 1 (Why are these the only regular polygons that tessellate by themselves?)

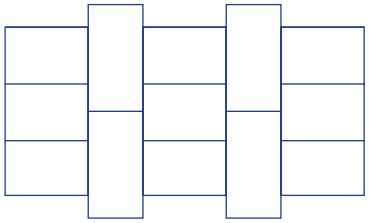
Use a template to show what happens when you try to use a different regular polygon – such as a pentagon, an octagon or a nonagon – to tessellate the plane. Copy and complete this table of the sizes of interior angles. Then explain why the equilateral triangle, square and hexagon are the only regular polygons that can be used by themselves to tessellate.

| Polygon | Number of sides | Interior angle sum | Size of each angle |
|---|---|---|---|
| Triangle | 3 | 180° | 60° |
| Quadrilateral | 4 | 360° | |
| Pentagon | | 540° | |
| | 6 | | |
| Heptagon | 7 | | |
| | | | 135° |
| | 9 | 1260° | |
| | 10 | | 144° |
| Dodecagon | 12 | | |

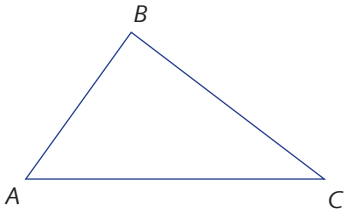
## Activity 2 (Tessellations with non-regular polygons)



If we consider polygons that are *not* regular, we can find many other single-polygon tessellations. For example, any rectangle tessellates the plane.

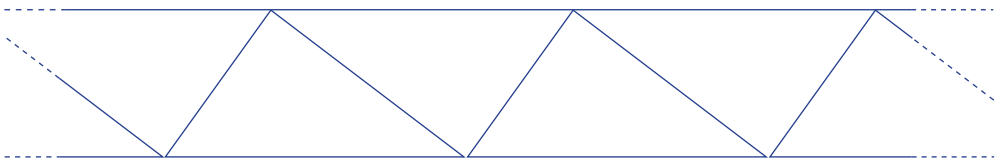


But even a rectangle can be used more inventively to tessellate.




It can be proved that *any* triangle can tessellate the plane by itself. Try doing this using a triangle like the one on the right.

Set it out along a line like this:

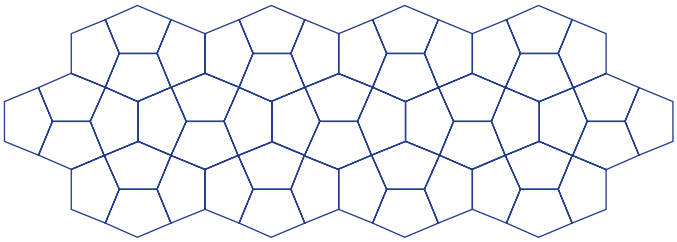


Use a similar construction to explain why any quadrilateral can be used to tessellate the plane.

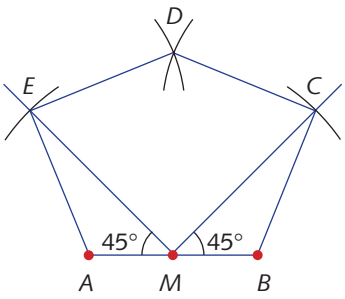

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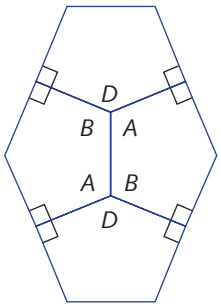
## Activity 3 (The Cairo tessellation)

The **Cairo tessellation** is so named because tiles such as these were used for many years on the streets of Cairo. Each tile is a pentagon with all sides of equal length.



Note that this pentagon is not regular, even though its sides are of equal length, because it does not have all its interior angles of the same size.




You can draw one such tile using your protractor, ruler and compasses.

*Step 1*: Draw the interval $AB$.

*Step 2*: Find the midpoint $M$ of $AB$.

*Step 3*: Draw rays from $M$ at 45° to $AB$.

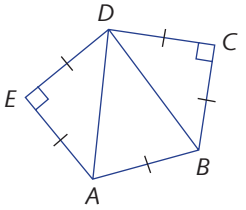
*Step 4*: Use your compasses set at the length of $AB$ to complete the pentagon.

The angles of the pentagon at $E$ and $C$ are 90°.

Draw one of these pentagons on card. Use it to form the Cairo tessellation.

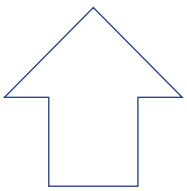
Why does this pentagon tessellate?

Another way of drawing such a pentagon is suggested by the diagram shown. Draw two identical right-angled isosceles triangles on card. Cut them out, join them at $D$ as shown, and rotate one around $D$ so that $AB$ is the same length as the equal sides of the isosceles triangles.




## Activity 4

Here is a heptagon that can be used to tessellate the plane.



Draw it on card and use it to cover a rectangular piece of paper at least 15 cm × 10 cm to show how the tessellation works. The angles of the heptagon are all one of 45°, 90° or 270°.

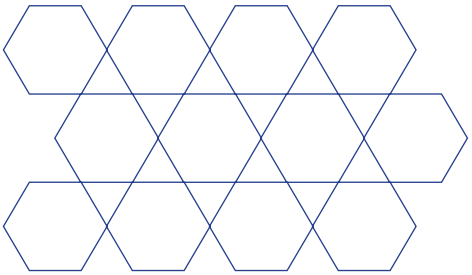

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## Activity 5

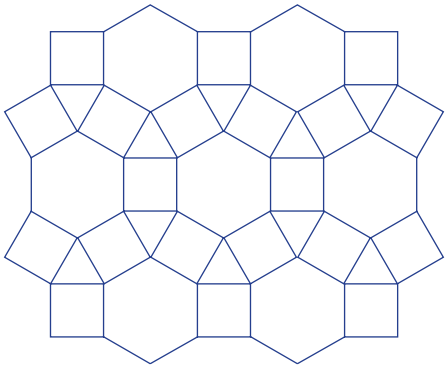
After Activity **1**, you probably noticed that, because the angles about a point add up to 360°, there are some *combinations* of regular polygons that can be used to tessellate the plane. These are called **semiregular tessellations**. One way of describing them is to look at the set of regular polygons that meet at a point and write down the numbers of sides these polygons have, 'in cyclic order' (that is, clockwise starting from the highest number), in square brackets. For example, the code for the second tessellation shown below is [6, 4, 3, 4].

Write the codes for the other seven semiregular tessellations.

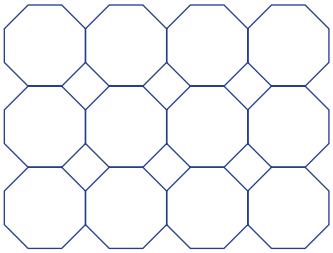
**1**



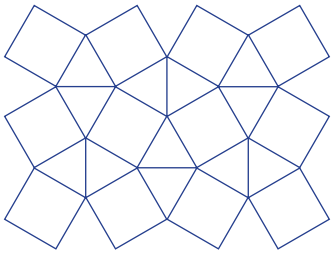
**2**



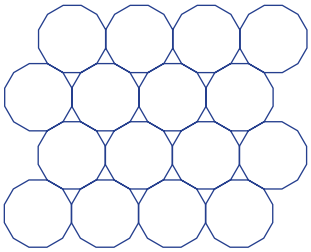
**3**



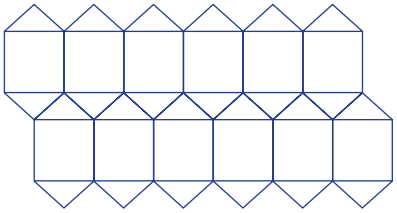
**4**



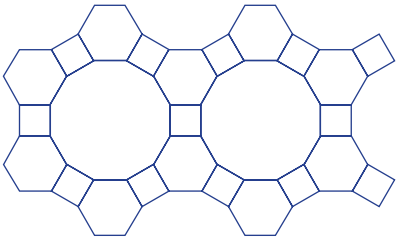
**5**



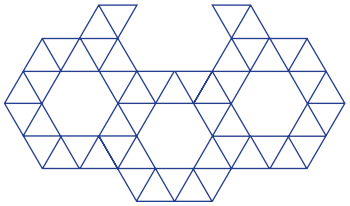
**6**



**7**



**8**




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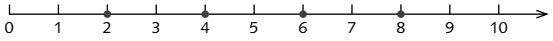
# Answers to exercises

## Chapter 1 answers

### Exercise 1A

**1** **a** 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
**b** 53, 54, 55, 56, 57, 58, 59, 60

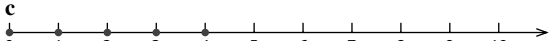
**2** **a**



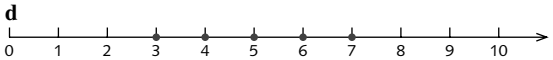

**b**



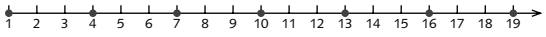

**c**




**d**




**3**




### Exercise 1B

**1** **a** 20 **b** 30 **c** 20 **d** 80
**e** 65 **f** 44 **g** 97 **h** 41
**i** 62 **j** 61 **k** 65 **l** 143

**2** **a** 43 **b** 45 **c** 36 **d** 45
**e** 94 **f** 84 **g** 98 **h** 117

**3** **a** 80 **b** 200 **c** 110
**d** 90 **e** 110 **f** 50

**4** **a** 300 **b** 610 **c** 350 **d** 810
**e** 460 **f** 1020 **g** 1010 **h** 615

**5** 99 L **6** 400 **7** 899 km **8** \$37

**9** 84 **10** 66 **11** 633

**12** **a** 45 **b** 4950

### Exercise 1C

**1** **a** 1351 **b** 784 **c** 981 **d** 1419 **e** 1269
**f** 3096 **g** 5223 **h** 1540 **i** 16085 **j** 10031

**2** **a** 1813 **b** 10130 **c** 2349
**d** 765 **e** 983 **f** 1015

**3** **a**
$$\begin{array}{r} 6\,\underline{5}\,\underline{8} \\ +\ \underline{3}\,\underline{3}\,\underline{9} \\ \hline \underline{9}\,\underline{9}\,\underline{7} \end{array}$$

**b**
$$\begin{array}{r} 8\,\underline{0}\,4 \\ +\ \underline{0}_1\,\underline{9}_1\,\underline{9} \\ \hline \underline{9}\,0\,\underline{3} \end{array}$$

**c**
$$\begin{array}{r} 9\,1\,9 \\ +\ 8_1\,9_1\,\underline{9} \\ \hline 1\,\underline{8}\,1\,\underline{8} \end{array}$$

**d**
$$\begin{array}{r} 6\,\underline{7}\,3 \\ +\ {}_1\,\underline{5}_1\,\underline{8}_1\,9 \\ \hline 1\,2\,6\,2 \end{array}$$

**e**
$$\begin{array}{r} 7\,\underline{0}\,1 \\ +\ {}_1\underline{3}\,\underline{9}\,7 \\ \hline 1\,0\,9\,\underline{8} \end{array}$$

**f**
$$\begin{array}{r} 3\,\underline{3}\,3 \\ +\ {}_1\,7_1\,\underline{7}_1\,\underline{7} \\ \hline \underline{1}\,1\,1\,0 \end{array}$$

**4** 478 + 487 + 748 + 784 + 847 + 874 = 4218

**5** **a** 994 **b** 5750

**6** 55752 km **7** \$2073 **8** 6670 km

**9** **a** E, D, A, C, B **b** 214
**c** A, B, D **d** 1535

**10** 663 + 792 + 587 = 2042

### Exercise 1D

**1** **a** 6 **b** 205 **c** 303 **d** 28
**e** 84 **f** 108 **g** 380 **h** 991

**2** **a** 8 **b** 0 **c** 1
**d** 56 **e** 38 **f** 101

**3** **a** 175 **b** 493 **c** 119 **d** 194

**4** **a** 3222 **b** 10640 **c** 2789

**5** **a**
$$\begin{array}{r} 7\,6 \\ -\ 2\,\underline{9} \\ \hline \underline{4}\,\underline{7} \end{array}$$

**b**
$$\begin{array}{r} 9\,\underline{3} \\ -\ \underline{4}\,\underline{7} \\ \hline \underline{4}\,6 \end{array}$$

**c**
$$\begin{array}{r} 2\,3\,\underline{3}\,\underline{6} \\ -\ \underline{2}\,\underline{2}\,\underline{9}\,\underline{7} \\ \hline 3\,\underline{9} \end{array}$$

**d**
$$\begin{array}{r} 1\,\underline{1}\,5\,\underline{3} \\ -\ 4\,\underline{1}\,\underline{9} \\ \hline 7\,\underline{3}\,\underline{4} \end{array}$$

**e**
$$\begin{array}{r} \underline{2}\,7\,6\,\underline{1} \\ -\ \underline{8}\,\underline{4}\,\underline{3} \\ \hline \underline{1}\,\underline{9}\,\underline{1}\,\underline{8} \end{array}$$

**f**
$$\begin{array}{r} 5\,3\,7\,\underline{6} \\ -\ 2\,\underline{1}\,\underline{4}\,\underline{8} \\ \hline \underline{3}\,\underline{2}\,\underline{2}\,\underline{8} \end{array}$$

**6** **a** 10 **b** 9, 27
**c** 68, 68 **d** 63, 81, 63

**7** \$3333 **8** 350 km **9** 1330

**10** 50 **11** 16388 **12** 9624

### Exercise 1E

**1** **a** 170 **b** 0 **c** 100
**d** 18000 **e** 12000 **f** 10000
**g** 73000 **h** 1000000 **i** 67430000

**2** **a** 600 **b** 260 **c** 4300
**d** 4900 **e** 3200 **f** 0
**g** 680 **h** 1200 **i** 420

**3** **a** 2 **b** 5

**4** 13 **5** 60 **6** 1200 **7** 18


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## Exercise 1F

**1** **a** 27 **b** 15 **c** 37 **d** 72 **e** 55
**f** 24 **g** 84 **h** 1 **i** 20

**2** **a** 870 **b** 7700 **c** 7800
**d** 108 **e** 918 **f** 8787

**3** **a** 5 **b** 15 **c** 23 **d** 21

**4** **a** 174 **b** 7700 **c** 780 **d** 600

**5** **a** 5 **b** 15 **c** 19 **d** 99

**6** 36 **7** 174

**8** **a** 16 **b** 48

**9** **a** 168 km **b** 48 km

**10** 15 **11** \$48

**12** **a** 41 **b** 65 **c** 18 **d** 70 **e** 72

## Exercise 1G

**1** **a** 1 **b** 2 **c** 2 **d** 4
**e** 3 **f** 3 **g** 6 **h** 6

**2** **a** $4 \times 10^1 + 6$, 6 **b** $6 \times 10^2 + 2 \times 10^1 + 3$, 600
**c** $5 \times 10^2 + 6 \times 10^1 + 9$, 60 **d** $6 \times 10^1 + 3$, 60
**e** $2 \times 10^2 + 8 \times 10^1 + 6$, 6 **f** $7 \times 10^2 + 6 \times 10^1$, 60

**3** **a** $2 \times 10^3 + 8 \times 10^1 + 3$, 3
**b** $3 \times 10^3 + 7 \times 10^2 + 5 \times 10^1 + 8$, 3000
**c** $5 \times 10^3 + 3 \times 10^1 + 6$, 30
**d** $4 \times 10^4 + 3 \times 10^3 + 1 \times 10^2 + 7 \times 10^1$, 3000
**e** $5 \times 10^4 + 7 \times 10^2 + 3 \times 10^1 + 2$, 30
**f** $2 \times 10^5 + 3 \times 10^4 + 5 \times 10^3 + 6 \times 10^2 + 7 \times 10^1 + 8$, 30000
**g** $2 \times 10^7 + 3 \times 10^6 + 6 \times 10^5 + 7 \times 10^4 + 8 \times 10^3 + 9 \times 10^2 + 7 \times 10^1 + 8$, 3000000

**4** 952, 925, 592, 529, 295, 259

**5** 999, 995, 992, 959, 955, 952, 929, 925, 922, 599, 595, 592, 559, 555, 552, 529, 525, 522, 299, 295, 292, 259, 255, 252, 229, 225, 222

## Exercise 1H

**1** **a** 212 **b** 152 **c** 448 **d** 340
**e** 2052 **f** 6552 **g** 1645 **h** 14042
**i** 45801 **j** 32636 **k** 822330 **l** 263922

**2** **a** 10872 **b** 15394 **c** 28858
**d** 40275 **e** 3645 **f** 34408
**g** 7990 **h** 412158 **i** 3559530
**j** 418338 **k** 527904 **l** 9959330

**3** **a** 234 **b** 10716
**c** 31089 **d** 37250

**4** 2924 **5** 3341

**6** **a**
$$\begin{array}{r} 86 \\ \times\ \ 7 \\ \hline 602 \end{array}$$
**b**
$$\begin{array}{r} 169 \\ \times\ \ 3 \\ \hline 507 \end{array}$$
**c**
$$\begin{array}{r} 216 \\ \times\ \ 23 \\ \hline 648 \\ 4320 \\ \hline 4968 \end{array}$$

**7** **a** 884 **b** 26520

**8** 415 kg **9** 862

## Exercise 1I

**1** **a** 9 **b** 200 **c** 6, 6

**2** **a** 24 **b** 10 **c** 36
**d** 56 **e** 18 **f** 24

**3** 7 **4** 12

**5** **a** 5, 2 **b** 7, 2 **c** 6, 1
**d** 10, 7 **e** 8, 1 **f** 12, 4

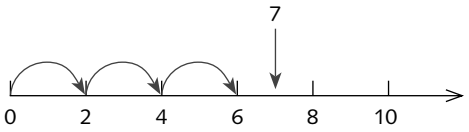
**6**



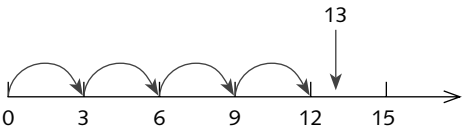
**7**



**8** **a** $7 = 3 \times 2 + 1$




**b** $13 = 4 \times 3 + 1$




**9** **a** 2 **b** 5 **c** 6
**d** 12 **e** 9 **f** 40

**10** **a** 14 **b** 8 **c** 23

**11** **a** 127 **b** 55 **c** 78
**d** 209 **e** 92 **f** 130

## Exercise 1J

**1** **a** 278 **b** 124 remainder 1
**c** 567 **d** 1078
**e** 121 **f** 912
**g** 136 **h** 2796
**i** 4360


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**2** **a** 131 **b** 365
**c** 83 **d** 53 remainder 7
**e** 67 remainder 3 **f** 813
**g** 11 257 remainder 1 **h** 2715

**3** \$850 **4** 45 boxes, 2 balls are left over

**5** 1 row of 96 biscuits, 2 rows of 48 biscuits, 3 rows with 32 biscuits, 4 rows with 24 biscuits, 6 rows with 16 biscuits, 8 rows with 12 biscuits, 12 rows with 8 biscuits, and so on

**6** 33 children **7** 37 people **8** 494 books

**9** 28 full cartons, 9 eggs are left over

**10** 761, 1 left over

**11** **a** 319 **b** 0

## Exercise 1K

**1** **a** 56 **b** 75 **c** 84 **d** 213

**2** **a** 42 **b** 92
**c** 76 **d** 17 remainder 9

**3** \$284 **4** \$526 **5** 45 cm

**6** 216 boxes, 20 golf balls are left over

**7** 47 children **8** 41 people

**9** 2 hours, 6 minutes and 8 seconds

## Exercise 1L

**1** **a** 32 **b** 12 **c** 5 **d** 7 **e** 9
**f** 20 **g** 10 **h** 0 **i** 0

**2** **a** 49 **b** 20 **c** 57
**d** 40 **e** 112 **f** 12

**3** **a** 4000 **b** 21 **c** 17 **d** 28
**e** 11 **f** 14 **g** 139 **h** 5
**i** 130 000 **j** 8

**4** **a** 5 **b** 19 **c** 0 **d** 23
**e** 1 **f** 440 **g** 190 **h** 6
**i** 246 **j** 113

**5** **a** $3 \times (6 + 4) = 30$ **b** $3 \times (7 - 6) \div 3 = 1$
**c** $8 \times (7 + 30 \div 5) = 104$ **d** $7 \times 3 \times (2 + 8) = 210$
**e** $(5 - 2) \times (1 + 23) \div 6 = 12$ **f** $(6 + 7) \times (11 + 1) = 156$

**6** **a** 100 **b** 100 **c** 8
**d** 40 **e** 4000

**7** **a** 18 **b** 14 **c** 13 **d** 45

**8** 743 **9** 213 **10** 720 **11** \$882

## Review exercise

**1** **a** 1305 **b** 24 **c** 27
**d** 10 **e** 2574 **f** 50
**g** 118 **h** 2680 **i** 118

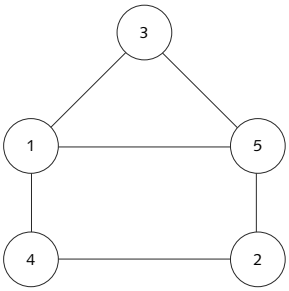
**2** 26 packets **3** 54 boxes **4** 170 people

**5** 15 buses **6** 2592 cans **7** 714 students

**8** 360 minutes **9** 72 **10** 377 hot dogs

## Challenge exercise

**1**

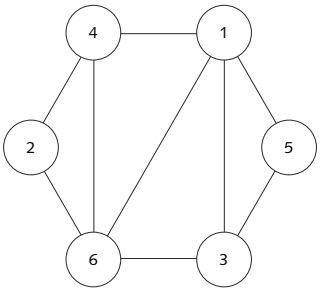



**2** 39 893, 39 993, 40 004, 40 104, 40 204

**3**

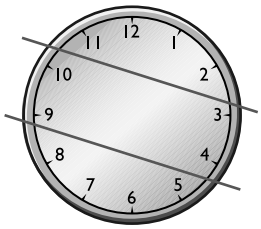
| | | |
|---|---|---|
| 19 | 18 | 8 |
| 4 | 15 | 26 |
| 22 | 12 | 11 |

**4**



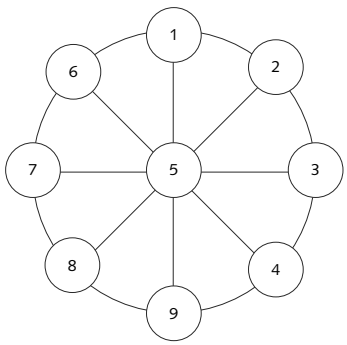

**5** One solution is $((6 + 6) - (6 \div 6)) \times 6 = 66$

**6**




This solution is unique.

**7**



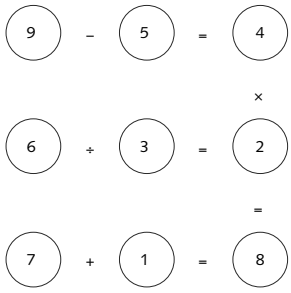

This has many solutions.


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**8** 36; this solution is unique.

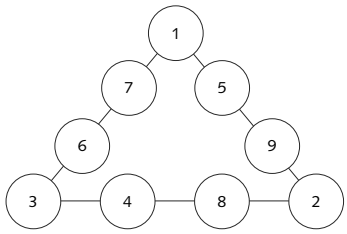
**9** One solution is $(6 \times 7 \times 2) + (3 \times 4) + 5 - 1 = 100$

**10**




This solution is unique.

**11**




This has many solutions.

**12** 1, 2, 4, 8, 8 or 1, 1, 3, 6, 12
or 1, 2, 2, 6, 12 or 1, 2, 3, 7, 10
or 1, 2, 3, 6, 11 or 1, 2, 4, 7, 9
or 1, 2, 4, 6, 10 or 1, 2, 4, 5, 11

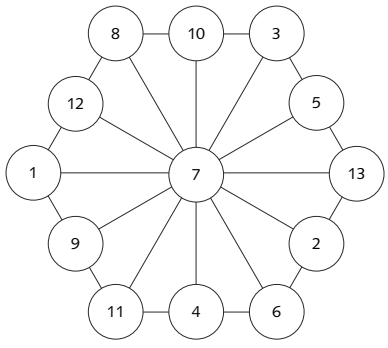
**13**

$$\begin{array}{r} 987 \\ \times\ 121 \\ \hline 987 \\ 19740 \\ 98700 \\ \hline 119427 \end{array}$$

**14**

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 6 | 2 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |

**15**




**16** 1, 3, 9, 27

**17** **a** 97531 + 86420 = 183951
**b** 13468 + 20579 = 34047
These solutions are not unique.

**18** 27 **19** 5050

# Chapter 2 answers

## Exercise 2A

**1** **a** 1, 2, 3 and 6 **b** 1, 2, 3, 4, 6 and 12
**c** 1, 2, 7 and 14 **d** 1, 3, 5 and 15
**e** 1 and 11

**2** **a** 1, 2, 4, 8 **b** 1, 2, 7, 14
**c** 1, 11 **d** 1, 2, 4, 8, 16, 32
**e** 1, 5, 25 **f** 1, 2, 3, 4, 6, 12
**g** 1, 2, 13, 26 **h** 1, 13
**i** 1, 3, 11, 33 **j** 1, 3, 9, 27, 81
**k** 1, 2, 3, 5, 6, 10, 15, 30 **l** 1, 2, 3, 6, 7, 14, 21, 42

**3** 6, 9, 27, 51 **4** 144, 192, 228

**5** **a** 0, 6, 12, 18, 24, 30, 36, 42, 48, 54, 60
**b** 0, 8, 16, 24, 32, 40, 48, 56, 64, 72, 80
**c** 0, 11, 22, 33, 44, 55, 66, 77, 88, 99, 110
**d** 0, 13, 26, 39, 52, 65, 78, 91, 104, 117, 130
**e** 0, 15, 30, 45, 60, 75, 90, 105, 120, 135, 150
**f** 0, 19, 38, 57, 76, 95, 114, 133, 152, 171, 190

**6** 68, 85, 170 **7** 3, 4, 6, 8, 9, 12, 24, 36

**8** **a** $12 \times 26$, $13 \times 24$
**b** $14 \times 28$
**c** $11 \times 13$
**d** $12 \times 45$, $15 \times 36$, $18 \times 30$, $20 \times 27$
**e** $13 \times 17$
**f** $12 \times 34$, $17 \times 24$

**9** 12: 1, 2, 3, 4, 6, 12; 18: 1, 2, 3, 6, 9, 18; common factors of 12 and 18: 1, 2, 3, 6

**10** 0, 6, 12, 18, 24, 30, 36, 42, 48, 54, 60; 0, 8, 16, 24, 32, 40, 48, 56, 64, 72, 80; common to both: 0, 24, 48

**11** **a** 49 **b** 98 **c** 203 **d** 301

**12** **a** 99 **b** 198 **c** 121 **d** 55
**e** 495 **f** 1001

**13** **a** 30; 15; 10; 7 remainder 2; 6
**b** 1, 2, 3, 5, 6, 10, 15, 30

**14** **a** 35; 17 remainder 1; 11 remainder 2; 8 remainder 3; 7; 5 remainder 5
**b** 1, 5, 7, 35

**15** **a** 1 is a factor of every number because every number is divisible by 1.
**b** Every number is a factor of 0 because any number multiplied by 0 equals 0.
**c** 1
**d** $0 = n \times 0$ where $n$ is any number.


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**16** factors of 6: 1, 2, 3; $1 + 2 + 3 = 6$
factors of 28: 1, 2, 4, 7, 14; $1 + 2 + 4 + 7 + 14 = 28$
factors of 496: 1, 2, 4, 8, 16, 31, 62, 124, 248;
$1 + 2 + 4 + 8 + 16 + 31 + 62 + 124 + 248 = 496$

**17** **a** It is a square number.
**b** 36 **c** 81

## Exercise 2B

**1** **a** 1, 3, 5, 15 **b** 2, 6, 10, 30
**c** 0, 6, 12, 18, 24, 30, 36, 42, 48, 54

**2** **a** 21, 23, 25, 27, 29, 31, 33
**b** 376, 378, 380, 382, 384, 386, 388, 390, 392
**c** 50 **d** 11

**3** **a** 215, 217 **b** 491, 493 **c** 513, 515

**4** Various answers are possible.

**5** **a** even **b** even **c** odd

**6** **a** **i** 48 **ii** 30 **iii** 35
**b** **i** even **ii** odd **iii** even

**7** **a** unreadable **b** readable
**c** unreadable **d** readable

**8** even

## Exercise 2C

**1** **a** 2, 3 **b** 3, 5 **c** 3, 7 **d** 2 **e** 5, 7

**2** **a** 2 and 3 **b** 3 and 5 **c** 3 and 7 **d** 5 and 11

**3** **a** 47 **b** 67 **c** 97

**4** 2 and any other prime number

**5** $32 = 3 + 29$, $34 = 3 + 31$, $36 = 5 + 31$,
$38 = 7 + 31$, $40 = 11 + 29$, $42 = 11 + 31$,
$44 = 13 + 31$, $46 = 17 + 29$, $48 = 11 + 37$,
$50 = 7 + 43$, $52 = 11 + 41$, $54 = 13 + 41$,
$56 = 13 + 43$, $58 = 5 + 53$, $60 = 7 + 53$,
$62 = 43 + 19$

**6** 11

## Exercise 2D

**1** $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, $2^5 = 32$, $2^6 = 64$,
$2^7 = 128$, $2^8 = 256$, $2^9 = 512$, $2^{10} = 1024$

**2** 3, 9, 27, 81, 243 **3** 5, 25, 125, 625, 3125

**4** **a** $4^5$ **b** $12^9$ **c** $5^2 \times 6^3$ **d** $7^3 \times 11^4$

**5** **a** 5 **b** 8 **c** 10 **d** 16 **e** 7
**f** 5 **g** 12 **h** 6 **i** 24

**6** **a** 5 **b** 9 **c** 4 **d** 1 **e** 2 **f** 3

**7** **a** $3^2 \times 5^2$ **b** $6^3 \times 7^4$ **c** $2^5 \times 3^5$
**d** $2^3 \times 3^3$ **e** $2 \times 5^2 \times 7^3$ **f** $3^3 \times 7^5$

**8** **a** 216 **b** 64 **c** 16
**d** 3125 **e** 125 **f** 16807

**9** **a** 18 **b** 36 **c** 100
**d** 400 **e** 16 **f** 64
**g** 600000 **h** 280000000

**10** **a** 60 **b** 18 **11** 1048576

## Exercise 2E

**1** **a** 210 **b** 80 **c** 90
**d** 120 **e** 360 **f** 540

**2** **a** 96 **b** 448 **c** 2128
**d** 336 **e** 1048 **f** 4032

**3** **a** 750 **b** 114 **c** 888 **d** 600
**e** 672 **f** 1024 **g** 108 **h** 296

**4** **a** 682 **b** 528 **c** 1089
**d** 783 **e** 3789

**5** **a** 86 **b** 174 **c** 127 **d** 34
**e** 27 **f** 51 **g** 33 **h** 21
**i** 31 **j** 127 **k** 42 **l** 56

**6** **a** 4000 **b** 2970 **c** 55
**d** 84 **e** 3303 **f** 240

## Exercise 2F

**1** **a** $2 \times 3$ **b** $2^3$ **c** $3^2$
**d** $2^2 \times 3$ **e** $2 \times 3^2$ **f** $2^3 \times 3$

**2** **a** $3 \times 5$ **b** $3 \times 5^2$ **c** $2^2 \times 3^2$
**d** $2^5 \times 3$ **e** $2^8$ **f** $29^2$

**3** **a** 900 **b** 10800 **c** 57600

**4** **a** $105 = 3 \times 5 \times 7$, $154 = 2 \times 7 \times 11$
**b** 7

**5** 30, 42, 66, 78

## Exercise 2G

**1** **a** 16 **b** 144 **c** 256
**d** 289 **e** 484 **f** 1089

**2** **a** 7 **b** 12 **c** 20
**d** 13 **e** 15 **f** 19
**g** 25 **h** 31 **i** 38
**j** 75

**3** **a** even **b** odd

**4** even

**5** odd


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**6** (*Note*: There may be other possibilities.)

**a** $14 = 1^2 + 2^2 + 3^2$ **b** $38 = 1^2 + 1^2 + 6^2$

**c** $45 = 2^2 + 4^2 + 5^2$ **d** $59 = 1^2 + 3^2 + 7^2$

**e** $70 = 3^2 + 5^2 + 6^2$ **f** $230 = 7^2 + 9^2 + 10^2$

**7** $10^2 - 7^2$

**8** $7^2 + 6^2$ or $2^2 + 9^2$ and $11^2 - 6^2$

## Exercise 2H

**1** **a** 24 **b** 72 **c** 17 **d** 60 **e** 49
**f** 180 **g** 12 **h** 72 **i** 120

**2** **a** 6 **b** 3 **c** 12 **d** 1 **e** 4
**f** 13 **g** 6 **h** 1 **i** 10

**3** **a** HCF is 112, LCM is 672
**b** HCF is 6, LCM is 126
**c** HCF is 15, LCM is 450

**4** 9:30 a.m. **5** 29 June **6** 5236 seconds

**7** 210

## Exercise 2I

**1** **a** yes **b** no **c** yes **d** yes

**2** **a** yes **b** yes **c** no
**d** no **e** yes

**3** **a** yes **b** yes **c** yes **d** no

**4** **a** yes **b** yes **c** no
**d** yes **e** yes

**5** **a** yes **b** yes **c** yes
**d** yes **e** yes

**6** **a** 792, 5838, 45891 and 19283746556000000001
**b** 792, 45891

**7** 2, 3, 4, 5, 6, 8, 10 **8** 2, 3, 6

**9** **a** 1002 **b** 1008 **c** 1004
**d** 1002 **e** 1008 **f** 1020

**10** (*Note*: There are many other possibilities.)

**a** 74424 **b** 78428 **c** 79425 **d** 72426
**e** 73422 **f** 74420 **g** 77424 **h** 76428
**i** 75420 **j** 73425 **k** 72420 **l** 76425

**11** **a** 1, 3, 7, 21, 49, 147
**b** 1, 3, 5, 15, 23, 69, 115, 345
**c** 1, 2, 3, 4, 6, 7, 8, 12, 14, 16, 21, 24, 28, 32, 42, 48, 56, 64, 84, 96, 112, 128, 168, 192, 224, 336, 384, 448, 672, 896, 1344, 2688

**12** **a** 108, 999 **b** 115, 989 **c** 207, 828

**13** **a** 67 **b** 33 **c** 167

**14** 2520

## Review exercise

**1** **a** 1, 2, 3, 6, 9, 18
**b** 1, 2, 3, 4, 6, 8, 12, 24
**c** 1, 5, 25
**d** 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
**e** 1, 2, 5, 10, 25, 50
**f** 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96

**2** **a** 0, 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99
**b** 0, 17, 34, 51, 68, 85
**c** 0, 33, 66, 99
**d** 0, 18, 36, 54, 72, 90
**e** 0, 15, 30, 45, 60, 75, 90
**f** 0, 21, 42, 63, 84

**3** **a**, **b** Various answers are possible.
**c** 3

**4** Various answers are possible.

**5** $6^1 = 6$, $6^2 = 36$, $6^3 = 216$, $6^4 = 1296$, $6^5 = 7776$, $6^6 = 46656$

**6** $9^1 = 9$, $9^2 = 81$, $9^3 = 729$, $9^4 = 6561$, $9^5 = 59049$

**7** **a** $3^3 \times 4^4$ **b** $8^7$
**c** $100^3$ **d** $3^3 \times 7^3$

**8** **a** 36 **b** 324 **c** 108
**d** 162 **e** 4096 **f** 1000

**9** **a** 68 **b** 136 **c** 85 **d** 204 **e** 340
**f** 425 **g** 109 **h** 124 **i** 52

**10** **a** 25 **b** 100 **c** 169
**d** 225 **e** 441 **f** 1849

**11** **a** 20 **b** 40 **c** 30
**d** 15 **e** 22 **f** 19

**12** **a** 54 **b** 99 **c** 153
**d** 198 **e** 252 **f** 297

**13** **a** 1008, 9999 **b** 1015, 9976 **c** 1044, 9918

**14** **a** 2 **b** 3 **c** 3 **d** 3
**e** 20 **f** 20 **g** 20

**15** **a** $2^9$ **b** $2^{10}$
**c** $5^3$ **d** $2^2 \times 5^3$
**e** $7 \times 11 \times 13$ **f** $7 \times 11 \times 13 \times 17$
**g** $7 \times 11 \times 13 \times 19$ **h** $2^5 \times 5^3$
**i** $2^{10} \times 5$ **j** $3^5$
**k** $5^2 \times 7^2$ **l** $2 \times 31 \times 43$

**16** **a** 1000 **b** 1001 **c** 29029
**d** 27000 **e** 14000 **f** 9800
**g** 343000 **h** 1225 **i** 2450

**17** 6, 10, 14, 15, 21, 22, 26, 33, 34, 35, 38, 39, 46


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## Challenge exercise

**1** 13 and 31, 17 and 71, 79 and 97

**2** **a** $7 \times 20 = 4 \times 20 + 3 \times 20 = 80 + 60 = 140$
**b** $7 \times 25 = 4 \times 25 + 3 \times 25 = 100 + 75 = 175$
**c** $72 \times 7 = (8 \times 9) \times 4 + (8 \times 9) \times 3 = 504$
**d** $78 \times 11 = (6 \times 13) \times 11 = 858$

**3** 114 is one such number.

**4** **a** 18, 20, 24, 30, 36
**b** The factors are:
1, 3, 5, 7, 9, 15, 21, 27, 35, 45, 63, 105, 135, 189, 315.
Sum of factors = 975 > 945.

**5** $65 = 1^2 + 8^2 = 4^2 + 7^2$, $85 = 2^2 + 9^2 = 6^2 + 7^2$

**6** **a** $73 = 1^3 + 2^3 + 4^3$ **b** $92 = 1^3 + 3^3 + 4^3$
**c** $99 = 2^3 + 3^3 + 4^3$

**7** **a** 2 **b** 2
**c** 1, 2, 3, 4 **d** 1, 3, 2, 4

**8** **a** $1^3 + 1^3 + 2^3$ **b** $1^3 + 12^3$
**c** $1^3 + 2^3 + 3^3$ **d** $1^3 + 1^3 + 2^3 + 3^3$
**e** $2^3 + 2^3 + 2^3 + 3^3$ **f** $2^3 + 3^3 + 4^3$
**g** $1^3 + 2^3 + 3^3 + 4^3$ **h** $3^3 + 5^3$
**i** $3^3 + 3^3 + 6^3$

**9** There are many possibilities.
**a** 120940 **b** 120945 **c** 120940
**d** 120945 **e** 120945

**10** $\square = 7$, $\Delta = 6$

**11** **a** 21 **b** 7 **c** 147

**12** **a** 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97
**b** 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199
**c** 14 (between 113 and 127)
**d** (3,5), (5,7), (11,13), (17, 19) (29, 31), (41, 43), (59, 61), (71, 73), (101, 103), (107, 109), (137, 139), (149, 151), (179, 181), (191, 193), (197, 199)
**e** The exception is (3, 5). Otherwise, all prime pairs are of the form $(6k - 1, 6k + 1)$, so their sum is $12k$ and product is $36k^2 - 1$.
**f** For any odd number $n$, one of the numbers $n$, $n + 2$ and $n + 4$ must be divisible by 3, so they can't all be primes, except when $n = 3$.

**13** **a** 2 **b** 24

**14** **a** $1^2 + 2^2 + 3^2 + 4^2$ **b** $1^2 + 3^2 + 5^2 + 6^2$
**c** $1^2 + 4^2 + 7^2 + 8^2$ **d** $2^2 + 2^2 + 8^2 + 12^2$
**e** $3^2 + 13^2 + 14^2 + 16^2$ **f** $4^2 + 5^2 + 17^2 + 18^2$

**15** $7^2 + 11^2$

**16** **a** 20002 **b** 10101 **c** 21012
**d** 50005 **e** 20202 **f** 10101
**g** 21112 **h** 10701

# Chapter 3 answers

## Exercise 3A

**1** $x + 3$

**2** **a** $a + 6$ **b** $p + 4$ **c** $p - 2$
**d** $y - 3$ **e** $x - 4$ **f** $x - 8$
**g** $p + q$ **h** $x + y + 2$ **i** $a + b + c$

**3** **a** $5 \times x$ **b** $x \times y$ **c** $3 \times x$ **d** $p \div q$

**4** **a** $7 \times x$ **b** $a \times b$ **c** $y - 6$
**d** $4 \times x + 3$ **e** $5 \times m - 3$ **f** $3 \times a + 2$

**5** **a** $x \div 3 + 2$ **b** $x \div 3 - 2$
**c** $p + q + r$ **d** $x \times y \times z$

**6** $n - 20$ **7** $5 \times x$ **8** $n \div 3$

**9** $\$(w + 1000)$ **10** $(x + 5)$ years old

**11** $x \div 4$ cm **12** $8 \times m$ **13** $10 - n$

**14** $n + 20$ **15** $s \div 3$ cm

## Exercise 3B

**1** **a** $5x$ **b** $2a$ **c** $mn$ **d** $6xy$
**e** $3x^2$ **f** $35x$ **g** $18ac$ **h** $6x^2y^2$
**i** $105x$ **j** $21x^2$ **k** $2x^2y^3$ **l** $3x + 9y$

**2** **a** $xy$ **b** $ab$ **c** $6p^2$
**d** $12x$ **e** $15a$ **f** $6p^2x^2$

**3** **a** $\frac{x}{4}$ **b** $\frac{x}{5}$ **c** $\frac{x}{7}$ **d** $\frac{z}{10}$
**e** $\frac{w}{z}$ **f** $\frac{q}{p}$ **g** $\frac{w}{x}$ **h** $\frac{x}{y}$

**4** **a** $5 \times a$ **b** $a \times a$ **c** $a \times b \times c$
**d** $7 \times a \times a$ **e** $7 \times x \times x \times y$

**5** **a** $x \div 3$ **b** $f \div 3$ **c** $x \div y$ **d** $mn \div p$

**6** **a** $7x$ **b** $x^2$ **c** $6ab$
**d** $21a^2$ **e** $\frac{x}{3}$ **f** $\frac{p}{q}$
**g** $7x + 5$ **h** $7m - 2$

**7** $\frac{x}{7}$ apples **8** $5x$ jelly beans

**9** $11x + 18y$ passengers **10** $\$(5x + 6z)$

**11** **a** $x^2$ **b** $x^2 + 3$ **c** $a^3$ **d** $a^3 + 3$


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## Exercise 3C

**1** **a** 7 **b** 9 **c** 10
**d** 23 **e** 103 **f** 1003
**g** 203 **h** 10003

**2** **a** 6 **b** 3 **c** 7 **d** 0

**3** **a** 20 **b** 8 **c** 40 **d** 348

**4** **a** 12 **b** 6 **c** 4 **d** 2

**5** **a** 4 **b** 9 **c** 81 **d** 100

**6** **a** 31 **b** 40 **c** 23 **d** 47
**e** 1 **f** 6 **g** 1 **h** 6

**7** **a** 9 **b** 18 **c** 8 **d** 4
**e** 12 **f** 16 **g** 4 **h** 16
**i** 14 **j** 12 **k** 40

**8** **a** 6 **b** 7 **c** 14 **d** 10

**9** **a** 2 **b** 4 **c** 1
**d** 0 **e** 10

**10** \$40 **11** \$300 **12** \$35 **13** 128

## Exercise 3D

**1** **a** like **b** like **c** like
**d** unlike **e** like **f** unlike
**g** unlike **h** like

**2** **a** $9x$ **b** $9x$ **c** $16x$
**d** $11x$ **e** $11x + 11y$ **f** $4x + 6y$
**g** $10x + 4y$ **h** $9x + 3y$ **i** $3x + 3y$

**3** **a** $11x$ **b** $15b$ **c** $15ab$
**d** $23ac$ **e** $3xy + 5xz$ **f** $8abc$
**g** $7x^2$ **h** $7x^2 + 2y^2$

**4** **a** $5x$ **b** $7xy$ **c** $4xy$
**d** $4a$ **e** $5x + 11y$ **f** $10x + 7y$
**g** $13xy$ **h** $8x^2$ **i** $8x^2 + 5x$
**j** $3x^2 + 4x$

**5** **a** $10y$ **b** $13xy$ **c** $7xz + 11xy$
**d** $11xy$ **e** $30xy$ **f** $17ab$

**6** **a** $5x + 11y$ **b** $3x + 8y$ **c** $2x + 15y$
**d** $95x + 20y$ **e** $70a + 30b$ **f** $80w$
**g** $6vw$ **h** $3xy$

**7** $n + (n + 1) + (n + 2) + (n + 3) + (n + 4) = 5n + 10$
$= 5(n + 2)$

## Exercise 3E

**1** **a** $3(x + 6)$ **b** $5(x - 7)$
**c** $4(x + 10)$ **d** $7(x - 11)$

**2** **a** $3x + 2$ **b** $3(2 + x)$
**c** $5x - 3$ **d** $5(x - 3)$

**3** **a** 16 **b** 13 **c** 6
**d** 9 **e** 17 **f** 28

**4** **a** 8 **b** 35 **c** 22
**d** 10 **e** 9 **f** 14

**5** **a** 30 **b** 12 **c** 9
**d** 13 **e** 10 **f** 45

**6** **a** $5m^2$ **b** $(5x)^2$ **c** $(2z)^3$ **d** $3a^3$

**7** **a** 80 **b** 400

**8** **a** 24 **b** 216

**9** **a** 81 **b** 38 **c** 55

**10** **a** $(n + 2)$ bananas **b** $5(n + 2)$ bananas

**11** $20(x + 4)$ seats

## Exercise 3F

**1** **a** $6a$ **b** $12x$ **c** $12m$ **d** $8x^2$
**e** $21x^2$ **f** $6xy$ **g** $6x^2y$ **h** $8x^2y^2$

**2** **a** $25n^2$ **b** $16z^2$ **c** $256z^2$ **d** $169z^2$

**3** **a** $12ab$ **b** $12x^2$ **c** $77ab$ **d** $66c^2$
**e** $24mn$ **f** $77m^2n^2$ **g** $12n^2$ **h** $42m^3$

**4** **a** $4x^5$ **b** $48x^3$ **c** $18x^3$
**d** $6x^2y^2$ **e** $8x^3y$ **f** $30y^2z^3$

**5** **a** $18x^4$ **b** $2x^2y^2$ **c** $100a^4$
**d** $100x^3y^2$ **e** $36x^3y$ **f** $20w^5$

## Exercise 3G

**1** **a** 13 **b** 301

**2** **a** $6x^2$ cm$^2$ **b** 150 cm$^2$ **c** 600 cm$^2$

**3** $(2xy + x^2)$ cm$^2$

**4** **a** $2n + 1$ **b** $2n + 2$

**5** **a** $3m + 2n$ cm$^2$ **b** $4m + 2n$ cm$^2$
**c** $3m + 2n$ cm$^2$ **d** $4m + 2n$ cm$^2$

**6** **a** yes **b** yes **c** yes **d** yes
**e** yes **f** yes **g** yes **h** yes
**i** no **j** yes **k** no **l** no

**7** **a** $b + 2$ **b** $a + 3$, $a + 6$ **c** $n - 2$

**8** **a** even **b** odd **c** even **d** odd
**e** odd **f** even **g** even **h** odd

**9** **a** $(8b + 4a)$ cm **b** $8ab$ cm$^2$


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**10** **a** $4\ell$ cm **b** $\ell^2$ cm$^2$

**11** **a** $(2x + 16)$ cm **b** $(6x + 12)$ cm$^2$

## Review exercise

**1** **a** $3 + x$ **b** $z - 6$ **c** $4x$
**d** $11 - t$ **e** $2b + 3$ **f** $4(z + 6)$
**g** $x^2 + 3$ **h** $\frac{x}{t} + 3$

**2** **a** 3 **b** 11 **c** 28 **d** 10
**e** 12 **f** 26 **g** 60 **h** 29

**3** 144 **4** 41

**5** $\frac{z}{4}$ metres **6** $\$(m + n + p)$ **7** $24 - n$

**8** **a** $mn$ **b** $6m^3$ **c** $7y^3$
**d** $24p^3$ **e** $30x^2y$ **f** $40z^3$

**9** **a** $\frac{x}{5}$ **b** $\frac{20}{y}$ **c** $\frac{m}{n}$

**10** **a** $11z$ **b** $\frac{x}{3}$ **c** $a^3$
**d** $x^4$ **e** $24b^2$ **f** $27x^3$

**11** **a** 53 **b** 10 **c** 79 **d** 125
**e** 20 **f** 254 **g** 625 **h** 3125

**12** **a** $9x$ **b** $5xy$ **c** $11x^2$
**d** $2xy$ **e** $4xy$ **f** $60x + 50y$

**13** **a** $6(8 + m)$ **b** $3(d - 7)$

**14** **a** $9n^2$ **b** $8n^3$ **c** $16n^3$ **d** $8x^4$

**15** **a** 20 **b** 16 **c** 39 **d** 14

**16** **a** $14x^2$ cm$^2$ **b** $3x^2$ cm$^2$ **c** $4x^2$ cm$^2$

**17** **a** 6 **b** 30 **c** 196 **d** 147

**18** **a** 103 **b** 10000 **c** 30 **d** 4

**19** **a** 16 **b** 5 **c** 9 **d** 384

**20** **a** $9a + b$ **b** $42a^2$
**c** $22a^2b$ **d** $4c^2d + 5cd$

**21** **a** $\frac{a^2}{2}$ cm$^2$ **b** $\frac{a^2}{4}$ cm$^2$

## Challenge exercise

**1** **a** $\frac{xy}{2}$ cm$^2$ **b** $xy$ cm$^2$ **c** $2xy$ cm$^2$

**2** **a** $2x - 5$ **b** $3(2x - 5) = 6x - 15$
**c** $6x - 15 + 15 = 6x$ **d** $6x - 5x = x$

**3** **a** ① $x^2$ cm$^2$ ② $6x$ cm$^2$ ③ $4x$ cm$^2$ ④ 24 cm$^2$
**b** $(x + 6)(x + 4)$ cm$^2$
**d** ① $4x$ cm$^2$ ② $(12 + 2x)$ cm$^2$
③ $(8 + 2x)$ cm ④ 20 cm
**e** $(4x + 20)$ cm

**4** **a** $(6x + 12)$ cm **b** $12x$ cm$^2$

**5** **a** rectangle $ABCD$, $18x$ cm$^2$; shaded rectangle, $6x$ cm$^2$
**b** $12x$ cm$^2$

**6** **a** 1, 4, 9, 6, 5, 0 **b** 1, 5, 7
**c** 2, 4, 6, 8, 0 **d** yes

**7** **a**

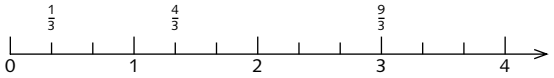
| $n$ | $n+2$ | $n-2$ | $(n+1)(n-2)$ | $n^2$ |
|---|---|---|---|---|
| 2 | 4 | 0 | 0 | 4 |
| 3 | 5 | 1 | 5 | 9 |
| 4 | 6 | 2 | 12 | 16 |
| 5 | 7 | 3 | 21 | 25 |
| 6 | 8 | 4 | 32 | 36 |
| 7 | 9 | 5 | 45 | 49 |

**c** $(n + 2)(n - 2) = n(n - 2) + 2(n - 2)$
$= n^2 - 2n + 2n - 4$
$= n^2 - 4$

# Chapter 4 answers

## Exercise 4A

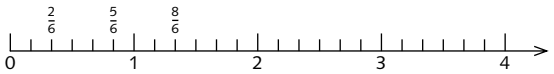
**1**




**2**




**3**




**4** **a** $\frac{7}{3}, \frac{16}{3}, \frac{21}{3}$ or $\frac{7}{1}$ **b** $\frac{14}{5}, \frac{26}{5}, \frac{48}{5}$
**c** $\frac{1}{4}, \frac{14}{4}, \frac{23}{4}$

**5, 6** Various answers are possible.

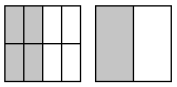
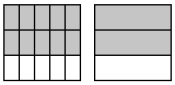
**7** **a** $\frac{12}{16} = \frac{3}{4}$ **b** $\frac{12}{9} = \frac{4}{3}$ **c** $\frac{7}{8}$ **d** $\frac{4}{8} = \frac{1}{2}$

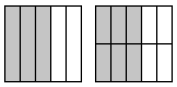
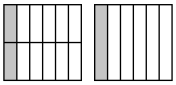
**8** **a** $\frac{11}{20}$ **b** $\frac{1}{4}$ **c** $\frac{6}{7}$
**d** $\frac{3}{4}$ **e** $\frac{14}{31}$ **f** $\frac{4}{12} = \frac{1}{3}$
**g** $\frac{43}{60}$ **h** $\frac{7}{18}$ **i** $\frac{1}{8}$
**j** $\frac{19}{23}$

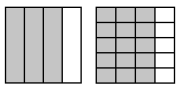
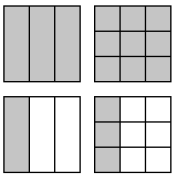
**9** **a** B **b** B


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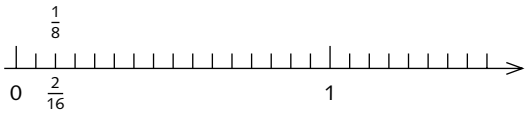
## Exercise 4B

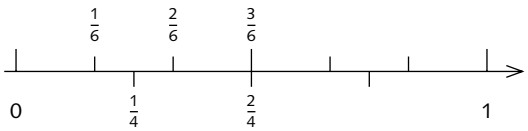
**1** **a**  **b** 

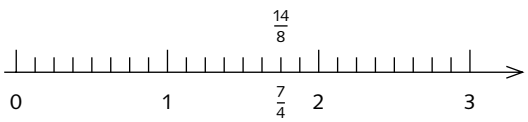
**c**  **d** 

**e**  **f** 

**2** **a** 

**b** 

**c** 

**d** 

**3** **b** $\times 3$ **c** $\times 2$ **d** $\times 4$

**4** **a** $\frac{1}{3} = \frac{5}{15} = \frac{25}{75} = \frac{40}{120}$ **b** $\frac{3}{4} = \frac{6}{8} = \frac{51}{68} = \frac{75}{100}$

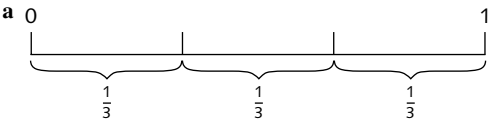
**c** $\frac{2}{3} = \frac{10}{15} = \frac{64}{96} = \frac{72}{108}$ **d** $\frac{1}{2} = \frac{6}{12} = \frac{17}{34} = \frac{51}{102}$

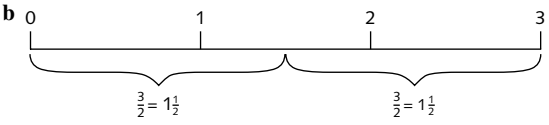
**e** $\frac{1}{8} = \frac{125}{1000} = \frac{8}{64} = \frac{2}{16}$ **f** $\frac{4}{5} = \frac{24}{30} = \frac{40}{50} = \frac{80}{100}$

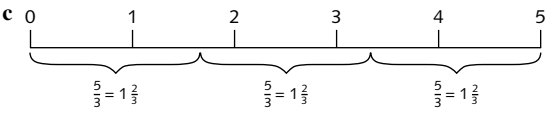
**5** **a** 50 **b** 9 **c** 120
**d** 15 **e** 11 **f** 36

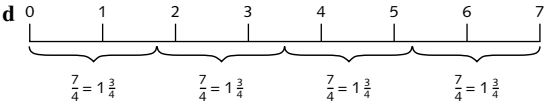
**6** **a** $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{3}{4}$
**d** $\frac{1}{3}$ **e** $\frac{1}{3}$ **f** $\frac{1}{4}$
**g** $\frac{1}{3}$ **h** $\frac{4}{5}$ **i** $\frac{3}{4}$
**j** $\frac{2}{3}$ **k** $\frac{1}{2}$ **l** $\frac{1}{8}$
**m** $\frac{1}{5}$ **n** 16 **o** $\frac{3}{2}$
**p** $\frac{7}{2}$ **q** 7 **r** $\frac{3}{5}$
**s** $\frac{3}{14}$ **t** $\frac{6}{13}$ **u** $\frac{3}{2}$

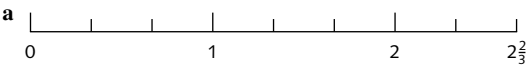
## Exercise 4C

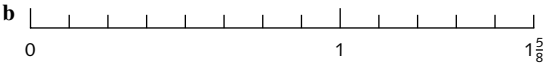
**1** **a** 

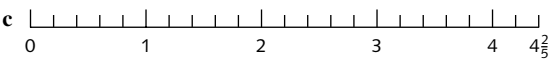
**b** 

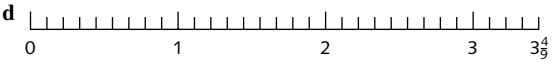
**c** 

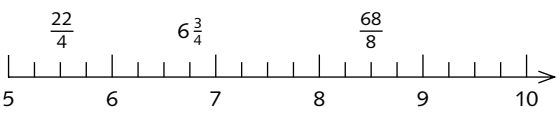
**d** 

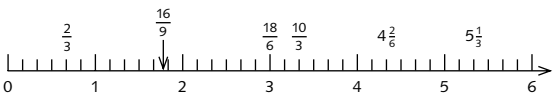
**2** **a** 

**b** 

**c** 

**d** 

**3** 

**4** 

**5** **a** $4\frac{4}{7}, 5\frac{6}{7}, 7\frac{3}{7}$ **b** $24\frac{1}{2}, 27\frac{5}{6}, 30\frac{1}{3}$

**6** **a** $3\frac{2}{3}$ **b** $5\frac{1}{3}$ **c** $4\frac{1}{2}$
**d** $3\frac{2}{3}$ **e** $4\frac{1}{3}$ **f** $1\frac{3}{4}$
**g** $4\frac{1}{27}$ **h** $4\frac{3}{4}$ **i** $5\frac{1}{2}$
**j** $8\frac{32}{35}$ **k** $11\frac{22}{25}$ **l** $8\frac{6}{25}$

**7** **a** $\frac{45}{7}$ **b** $\frac{19}{4}$ **c** $\frac{43}{5}$
**d** $\frac{31}{11}$ **e** $\frac{28}{5}$ **f** $\frac{104}{9}$
**g** $\frac{103}{10}$ **h** $\frac{43}{2}$ **i** $\frac{101}{3}$
**j** $\frac{23}{4}$ **k** $\frac{27}{7}$ **l** $\frac{21}{8}$

**8** **a** $\frac{88}{5}$ **b** $\frac{881}{30}$ **c** $\frac{1105}{21}$ **d** $\frac{609}{13}$


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**9** **a** $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1\frac{1}{4}, 1\frac{1}{2}, 1\frac{3}{4}, 2, 2\frac{1}{4}, 2\frac{1}{2}, 2\frac{3}{4}$

**b** $\frac{1}{3}, \frac{2}{3}, 1, 1\frac{1}{3}, 1\frac{2}{3}, 2, 2\frac{1}{3}, 2\frac{2}{3}, 3, 3\frac{1}{3}, 3\frac{2}{3}, 4$

**c** $\frac{3}{8}, \frac{3}{4}, 1\frac{1}{8}, 1\frac{1}{2}, 1\frac{7}{8}, 2\frac{1}{4}, 2\frac{5}{8}, 3, 3\frac{3}{8}, 3\frac{3}{4}$

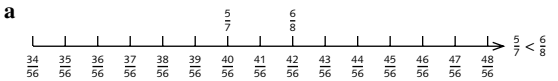
**d** $\frac{4}{9}, \frac{7}{9}, 1\frac{1}{9}, 1\frac{4}{9}, 1\frac{7}{9}, 2\frac{1}{9}, 2\frac{4}{9}, 2\frac{7}{9}, 3\frac{1}{9}, 3\frac{4}{9}$

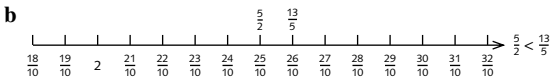
**10** $\frac{3}{4}$ 

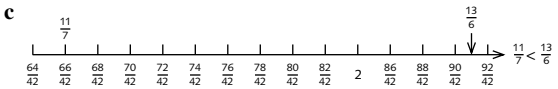
## Exercise 4D

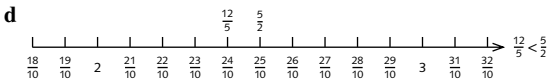
**1** **a – c** Various answers are possible.

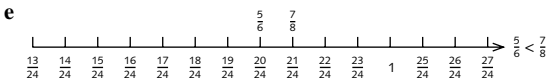
**2** Number lines are not required.

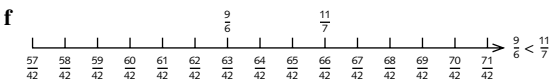
**a** 


**b** 


**c** 


**d** 


**e** 


**f** 


**3** **a** $\frac{13}{6}$ **b** $\frac{47}{8}$ **c** $\frac{3}{2}$

**d** $\frac{107}{25}$ **e** $\frac{130}{64}$ **f** $\frac{143}{12}$

**4** **a** $\frac{2}{3}$ **b** $\frac{9}{8}$ **c** $\frac{8}{17}$

**d** $\frac{1}{8}$ **e** $\frac{17}{27}$

**5** **a** $\frac{1}{4}, \frac{3}{8}, \frac{1}{2}, \frac{3}{4}, \frac{9}{8}, \frac{10}{8}$ **b** $\frac{3}{6}, \frac{5}{6}, \frac{8}{9}, \frac{4}{3}, \frac{9}{6}, \frac{5}{3}$

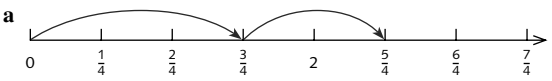
**c** $\frac{1}{3}, \frac{4}{5}, \frac{4}{3}, \frac{21}{15}, \frac{11}{6}, \frac{19}{10}$ **d** $\frac{3}{7}, \frac{5}{9}, \frac{3}{5}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}$

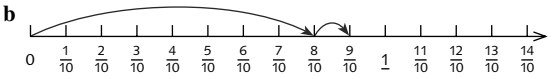
**e** $\frac{1}{4}, \frac{1}{3}, \frac{4}{11}, \frac{8}{16}, \frac{7}{12}, \frac{3}{5}$ **f** $\frac{3}{8}, \frac{2}{5}, \frac{11}{20}, \frac{23}{40}, \frac{7}{10}, \frac{3}{4}$

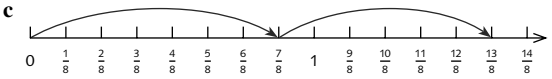
**6** **a** $\frac{7}{16}$ **b** $\frac{5}{6}$ **c** $\frac{17}{15}$ **d** $\frac{157}{20}$

**7** $\frac{99}{100}$ is larger as it is closer to 1.

## Exercise 4E

**1** **a** 


**b** 


**c** 


**2** **a** $\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$ **b** $\frac{5}{16} + \frac{8}{16} = \frac{13}{16}$

**c** $\frac{32}{100} + \frac{41}{100} = \frac{73}{100}$

**3** **a** $\frac{7}{10}$ **b** $\frac{5}{6}$ **c** $\frac{7}{8}$ **d** $\frac{11}{12}$

**e** $1\frac{7}{25}$ **f** $1\frac{1}{30}$ **g** $1\frac{4}{21}$ **h** $1\frac{2}{15}$

**4** **a** $1\frac{5}{8}$ **b** $1\frac{1}{2}$ **c** $1\frac{2}{5}$

**d** $1\frac{5}{12}$ **e** $1\frac{2}{5}$ **f** $1\frac{1}{4}$

**g** $1\frac{1}{30}$ **h** $1\frac{2}{3}$ **i** $1\frac{44}{45}$

**j** $1\frac{19}{40}$ **k** $1\frac{11}{30}$ **l** $1\frac{87}{136}$

**5** **a** $\frac{1}{8}$ **b** $\frac{1}{12}$ **c** $\frac{4}{25}$

**d** $\frac{1}{8}$ **e** $\frac{5}{8}$ **f** $1\frac{1}{2}$

**g** $\frac{3}{10}$ **h** $\frac{1}{9}$ **i** $\frac{4}{15}$

**j** $\frac{23}{35}$ **k** $\frac{19}{60}$ **l** $\frac{47}{72}$

**6** **a** $1\frac{1}{5}$ **b** $\frac{15}{17}$ **c** $2\frac{1}{4}$

**d** $\frac{6}{7}$ **e** $\frac{3}{8}$ **f** $\frac{2}{9}$

**g** $2\frac{5}{8}$ **h** $\frac{1}{2}$ **i** $\frac{37}{60}$

**j** $\frac{61}{126}$ **k** $\frac{7}{9}$ **l** $2\frac{239}{264}$

**7** **a – c** Various answers are possible.

**8** **a** $\frac{4}{7}$ **b** $\frac{1}{4}$ **c** $\frac{13}{32}$ **d** $\frac{101}{144}$

**9** **a** $3\frac{1}{2}$ **b** 6 **c** $7\frac{1}{2}$ **d** $6\frac{1}{2}$

**e** $11\frac{13}{21}$ **f** $8\frac{2}{25}$ **g** $5\frac{7}{15}$ **h** $13\frac{13}{20}$

## Exercise 4F

**1** $\frac{11}{15}$ L **2** $\frac{7}{12}$ **3** $\frac{11}{12}$


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**4** $\frac{8}{15}$ **5** $\frac{3}{5}$ L **6** $\frac{8}{9}$

**7** $\frac{5}{8}$ **8** $\frac{1}{6}$ **9** $\frac{9}{16}$

**10** **a** $\frac{2}{7}$ **b** $\frac{1}{3}$ **c** $\frac{13}{21}$

**11** **a** $\frac{7}{18}$ **b** $\frac{31}{54}$ **c** $\frac{23}{54}$

## Exercise 4G

**1** **a** $1\frac{1}{5}$ **b** $1\frac{1}{3}$ **c** $3\frac{1}{2}$

**d** $5\frac{3}{8}$ **e** $1\frac{3}{4}$ **f** $1\frac{1}{8}$

**g** $5\frac{3}{20}$ **h** $\frac{11}{40}$ **i** $\frac{7}{40}$

**2** **a** $\frac{25}{72}$ **b** $\frac{7}{10}$ **c** $\frac{9}{20}$

**d** $\frac{3}{4}$ **e** $1\frac{7}{9}$ **f** $\frac{5}{24}$

**g** $41\frac{8}{15}$ **h** $2\frac{27}{35}$ **i** $\frac{25}{156}$

**j** $2\frac{53}{110}$ **k** $55\frac{39}{44}$ **l** $\frac{13}{15}$

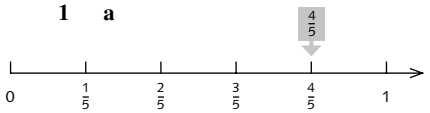
**3** $77\frac{39}{44}$ **4** $26\frac{29}{84}$ **5** $4\frac{1}{2}$

**6** **a** $2\frac{1}{8}$ **b** $2\frac{1}{5}$

**7** $3\frac{1}{12}$

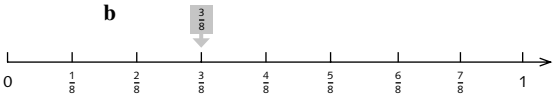
## Review exercise

**1** **a**






**b**






**2** **a** $\frac{21}{31}$ **b** $\frac{5}{26}$ **c** $\frac{5}{10} = \frac{1}{2}$ **d** $\frac{343}{500}$

**3** **a** 4 **b** 8 **c** 2

**d** 6 **e** 36 **f** 2

**4** **a** 6 **b** 12 **c** 120

**5** **a** $\frac{3}{4}$ **b** $1\frac{1}{3}$ **c** $1\frac{1}{20}$

**d** $1\frac{5}{27}$ **e** $1\frac{3}{4}$ **f** $\frac{1}{3}$

**6** **a** $1\frac{13}{24}$ **b** $\frac{11}{12}$ **c** $1\frac{2}{5}$

**d** $\frac{73}{230}$ **e** $\frac{31}{33}$ **f** $\frac{21}{22}$

**g** $1\frac{28}{33}$ **h** $1\frac{89}{110}$

**7** **a** $6\frac{3}{8}$ **b** $14\frac{19}{20}$ **c** $8\frac{1}{56}$

**d** $7\frac{5}{24}$ **e** $4\frac{1}{12}$ **f** $7\frac{1}{21}$

**g** $3\frac{14}{33}$ **h** $30\frac{1}{10}$

**8** **a** $\frac{3}{40}$ **b** $\frac{33}{40}$ **c** $\frac{1}{12}$

**d** $\frac{11}{240}$ **e** $\frac{13}{30}$ **f** $\frac{97}{230}$

**g** $1\frac{7}{12}$ **h** $\frac{19}{60}$

**9** **a** $2\frac{1}{3}$ **b** $3\frac{1}{2}$ **c** $3\frac{5}{24}$

**d** $4\frac{1}{12}$ **e** $2\frac{7}{12}$ **f** $3\frac{3}{4}$

**g** $\frac{5}{6}$ **h** $5\frac{53}{60}$

**10** **a** $\times 27$ **b** $\times 5$ **c** $\times 3$ **d** $\times 7$

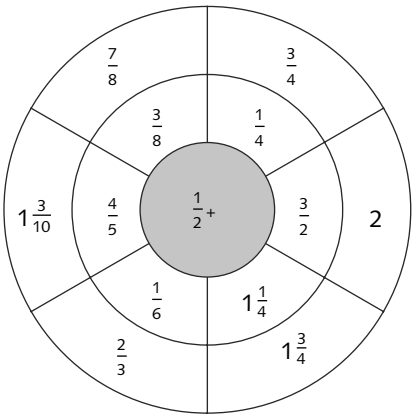
**11** **a** +, −

**b** −, −

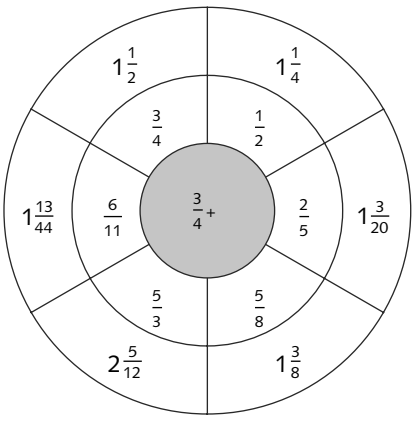
**c** −, −, + or +, +, −

**d** +, −, −

**12** **a**




**b**




**13** $\frac{1}{16}$ **14** $\frac{5}{6}$

**15** **a** $\frac{13}{25}$ **b** $\frac{3}{5}$



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## Challenge exercise

**1** $\frac{3005}{3006}$ is larger.

**2** $\frac{1}{8}+\frac{1}{24}, \frac{1}{9}+\frac{1}{18}, \frac{1}{10}+\frac{1}{15}, \frac{1}{12}+\frac{1}{12}$

**3** $\frac{1}{9}+\frac{1}{72}, \frac{1}{10}+\frac{1}{40}, \frac{1}{12}+\frac{1}{24}, \frac{1}{16}+\frac{1}{16}$

**4** Various answers are possible.

**5** $x=\frac{12}{5}$

**6** $\frac{13}{30}$ and $\frac{37}{60}$

**7** **a**

| | | |
|---|---|---|
| $2\frac{1}{2}$ | $2\frac{1}{3}$ | $\frac{2}{3}$ |
| 0 | $1\frac{5}{6}$ | $3\frac{2}{3}$ |
| 3 | $1\frac{1}{3}$ | $1\frac{1}{6}$ |

**b**

| | | |
|---|---|---|
| 3 | $2\frac{3}{10}$ | $2\frac{4}{5}$ |
| $2\frac{1}{2}$ | $2\frac{7}{10}$ | $2\frac{9}{10}$ |
| $2\frac{3}{5}$ | $3\frac{1}{10}$ | $2\frac{2}{5}$ |

**8** **a** $\frac{1}{4}$ **b** $\frac{1}{8}$ **c** $\frac{1}{16}$ **d** $\frac{1}{128}$

**e** The answer is always the smallest fraction in the brackets. This means that the sum of the fractions is getting closer and closer to 1.

**9** 45 marbles **10** 29 black sheep

# Chapter 5 answers

## Exercise 5A

**1** **a** $\frac{3}{20}$ **b** $\frac{15}{32}$ **c** $\frac{1}{16}$ **d** $\frac{2}{9}$ **e** $\frac{15}{28}$ **f** $\frac{15}{88}$ **g** $4\frac{9}{10}$ **h** $\frac{9}{14}$ **i** $\frac{5}{6}$

**2** **a** $2\frac{6}{7}$ **b** $1\frac{10}{11}$ **c** $2\frac{11}{12}$ **d** $7\frac{1}{3}$ **e** $3\frac{3}{4}$ **f** $1\frac{1}{11}$

**3** **a** $\frac{1}{6}$ **b** $\frac{1}{9}$ **c** $\frac{15}{32}$ **d** 1 **e** $\frac{3}{4}$ **f** $\frac{8}{27}$ **g** $2\frac{2}{3}$ **h** $10\frac{2}{5}$

**4** **a** 1 **b** $\frac{5}{21}$ **c** $\frac{3}{8}$ **d** $\frac{1}{35}$ **e** $\frac{1}{3}$ **f** $1\frac{2}{5}$ **g** $\frac{1}{2}$ **h** $\frac{1}{5}$

**5** **a** $\frac{1}{36}$ **b** $\frac{15}{64}$ **c** $\frac{7}{32}$ **d** $\frac{1}{4}$ **e** $\frac{5}{18}$ **f** $\frac{4}{27}$ **g** $\frac{1}{9}$ **h** 1 **i** $\frac{2}{9}$ **j** $\frac{2}{15}$ **k** $\frac{4}{9}$ **l** $\frac{3}{10}$

## Exercise 5B

**1** **a** $6\frac{2}{3}$ **b** $4\frac{2}{3}$ **c** $2\frac{2}{5}$ **d** $8\frac{3}{4}$ **e** $3\frac{3}{8}$ **f** $9\frac{1}{6}$ **g** $10\frac{1}{2}$ **h** $13\frac{1}{5}$

**2** **a** $1\frac{1}{20}$ **b** $1\frac{7}{15}$ **c** $\frac{24}{55}$ **d** $\frac{9}{22}$ **e** $\frac{15}{16}$ **f** $1\frac{1}{6}$ **g** $\frac{55}{84}$ **h** $\frac{65}{77}$

**3** **a** 2 **b** $\frac{1}{2}$ **c** $1\frac{1}{2}$ **d** $\frac{4}{5}$ **e** $1\frac{1}{5}$ **f** $1\frac{1}{2}$ **g** $\frac{20}{21}$ **h** $\frac{65}{77}$ **i** $1\frac{1}{2}$ **j** $\frac{3}{4}$ **k** $\frac{2}{9}$ **l** $1\frac{1}{8}$

**4** **a** $\frac{1}{2}$ **b** 1 **c** $\frac{9}{16}$ **d** $\frac{392}{729}$ **e** $\frac{16}{81}$ **f** $\frac{25}{64}$ **g** $\frac{2}{9}$ **h** $\frac{1}{2}$ **i** $1\frac{2}{3}$ **j** $3\frac{3}{5}$ **k** $\frac{2}{5}$ **l** $\frac{4}{7}$ **m** 4 **n** $\frac{1}{2}$ **o** $\frac{2}{5}$ **p** $\frac{2}{45}$

**5** 6 **6** 16 **7** 32

## Exercise 5C

**1** **a** $34\frac{3}{8}$ **b** 18 **c** $2\frac{26}{27}$ **d** $1\frac{2}{9}$ **e** $\frac{9}{14}$ **f** $3\frac{3}{20}$ **g** $3\frac{11}{27}$ **h** $\frac{9}{20}$ **i** $16\frac{2}{3}$ **j** 34 **k** $14\frac{4}{7}$ **l** 18

**2** **a** 4 **b** $3\frac{1}{5}$ **c** $1\frac{8}{9}$ **d** $4\frac{1}{2}$ **e** $6\frac{1}{24}$ **f** $1\frac{13}{20}$ **g** $1\frac{13}{51}$ **h** $5\frac{11}{25}$


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**3** **a** 2 **b** $2\frac{8}{9}$ **c** $2\frac{29}{35}$ **d** $1\frac{37}{47}$

**e** $\frac{64}{75}$ **f** $3\frac{9}{25}$ **g** $\frac{85}{288}$ **h** $\frac{3}{4}$

**i** $\frac{76}{81}$ **j** $\frac{3}{20}$ **k** $1\frac{1}{5}$ **l** $1\frac{29}{57}$

**4** $6\frac{1}{4}$ L

**5** **a** $2\frac{1}{6}$ **b** $3\frac{13}{16}$ **c** $5\frac{17}{40}$

**6** $\frac{5}{18}$ m **7** $\frac{17}{24}$ m **8** \$7.50

**9** **a** $13\frac{1}{4}$ **b** $1\frac{25}{28}$

## Exercise 5D

**1** 64

**2** **a** 560 **b** $\frac{7}{12}$

**3** **a** 4375 **b** $\frac{1}{8}$

**4** **a** $\frac{2}{5}$ **b** 25

**5** **a** $\frac{1}{4}$

**b** Jan's is \$56 000, David's is \$16 000, Greg's is \$24 000

**6** **a** $\frac{1}{3}$

**b** Back stalls has 450 seats, front stalls has 270 seats, balcony has 360 seats

**7** $11\frac{1}{4}$ L

**8** **a**

| | | |
|---|---|---|
| $\frac{1}{6}$ | $\frac{7}{12}$ | $\frac{1}{2}$ |
| $\frac{3}{4}$ | $\frac{5}{12}$ | $\frac{1}{12}$ |
| $\frac{1}{3}$ | $\frac{1}{4}$ | $\frac{2}{3}$ |

**b**

| | | |
|---|---|---|
| 2 | $\frac{3}{4}$ | 1 |
| $\frac{1}{4}$ | $1\frac{1}{4}$ | $2\frac{1}{4}$ |
| $1\frac{1}{4}$ | $1\frac{3}{4}$ | $\frac{1}{2}$ |

**9** **a** $\frac{2}{5}$ **b** 20

**10** **a** 400 girls **b** 1000 students

**11** \$4608 **12** $\frac{1}{6}$

## Exercise 5E

**1** **a** $\frac{19}{24}$ **b** $\frac{7}{15}$ **c** $\frac{13}{15}$

**2** **a** $1\frac{5}{16}$ **b** $1\frac{1}{8}$ **c** $\frac{5}{8}$ **d** $\frac{59}{72}$

**e** $1\frac{5}{18}$ **f** $\frac{5}{16}$ **g** $\frac{1}{4}$ **h** $1\frac{1}{6}$

**i** $4\frac{7}{12}$ **j** $\frac{19}{24}$ **k** $\frac{3}{10}$ **l** $\frac{3}{8}$

**m** $\frac{5}{6}$ **n** $1\frac{3}{7}$ **o** $\frac{23}{210}$ **p** $3\frac{17}{30}$

## Review exercise

**1** **a** $\frac{1}{2}$ **b** $10\frac{1}{3}$ **c** $\frac{1}{4}$ **d** $\frac{8}{25}$

**e** 66 **f** $17\frac{7}{8}$ **g** $1\frac{19}{20}$ **h** $28\frac{1}{3}$

**2** **a** $1\frac{3}{5}$ **b** $2\frac{3}{11}$ **c** $18\frac{1}{2}$ **d** $3\frac{2}{3}$

**3** **a** $8\frac{1}{3}$ **b** $4\frac{8}{15}$ **c** $36\frac{2}{3}$ **d** $1\frac{2}{17}$

**4** 42 **5** 36 **6** $1\frac{1}{2}$ m **7** $\frac{1}{6}$

**8** **a** 12 m **b** \$15 **c** 1400 kg

**9** $\frac{3}{10}$ **10** 385 pages **11** \$39.20 **12** 390 ha

## Challenge exercise

**1** $x = \frac{1}{2} \times 3 = \frac{3}{2}$ **2** $x = 3 \times \frac{1}{4} = \frac{3}{4}$

**3** $\frac{1}{5}$ **4** 2

**5** **a** $1 + \dfrac{1}{1 + \frac{1}{2}} = 1 + \dfrac{1}{\frac{3}{2}} = 1 + \frac{2}{3} = 1\frac{2}{3}$

**b** $1 + \dfrac{1}{1 + \dfrac{1}{1 + \frac{1}{2}}} = 1 + \dfrac{1}{1 + \dfrac{1}{\frac{3}{2}}} = 1 + \dfrac{1}{1 + \frac{2}{3}}$

$= 1 + \dfrac{1}{\frac{5}{3}} = 1 + \frac{3}{5} = 1\frac{3}{5}$

**6** **a** $3\frac{1}{2} \times \frac{2}{3} = 2\frac{1}{3}$ **b** $1\frac{2}{3} \times 1\frac{2}{5} = 2\frac{1}{3}$

**7** 6

**8** $\frac{1}{n}$ and $\frac{1}{n+1}$ for any non-zero $n$.

**9** 2 L, 1 L, $\frac{1}{2}$ L, $\frac{1}{4}$ L, $\frac{1}{8}$ L (that is, half of what was there at the start of each night). No, there would always have been a bit left.

**10** **a** Nathan **b** $35\frac{47}{48}$ km **c** $3\frac{5}{24}$ km

**11** day 1, 240; day 2, 288; day 4, 315; day 5, 21

**12** 1 **13** $a = 1$, $b = 5$, $c = 2$

**14** 450 passengers



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# Chapter 6 answers

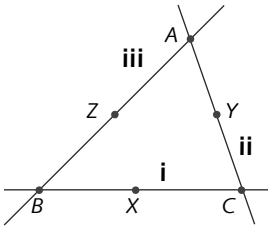
## Exercise 6A

**1** $AB$ or $BA$, $BC$ or $CB$, $CA$ or $AC$

**2** **a** $AB \parallel DC$ **b** $BC \parallel AD$ **c** $B$ **d** $C$
**e** No, the two lines are parallel.

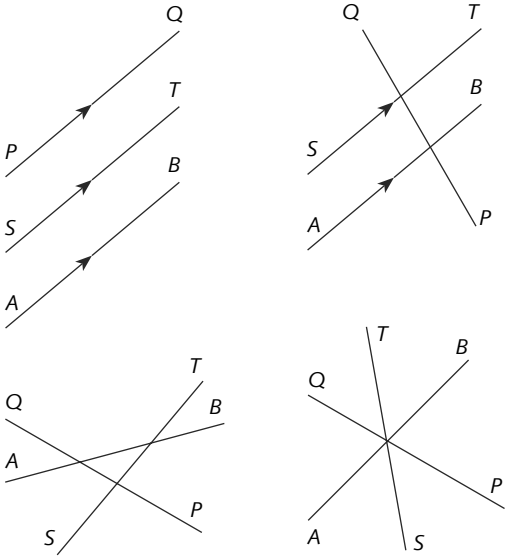
**3** **a** $AX$, $BX$, $CX$, $YX$ **b** $AC \parallel XY$
**c** $B$ **d** $CX$ **e** $A$, $B$, $C$
**f** None, the two lines are parallel.

**4** **a**

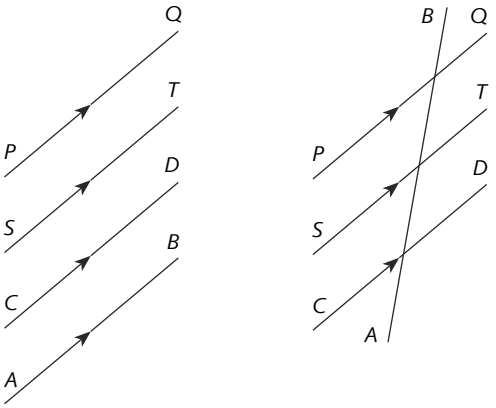


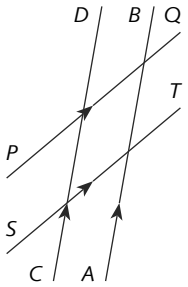

**b** The three lines are concurrent.

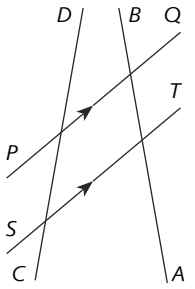
**8**

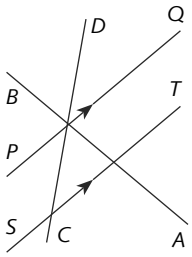


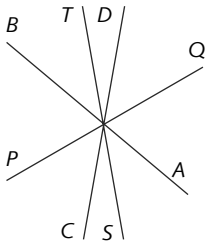

**9**

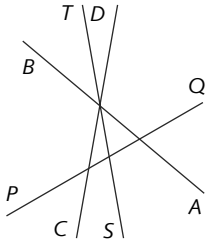


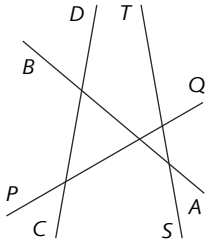



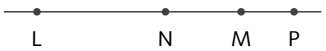




## Exercise 6B

**1** **a** **i** $A$, $B$, $C$ and $D$ **ii** $B$, $C$ and $D$
**iii** $B$ and $C$ **iv** $A$, $B$, $C$ and $D$
**v** $A$, $B$ and $C$ **vi** $A$, $B$, $C$ and $D$
**b** opposite
**c** the interval $BC$

**2** **a** concurrent **b** yes **c** no
**d** ray $PA$ and ray $PK$
**e** ray $AP$, ray $AB$, ray $AO$ and ray$AK$

**3** **a**




**b** yes **c** no

**4** **a** $\angle ABC$ **b** $\angle KLM$ **c** $\angle XVZ$

**5** **a** $\angle FGH$, $\angle GHF$, $\angle HFG$
**b** $\angle RST$, $\angle STU$, $\angle TUR$, $\angle URS$

**6** **a** $\angle AOB$ **b** $\angle LOM$, $\angle LMN$
**c** $\angle FVH$ **d** $\angle AMD$
**e** $\angle ALX$, $\angle LMB$ **f** $\angle PSQ$, $\angle SRQ$

**7** **a** $\angle BAD$, $\angle BCD$, $\angle ADC$, $\angle ADB$, $\angle CDB$, $\angle ABC$, $\angle ABD$, $\angle CBD$
**b** $\angle BAC$, $\angle CAD$, $\angle DAB$, $\angle ABC$, $\angle CBD$, $\angle DBA$, $\angle ACB$, $\angle BCD$, $\angle DCA$, $\angle ADB$, $\angle BDC$, $\angle CDA$


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## Exercise 6C

**1** **a** 75° **b** 55° **c** 18° **d** 2° **e** 34°

**2** **a** 20° **b** 80° **c** 105° **d** 92° **e** 34°

**3** **a** acute **b** obtuse **c** reflex **d** right

**4** **a** 180° **b** 90° **c** 30° **d** 60°
**e** 120° **f** 120° **g** 60° **h** 150°

**5** **a** 360° **b** 180° **c** 270° **d** 30°
**e** 300° **f** 150° **g** 720° **h** 540°

**6** **a** 52° **b** 122°

**7** **a** **i** 42° **ii** 122° **iii** 56° **iv** 134°
**b** **i** 318° **ii** 238° **iii** 304° **iv** 226°

**8** Various answers are possible.

**9** **a** $\angle AOB = 27°$, $\angle BOC = 34°$, $\angle COD = 44°$
**b** $\angle AOD = 105°$
**c** $\angle AOB$, $\angle BOC$ and $\angle COD$
**d** 255°
**e** 333°

**10** **a** $\angle ALP = 40°$, $\angle ALQ = 140°$
**b** supplementary
**c** $\angle ALP = 40°$, $\angle BLQ = 40°$
**d** equal
**e** 360°

**11** **a** $\angle PQA = 63°$, $\angle PRL = 63°$
**b** equal
**c** $\angle AQR = 117°$, $\angle LRQ = 63°$
**d** supplementary
**e** $\angle BQR = 63°$, $\angle QRL = 63°$
**f** equal

**12** **a** $\angle URS = 56°$, $\angle UTS = 56°$
**b** equal
**c** $\angle RUT = 124°$, $\angle STU = 56°$
**d** supplementary
**e** 360°
**f** $\angle RUT$ and $\angle RST$ are obtuse, $\angle URS$ and $\angle UTS$ are acute.

## Exercise 6D

**5** **a** 75° (right angles at $O$)
**b** 115° (adjacent angles at $O$)
**c** 155° (straight angle at $O$)
**d** 123° (adjacent angles at $O$)
**e** 50° (straight angle at $O$)
**f** 180° (straight angle at $O$)
**g** 150° (revolution at $O$)
**h** 170° (revolution at $O$)
**i** 130° (adjacent angles at $O$)
**j** 20° (revolution at $O$)
**k** 30° (straight angle at $O$)
**l** 30° (revolution at $O$)

**6** **a** $\angle SQT$ **b** $\angle VQU$ **c** $\angle AQB$ **d** $\angle NQO$

**7** **a** $\alpha = 70°$ (vertically opposite angles at $B$)
**b** $\beta = 90°$ (vertically opposite angles at $K$),
$\theta = 90°$ (straight angle at $K$)
**c** $\alpha = 50°$ (vertically opposite angles at $V$),
$\beta = 130°$ (straight angle at $V$)
**d** $\alpha = 50°$ (straight angle at $M$),
$\beta = 90°$ (vertically opposite angles at $M$),
$\gamma = 40°$ (vertically opposite angles at $M$),
$\theta = 50°$ (straight angle at $M$)
**e** $\alpha = 60°$ (vertically opposite angles at $O$),
$\beta = 60°$ (straight angle at $O$),
$\gamma = 60°$ (vertically opposite angles at $O$),
$\theta = 60°$ (straight angle at $O$)
**f** $\theta = 75°$ (vertically opposite angles at $V$)
**g** $\alpha = 50°$ (straight angle at $W$),
$\beta = 50°$ (straight angle at $W$)
**h** $\alpha = 70°$ (vertically opposite angles at $M$),
$\beta = 110°$ (straight angle at $M$),
$\gamma = 30°$ (straight angle at $D$),
$\theta = 110°$ (straight angle at $D$)

**8** **a** $\alpha = 60°$ (straight angle at $B$)
**b** $\beta = 40°$ (straight angle at $K$)
**c** $\theta = 20°$ (vertically opposite angles at $V$)
**d** $\beta = 135°$ (revolution at $O$)
**e** $\alpha = 72°$ (revolution at $V$)
**f** $\alpha = 30°$ (straight angle at $W$)
**g** $\gamma = 30°$ (straight angle at $F$),
$\theta = 120°$ (straight angle at $F$)
**h** $\alpha = 65°$ (straight angle at $K$)

## Exercise 6E

**1** **a** corresponding angles **b** alternate angles
**c** co-interior angles **d** alternate angles
**e** co-interior angles **f** corresponding angles
**g** co-interior angles **h** alternate angles
**i** corresponding angles **j** co-interior angles
**k** corresponding angles **l** alternate angles

**3** **a** **i** $\angle PQB$ **ii** $\angle MLQ$
**iii** $\angle TNO$ **iv** $\angle VCD$
**b** **i** $\angle AQR$ **ii** $\angle PLK$
**iii** $\angle SNM$ **iv** $\angle WCB$
**c** **i** $\angle BQR$ **ii** $\angle QLK$
**iii** $\angle TNM$ **iv** $\angle VCB$

**4** **e** If a transversal crosses two parallel lines, the alternate angles are equal.


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**5** **e** If a transversal crosses two parallel lines, the corresponding angles are equal.

**6** **e** If a transversal crosses two parallel lines, the co-interior angles are supplementary.

**7** **a** $\theta = 70°$ (corresponding angles, $AB \parallel CD$.)

**b** $\theta = 130°$ (alternate angles, $PQ \parallel RS$)

**c** $\theta = 130°$ (co-interior angles, $AB \parallel CD$)

**d** $\alpha = 42°$ (alternate angles, $RS \parallel TU$)

**e** $\beta = 84°$ (co-interior angles, $LM \parallel ON$)

**f** $\gamma = 75°$ (corresponding angles, $BK \parallel CL$)

**g** $\theta = 64°$ (co-interior angles, $DG \parallel EF$)

**h** $\alpha = 52°$ (corresponding angles, $BX \parallel CY$)

**i** $\alpha = 24°$ (alternate angles, $FG \parallel LM$)

**j** $\beta = 90°$ (corresponding angles, $RU \parallel ST$)

**k** $\beta = 69°$ (alternate angles, $CA \parallel BD$)

**l** $\gamma = 142°$ (co-interior angles, $AX \parallel BY$)

**8** **a** $\alpha = 70°$ (corresponding angles, $AB \parallel CD$),
$\beta = 70°$ (alternate angles, $AB \parallel CD$),
$\gamma = 110°$ (co-interior angles, $AB \parallel CD$).

**b** $\alpha = 105°$ (co-interior angles, $PQ \parallel SR$),
$\beta = 75°$ (co-interior angles, $PS \parallel QR$),
$\gamma = 105°$ (co-interior angles, $PQ \parallel SR$)

**c** $\beta = 98°$ (co-interior angles, $FI \parallel GH$),
$\gamma = 98°$ (corresponding angles, $FG \parallel IH$)

**d** $\theta = 90°$ (co-interior angles, $MN \parallel PO$),
$\gamma = 138°$ (co-interior angles, $MN \parallel PO$)

**e** $\alpha = 110°$ (corresponding angles, $AB \parallel PQ$),
$\beta = 110°$ (corresponding angles, $AB \parallel VW$),
$\gamma = 110°$ (alternate angles, $AB \parallel PQ$),
$\theta = 110°$ (alternate angles, $AB \parallel VW$)

**f** $\alpha = 71°$ (corresponding angles, $AC \parallel BX$),
$\beta = 50°$ (alternate angles, $AC \parallel BX$)

**g** $\alpha = 125°$ (corresponding angles, $AC \parallel DF$),
$\beta = 125°$ (corresponding angles, $AG \parallel BH$),
$\gamma = 125°$ (corresponding angles, $AC \parallel DF$)

**h** $\gamma = 43°$ (alternate angles, $CB \parallel LM$),
$\theta = 76°$ (alternate angles, $CB \parallel LM$)

**9** **a** $\angle WAV = 137°$ (co-interior angles, $WB \parallel AV$),
$\angle AVB = 43°$ (co-interior angles, $AW \parallel BV$)

**b** $\angle PBV = 85°$ (alternate angles, $PQ \parallel VB$),
$\angle AVB = 85°$ (alternate angles, $AV \parallel BP$)

**c** $\angle QAV = 35°$ (alternate angles, $PQ \parallel AV$),
$\angle AVB = 35°$ (alternate angles, $AQ \parallel BC$)

**d** $\angle VAR = 45°$ (alternate angles, $RS \parallel VA$),
$\angle AVB = 135°$ (co-interior angles, $RA \parallel BV$)

## Exercise 6F

**1** **a** $\alpha = 20°$ (vertically opposite angles at $Q$),
$\beta = 20°$ (corresponding angles, $AB \parallel CD$),
$\gamma = 160°$ (straight angle at $R$)

**b** $\alpha = 60°$ (corresponding angles, $AB \parallel EF$),
$\beta = 60°$ (vertically opposite angles at $S$),
$\gamma = 60°$ (corresponding angles, $CD \parallel EF$),
$\theta = 120°$ (straight angle at $S$)

**c** $\alpha = 70°$ (straight angle at $B$),
$\theta = 110°$ (alternate angles, $AC \parallel DE$)

**d** $\theta = 110°$ (corresponding angles, $AX \parallel BY$),
$\beta = 70°$ (straight angle at $A$)

**e** $\alpha = 70°$ (alternate angles, $AB \parallel JK$),
$\beta = 30°$ (alternate angles, $AB \parallel JK$),
$\gamma = 80°$ (straight angle at $C$)

**f** $\alpha = 65°$ (corresponding angles, $AC \parallel BM$),
$\beta = 80°$ (alternate angles, $AC \parallel BM$),
$\gamma = 35°$ (straight angle at $B$)

**g** $\alpha = 150°$ (co-interior angles, $AB \parallel CD$),
$\theta = 120°$ (revolution at $D$)

**h** $\alpha = 90°$ (alternate angles, $PQ \parallel AB$),
$\beta = 60°$ (alternate angles, $RS \parallel AB$),
$\gamma = 30°$ (straight angle at $O$)

**2** **a** $\angle NOQ = 45°$ (corresponding angles, $NR \parallel OQ$),
$\angle POQ = 135°$ (straight angle at $O$)

**b** $\angle FQR = 137°$ (straight angle at $Q$),
$\angle POQ = 137°$ (corresponding angles, $PO \parallel FG$)

**c** $\angle PON = 135°$ (co-interior angles, $PA \parallel ON$),
$\angle QON = 145°$ (co-interior angles, $QB \parallel ON$),
$\angle POQ = 80°$ (revolution at $O$)

**d** $\angle POY = 160°$ (alternate angles, $XP \parallel OY$),
$\angle QOY = 150°$ (alternate angles, $ZQ \parallel OY$),
$\angle POQ = 50°$ (revolution at $O$)

**3** **a** After some argument, $\angle VOW = 110°$.

**b** Construct $XO$ parallel to $VP$. Then, after some argument, $\angle VOW = 135°$.

**c** Construct $XV$ parallel to $AB$. Then, after some argument, $\angle VOW = 70°$.

**d** Construct $XM$ parallel to $VO$. Then, after some argument, $\angle VOW = 35°$.

**e** Construct $OX$ parallel to $AC$. Then, after some argument, $\angle VOW = 65°$.

**f** Construct $XO$ parallel to $VL$. Then, after some argument, $\angle VOW = 52°$.

**g** Construct $XO$ parallel to $GV$. Then, after some argument, $\angle VOW = 102°$.

**h** Construct $XO$ parallel to $AB$. Then, after some argument, $\angle VOW = 50°$.

**4** **a** $\theta = 60°$ (corresponding angles, $BL \parallel CM$)

**b** $\alpha = 90°$ (co-interior angles, $PQ \parallel RS$)

**c** $\beta = 72°$ (corresponding angles, $AR \parallel BS$)

**d** $\gamma = 40°$ (co-interior angles, $FG \parallel IH$)

**e** $\theta = 45°$ (corresponding angles, $AB \parallel DC$)

**f** $\alpha = 60°$ (co-interior angles, $SR \parallel PQ$)

**g** $\angle GBD = \beta$ (corresponding angles, $BG \parallel CH$),
$\beta = 60°$ (straight angle at $B$)

**h** $\gamma = 90°$ (co-interior angles, $LM \parallel NO$),
$\theta = 70°$ (co-interior angles, $LM \parallel NO$)

## Exercise 6G

**1** **a** $AB \parallel CZ$ (alternate angles are equal)

**b** $FP \parallel GQ$ (corresponding angles are equal)

**c** $HJ \parallel KM$ (co-interior angles are supplementary)


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**d** $RS \parallel UT$ (co-interior angles are supplementary), $RU \parallel ST$ (co-interior angles are supplementary)

**e** $AD \parallel EH$ (corresponding angles are equal)

**f** $DE \parallel GF$ (alternate angles are equal)

**g** $KL \parallel NM$ (co-interior angles are supplementary)

**h** $DF \parallel AC$ (corresponding angles are equal), $AV \parallel BF$ (alternate angles are equal)

**2** **a** $AB \parallel CD$ (alternate angles are equal)
Hence, $\theta = 98°$ (alternate angles, $AB \parallel CD$)

**b** $AB \parallel CD$ (corresponding angles are equal)
Hence, $\beta = 35°$ (corresponding angles, $AB \parallel CD$)

**c** $AB \parallel CD$ (co-interior angles are supplementary)
Hence, $\alpha = 115°$ (co-interior angles, $AB \parallel CD$)

**d** $AB \parallel CD$ (alternate angles are equal)
Hence, $\gamma = 30°$ (alternate angles, $AB \parallel CD$)

**3** **a** $\theta = 50°$ **b** $\alpha = 70°$ **c** $\beta = 55°$
**d** $\gamma = 20°$ **e** $\alpha = 20°$ **f** $\alpha = 50°$
**g** $\beta = 50°$ **h** $\alpha = 30°$

**4** **a** True. If you can walk, then you can run; false.

**b** False. If a number is greater than 1000, then it is greater than 10; true.

**c** False. If a man lives in Melbourne, then he lives in Australia; true.

**d** False. If the last digit of a number is 5, then is divisible by 5; true.

**e** True. If a woman is a mother, then she has a daughter; false.

**f** True. If a whole number is less than 1000, then it has fewer than four digits; true.

**g** False. If a dog can stand on its hind legs, then it has black and white hair; false.

**h** False. If the area of a rectangle is 40 $cm^2$, then one of its sides has length 8 cm; false.

## Review exercise

**1** **a** $\theta = 75°$ (corresponding angles, $AB \parallel CD$)

**b** $\beta = 90°$ (co-interior angles, $PQ \parallel RS$)

**c** $\alpha = 55°$ (corresponding angles, $AB \parallel CD$)

**d** $\alpha = 20°$ (vertically opposite angles)
$\angle QRD = 160°$ (co-interior angles, $AB \parallel CD$)
$\gamma = 160°$ (vertically opposite angles)
$\beta = 20°$ (supplementary angles)

**e** $\beta = 90°$ (co-interior angles, $LM \parallel NO$)

**2** **a** 60° **b** 27° **c** 16° **d** 6°

**3** **a** 53° **b** 104° **c** 46° **d** 165°

**4** $\angle AOC = 58°$, reflex angle $\angle AOC = 302°$

**5** $\angle AOC = 10°$, reflex angle $\angle AOD = 350°$

**6** **a** 45° **b** 120° **c** 66° **d** 100°

**7** **a** $\alpha = 30°$ **c** $\alpha = 65°$
**b** $\alpha = 35°$ **d** $\alpha = 93°$

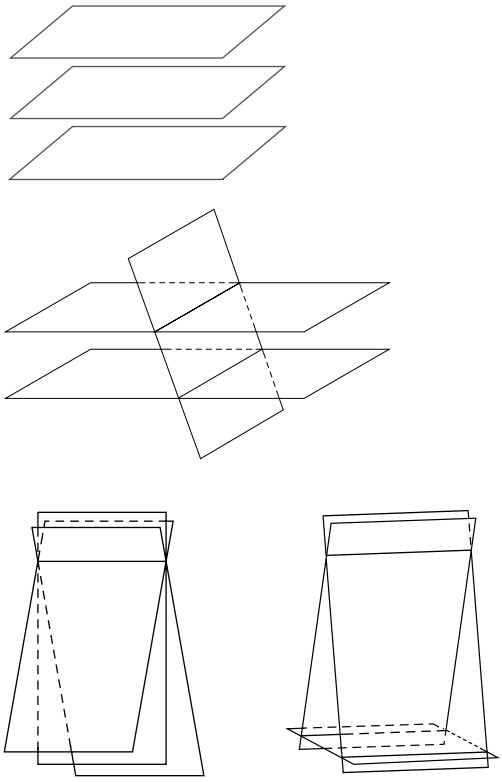
## Challenge exercise

**1** 95°

**2** **a** 40° **b** $x = \gamma + \beta - \alpha$

**3** $x = 180° - \gamma + \alpha - \beta$

**4**



The fifth possibility involves three planes intersecting at a point. Draw it!

**5** **c** 900° **6** 82.5° **7** $16\frac{4}{11}$ minutes

# Chapter 7 answers

## Exercise 7A

**1** **a** $\frac{x}{5}$ **b** $\frac{x}{5} + 3$ **c** $\frac{m+7}{3}$
**d** $\frac{p-6}{3}$ **e** $6\left(\frac{m}{11} - 3\right)$

**2** **a** $\frac{b+2}{2}$ **b** $\frac{c-5}{4}$
**c** $\frac{y+11}{4}$ **d** $\frac{23-x}{7}$

**3** **a** 10 **b** 11 **c** 5 **d** 2

**4** **a** $\frac{1}{2}$ **b** 1 **c** 7 **d** 1

**5** **a** $\frac{n}{5}$ bananas **b** $\left(\frac{n}{5} + 3\right)$ bananas


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**6** **a** $(n + 1000)$ tonnes **b** $\frac{n + 1000}{10}$ tonnes

**7** **a** $x - 2$

**b** $\frac{1}{x - 2}$ hours or $\frac{60}{x - 2}$ minutes

**8** **a** $\$\frac{y}{4}$ **b** $\$\left(50 - \frac{y}{4}\right)$

## Exercise 7B

**1** **a** $\frac{7x}{3}$ **b** $\frac{3x}{4}$ **c** $\frac{3x}{2} + 5$

**d** $\frac{5x - 2}{5}$ **e** $\frac{3x}{4}$ **f** $\frac{7x}{8} + 6$

**g** $\frac{3x}{8} + 7$ **h** $7\left(\frac{x}{3} + 7\right)$ **i** $9\left(\frac{x}{8} - 5\right)$

**j** $4\left(\frac{2x}{5} - 3\right)$ **k** $\frac{3}{7}\left(23 - \frac{4x}{9}\right)$ **l** $\frac{2}{13}\left(\frac{2}{3} - \frac{4x}{9}\right)$

**m** $\frac{2x^2}{3}$

**2** **a** 9 **b** 0 **c** 6

**3** **a** 2 **b** $10\frac{2}{5}$ **c** $1\frac{2}{5}$ **d** 1

**4** **a** 44 **b** 18 **c** 24
**d** 4 **e** 2 **f** $25\frac{9}{11}$
**g** 12 **h** 36

**5** **a** $5\frac{1}{2}$ **b** $10\frac{1}{2}$ **c** $8\frac{1}{2}$ **d** $1\frac{1}{4}$
**e** $\frac{1}{10}$ **f** $2\frac{1}{2}$ **g** $\frac{1}{10}$ **h** $\frac{1}{2}$

**6** $\frac{x}{3}$ cm; $7\frac{1}{3}$ cm

**7** **a** $\frac{x}{5}$ m

**b** **i** 4 m **ii** $8\frac{2}{5}$ m **iii** $19\frac{1}{5}$ m

**8** $\frac{x}{4}$, $1\frac{3}{4}$ **9** $\frac{4x}{7} - 5$, $\frac{5}{7}$

**10** **a** 66 **b** $4\frac{5}{6}$ **c** $6\frac{1}{4}$ **d** 34

**11** \$9

**12** **a** $\frac{12}{x}$ m **b** $1\frac{1}{5}$ m

**13** $\$\frac{90}{x}$

**14** **a** $3x^2$ cm **b** 48 cm

**15** **a** $\frac{6x^2}{5}$ cm **b** $19\frac{1}{5}$ cm

**16** **a** $\frac{1}{15}$ **b** 2 **c** $3\frac{2}{15}$ **d** $2\frac{1}{15}$
**e** $\frac{2}{5}$ **f** $1\frac{1}{9}$ **g** $\frac{9}{10}$ **h** $1\frac{4}{5}$

## Exercise 7C

**1** **a** $\frac{x}{2}$ **b** $4x$ **c** $\frac{x}{5}$ **d** $2e$
**e** $4x$ **f** $\frac{a}{3}$ **g** $5y$ **h** $\frac{1}{4}$

**2** **a** 6 **b** $8x$ **c** $24y$
**d** $\frac{np}{5}$ **e** $3x$ **f** $18x$
**g** $18x$ **h** $\frac{9ab}{2}$ **i** $\frac{9x}{a}$
**j** $\frac{6abx}{cd}$ **k** $\frac{4e}{3yz}$ **l** $\frac{17ab}{3d}$

**3** **a** $a$ **b** $a$ **c** $a^2$ **d** 4
**e** $3ab$ **f** $\frac{5b}{a}$ **g** $\frac{abc}{efg}$

**4** **a** $12xy$ **b** $\frac{5xy^2}{4a}$ **c** $\frac{9x^2y}{4}$

**5** **a** $a$ **b** $\frac{ab}{4}$ **c** $\frac{2a^2}{15}$
**d** $\frac{a^2}{3}$ **e** $\frac{ab}{6}$ **f** $\frac{2a^2}{5}$
**g** $25a$ **h** $\frac{a}{5}$

**6** **a** 2 **b** 6 **c** 30
**d** $1\frac{1}{2}$ **e** 1 **f** $\frac{2}{3}$

**7** $\frac{x^2}{3}$ cm$^2$, 27 cm$^2$ **8** $\frac{x^2}{2}$ cm$^2$, 72 cm$^2$

**9** **a** \$0.40 **b** $\$\frac{2z}{5}$

**10** $\$\frac{2x}{3}$ **11** $\$\frac{9x}{5}$ **12** $\$\frac{4x}{3}$

**13** **a** $\frac{6x}{y}$ **b** 50

## Review exercise

**1** **a** $\frac{x}{10}$ **b** $\frac{m}{3} + 4$ **c** $\frac{6 + x}{5}$
**d** $\frac{z}{7} - 4$ **e** $\frac{7m}{3}$ **f** $\frac{5a}{2}$
**g** $\frac{8b}{7} + 5$ **h** $10\left(\frac{c}{7} - 5\right)$ **i** $\frac{2}{5}\left(\frac{2p}{5} - 6\right)$
**j** $\frac{3}{8}\left(23 - \frac{4p}{11}\right)$ **k** $\frac{3}{5}\left(\frac{3r}{5} - 3\right)$ **l** $\frac{3r^2}{4}$

**2** **a** 10 **b** 14 **c** $4\frac{4}{7}$


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**3** **a** 10 **b** $6\frac{2}{5}$ **c** $3\frac{9}{10}$
**d** $15\frac{1}{5}$ **e** $7\frac{1}{5}$ **f** 5
**g** $6\frac{2}{5}$ **h** $6\frac{2}{3}$

**4** **a** $\frac{6}{7}$ **b** $2\frac{3}{5}$ **c** $5\frac{1}{2}$ **d** $\frac{3}{7}$

**5** $\left(\frac{x}{18} + \frac{6y}{35}\right)$ L

**6** **a** $2x$ **b** $6n$ **c** $24p$ **d** $\frac{yz}{5}$
**e** $4n$ **f** $11a$ **g** $18a$ **h** $3ab$

**7** **a** $\frac{1}{20}$ **b** $2\frac{2}{5}$ **c** $3\frac{17}{20}$ **d** $4\frac{1}{5}$
**e** $4\frac{1}{20}$ **f** $\frac{3}{5}$ **g** $1\frac{1}{15}$ **h** $3\frac{3}{4}$

**8** **a** $\frac{25}{56}$ **b** $\frac{6}{7}$ **c** $\frac{25}{28}$ **d** $3\frac{51}{56}$
**e** $\frac{73}{112}$ **f** $2\frac{1}{24}$ **g** $\frac{2977}{3136}$ **h** $4\frac{1}{8}$

**9** **a** $x$ **b** $\frac{2}{3}$ **c** $9a$
**d** $\frac{9a}{7}$ **e** $\frac{1}{44a}$ **f** $\frac{5}{12}$

**10** **a** $z$ **b** $3x^2$ **c** $5mn^3$ **d** 1

**11** $\frac{2x^2}{5}$ cm$^2$, 40 cm$^2$ **12** $\frac{15x^2}{16}$ m$^2$, 135 m$^2$

## Challenge exercise

**1** **a** 8 cm **b** $\frac{x}{3}$ cm **c** $\frac{x+4}{3}$ cm **d** $\frac{3}{x}$ cm

**2** **a** $2\left(\frac{x}{2}+2\right) = (x+4)$ cm
**b** $2\left(\frac{x+3}{2}+2\right) = (x+7)$ cm

**3** **a** 180 km **b** $60n$ km
**c** $3x$ km **d** $xn$ km

**4** **a** **i** 140 km **ii** $(50 + 30n)$ km
**b** **i** 375 km **ii** $(500 - 25n)$ km

**5** **a** 60 km/h **b** $\frac{n}{3}$ km/h **c** $\frac{n}{m}$ km/h

**6** $\frac{n}{50}$ h

**7** **a** $BC = \frac{16}{x}$ cm **b** perimeter $= 2x + \frac{32}{x}$ cm
**c** 16 cm **d** $128\frac{1}{2}$ cm

**8** **a** $(20 - d)$ km **b** **i** $\frac{d}{60}$ h **ii** $\frac{20-d}{80}$ h
**c** total time $= \frac{d}{60} + \frac{20-d}{80} = \frac{d+60}{240}$ h
**d** **i** $16\frac{1}{2}$ min **ii** 18 min **iii** 20 min

# Chapter 8 answers

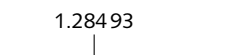
## Exercise 8A

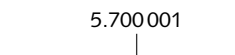
**1** $2 \times 10^2 + 7 \times 10 + 6 + \frac{3}{10} + \frac{5}{10^2} + \frac{7}{10^4}$

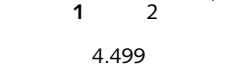
**2** $\frac{4276}{1000}$ **3** 0.057

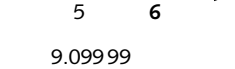
**4** **a** 2 **b** 0.02 **c** 0.2
**d** 20 **e** 0.002 **f** 0.000 02

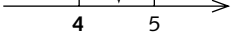
**5** Any 10 of the following:
1.023, 1.032, 1.203, 1.230, 1.302, 1.320, 2.013, 2.031, 2.103, 2.130, 2.301, 2.310, 3.012, 3.021, 3.102, 3.120, 3.201, 3.210, 10.23, 10.32, 12.03, 12.30, 13.02, 13.20, 20.13, 20.31, 21.03, 21.30, 23.01, 23.10, 30.12, 30.21, 31.02, 31.20, 32.01, 32.10, 102.3, 103.2, 120.3, 123.0, 130.2, 132.0, 201.3, 203.1, 210.3, 213.0, 230.1, 231.0, 301.2, 302.1, 310.2, 312.0, 320.1, 321.0

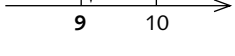
**6** (*Note*: Answers are in **bold** type.)
**a** 3.09


**b** 3.9


**c** 1.284 93


**d** 5.700 001


**e** 4.499


**f** 9.099 99



**7** **a** 0.4 **b** 1.93 **c** 6.8 **d** 5.63
**e** 7.3412 **f** 5.1 **g** 8.999 78 **h** 3.67
**i** 297.2357 **j** 11.3 **k** 434.897 69 **l** 3.56

**8** **a** 0.2, 2.3, 4.08, 7.2, 8.153
**b** 1.09, 1.30, 1.39, 1.9, 1.93
**c** 7.000 023, 7.0748, 7.230 000, 7.410 9572, 7.99
**d** 6.4, 6.66, 6.888, 6.9234, 6.985 74

**9** **a** 4 **b** 5 **c** 5 **d** 5 **e** 4 **f** 4
**g** 4 **h** 4 **i** 4 **j** 4 **k** 5 **l** 4

**10** **a** 600 thousandths **b** 60 hundredths
**c** 0.6 units **d** 0.06 tens
**e** 0.006 hundreds **f** 0.0006 thousandths

**11** **a** 890 thousandths **b** 8.9 tenths
**c** 0.89 units **d** 0.089 tens
**e** 0.0089 hundreds **f** 0.000 89 thousands

**12** **a** 72.3 hundredths **b** 7.23 tenths
**c** 0.723 units **d** 0.0723 tens
**e** 0.007 23 hundreds **f** 0.000 723 thousands


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## Exercise 8B

**1** **a** $\frac{3}{4}$ **b** $15\frac{1}{4}$ **c** $\frac{17}{50}$ **d** $4\frac{1}{8}$
**e** $\frac{1}{40}$ **f** $\frac{9}{1000}$ **g** $\frac{403}{500}$ **h** $8\frac{3}{4}$
**i** $\frac{1}{80}$ **j** $54\frac{5}{8}$

**2** **a** 0.5 **b** 0.43 **c** 0.05
**d** 0.005 **e** 0.057 **f** 0.03
**g** 0.27 **h** 0.0009 **i** 23
**j** 2.3 **k** 16.96 **l** 16.096
**m** 24.035 **n** 101.101 **o** 101.0101

**3** **a** 5.5 **b** 12.5 **c** 6.5 **d** 8.6
**e** 57.4 **f** 95.25 **g** 54.75 **h** 5.4

**4** **a** 1.1 **b** 0.92 **c** 0.6
**d** 1.05 **e** 0.132 **f** 0.815
**g** 0.016 **h** 1.04 **i** 1.068
**j** 2.7 **k** 5.4 **l** 19.12
**m** 2.62 **n** 0.12 **o** 0.016

**5** **a** 2 **b** 7 **c** 6
**d** 4 **e** 5 **f** 3

**6**

| **Decimal** | 0.01 | 0.125 | 0.25 | 0.2 | 0.5 |
|---|---|---|---|---|---|
| **Fraction** | $\frac{1}{100}$ | $\frac{1}{8}$ | $\frac{1}{4}$ | $\frac{1}{5}$ | $\frac{1}{2}$ |

## Exercise 8C

**1** **a** 5.9 **b** 7.3 **c** 8.9
**d** 6.32 **e** 9.18 **f** 9.97
**g** 9.562 **h** 9.8997 **i** 8.008
**j** 11.815 **k** 10.1993 **l** 88.0594
**m** 39.887 01 **n** 126.9739 **o** 237.1128

**2** **a** 4.4 **b** 8.1 **c** 1.1 **d** 1.77
**e** 5.25 **f** 15.5 **g** 5.9 **h** 11.9
**i** 3.478 **j** 8.255 **k** 80.375 **l** 7.397 336 776

**3** 25.26 kg **4** 0.26 m

**5** 0.83 s, 1.16 s, 1.69 s **6** 2.25 km

**7** 8800 m or 8.8 km **8** 12.72 m

## Exercise 8D

**1** **a** 8 **b** 83 **c** 0.8
**d** 5 **e** 0.5 **f** 0.05
**g** 928.9432 **h** 3920.01

**2** **a** 34 **b** 478 **c** 1.3
**d** 80 **e** 0.4 **f** 2830.4
**g** 430.9387 **h** 20 120.1201

**3** **a** 0.12 **b** 0.008
**c** 0.0002 **d** 20.120 1201

**4** **a** 2.406 **b** 0.1406
**c** 0.55703 **d** 0.020 006

**5** **a** 507 **b** 789 028
**c** 2.300 001 **d** 234 560
**e** 0.010 462 **f** 0.6
**g** 25 000 **h** 30
**i** 1.2 **j** 3.435
**k** 402.521 **l** 10

**6** **a** 0.82 **b** 0.082
**c** 8.2 **d** 85 700
**e** 8570 **f** 857
**g** 0.502 **h** 0.0502
**i** 0.005 02 **j** 0.000 005 43
**k** 0.000 0543 **l** 0.000 543

**7** $32.58

**8** **a** 680 min **b** 40.8 s **c** 1000

**9** **a** 1000 mm
**b** 1 000 000 mm
**c** 36 400 000 mm
**d** 0.000 276 km
**e** 1 000 000 000 000 square mm
**f** 13 060 000 000 000 square mm
**g** 0.000 000 045 67 8 square km

## Exercise 8E

**1** **a** 0.03 **b** 0.16 **c** 0.21
**d** 0.024 **e** 0.12 **f** 0.34
**g** 0.048 **h** 0.11 **i** 0.0666
**j** 0.044 **k** 0.011 **l** 0.648

**2** **a** 0.7 **b** 0.08 **c** 1.4
**d** 0.024 **e** 1.2 **f** 3
**g** 0.48 **h** 1.6 **i** 0.666
**j** 0.0044 **k** 0.011 **l** 6.48

**3** **a** 1.8 **b** 0.08 **c** 0.0008
**d** 0.98 **e** 0.096 **f** 10
**g** 0.12 **h** 0.36 **i** 0.0009
**j** 0.042 **k** 16.856 **l** 61.854


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**4**

| x | 0.009 | 0.09 | 0.9 | 9 | 90 | 900 | 9000 |
|---|---|---|---|---|---|---|---|
| **0.008** | 0.000072 | 0.00072 | 0.0072 | 0.072 | 0.72 | 7.2 | 72 |
| **0.08** | 0.00072 | 0.0072 | 0.072 | 0.72 | 7.2 | 72 | 720 |
| **0.8** | 0.0072 | 0.072 | 0.72 | 7.2 | 72 | 720 | 7200 |
| **8** | 0.072 | 0.72 | 7.2 | 72 | 720 | 7200 | 72000 |
| **80** | 0.72 | 7.2 | 72 | 720 | 7200 | 72000 | 720000 |
| **800** | 7.2 | 72 | 720 | 7200 | 72000 | 720000 | 7 200000 |
| **8000** | 72 | 720 | 7200 | 72000 | 720000 | 7 200000 | 72000000 |

**5** **a** 0.3 **b** 1.8 **c** 6.3
**d** 2.06 **e** 0.0042 **f** 0.152
**g** 0.1476 **h** 0.0606 **i** 0.043

**6** **a** 1.2 **b** 1 **c** 4.3
**d** 0.05 **e** 0.26 **f** 0.108

**7** **a** 0.15 **b** 0.1 **c** 0.63
**d** 0.005 **e** 0.043 **f** 0.0206

**8** **a** 0.023 **b** 0.1 **c** 0.01
**d** 0.00005 **e** 0.0005 **f** 0.00407

**9** **a** \$12.15 **b** \$0.70 **c** \$14.90
**d** \$1.87 **e** \$10.89

## Exercise 8F

**1** **a** 0.5 **b** 0.9 **c** 9.1 **d** 0.02
**e** 0.2 **f** 0.8 **g** 3.1 **h** 0.032
**i** 0.33 **j** 0.0064 **k** 0.048 **l** 4.94
**m** 33.85 **n** 0.0058 **o** 0.000345 **p** 0.001354

**2** **a** 0.4 **b** 0.3 **c** 3 **d** 0.2
**e** 201 **f** 3 **g** 0.5 **h** 0.4

**3** **a** 0.4 **b** 0.2 **c** 0.9 **d** 1.2
**e** 12 **f** 24 **g** 3.2 **h** 0.25
**i** 3.1 **j** 152.7 **k** 0.32425 **l** 3579.9375

**4** **a** 120, 1200, 12000 **b** 100, 1000, 10000
**c** 430, 4300, 43000 **d** 5, 50, 500
**e** 26, 260, 2600 **f** 1000, 10000, 100000

**6** \$1.13 **7** 121

## Exercise 8G

**1** **a** 0.375 **b** 1.75 **c** 0.05
**d** 0.35 **e** 0.025

**2** **a** $0.\dot{3}$ **b** $0.\dot{6}$ **c** $0.\dot{1}$ **d** $0.\dot{2}$
**e** $0.\dot{3}$ **f** $0.\dot{4}$ **g** $0.\dot{5}$ **h** $0.\dot{6}$
**i** $0.\dot{7}$ **j** $0.\dot{8}$

**3** **a** $0.\dot{0}\dot{9}$ **b** $0.\dot{1}\dot{8}$ **c** $0.\dot{2}\dot{7}$ **d** $0.\dot{3}\dot{6}$
**e** $0.\dot{4}\dot{5}$ **f** $0.\dot{5}\dot{4}$ **g** $0.\dot{6}\dot{3}$ **h** $0.\dot{7}\dot{2}$
**i** $0.\dot{8}\dot{1}$ **j** $0.\dot{9}\dot{0}$

**4** **a** $0.1\dot{6}$ **b** $0.\dot{3}$ **c** $0.\dot{5}$ **d** $0.\dot{6}$
**e** $0.8\dot{3}$ **f** $0.08\dot{3}$ **g** 0.25 **h** $0.41\dot{6}$
**i** $0.58\dot{3}$ **j** 0.75 **k** $0.916\dot{6}$

**5** $0.\dot{1}42857\dot{1}$, $0.\dot{2}857\dot{1}4$, $0.\dot{4}2857\dot{1}$, $0.\dot{5}7142\dot{8}$, $0.\dot{7}1428\dot{5}$, $0.\dot{8}5714\dot{2}$

**6** **a** $0.6\dot{8}5714\dot{2}$ **b** $0.9\dot{6}$ **c** $4.\dot{5}\dot{4}$
**d** $1.31\dot{6}$ **e** $0.92\dot{4}2857\dot{1}$ **f** $0.36\dot{8}5714\dot{2}$
**g** $5.1\dot{6}$ **h** $5.\dot{3}\dot{6}$ **i** $4.7\dot{8}\dot{1}$
**j** $4.6\dot{8}$ **k** $157.\dot{3}$ **l** $0.39\dot{3}$

## Exercise 8H

**1** **a** 3.15 **b** 9.50 **c** 8.00 **d** 2.15
**e** 6.45 **f** 7.99 **g** 12.355 **h** 6.454
**i** 7.992 **j** 12.3548 **k** 6.4541 **l** 7.9920

**5**

| ÷ | 0.006 | 0.06 | 0.6 | 6 | 60 | 600 | 6000 |
|---|---|---|---|---|---|---|---|
| **0.024** | 4 | 0.4 | 0.04 | 0.004 | 0.0004 | 0.00004 | 0.000004 |
| **0.24** | 40 | 4 | 0.4 | 0.04 | 0.004 | 0.0004 | 0.00004 |
| **2.4** | 400 | 40 | 4 | 0.4 | 0.04 | 0.004 | 0.0004 |
| **24** | 4000 | 400 | 40 | 4 | 0.4 | 0.04 | 0.004 |
| **240** | 40000 | 4000 | 400 | 40 | 4 | 0.4 | 0.04 |
| **2400** | 400000 | 40000 | 4000 | 400 | 40 | 4 | 0.4 |
| **24000** | 4000000 | 400000 | 40000 | 4000 | 400 | 40 | 4 |


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**2** **a** 0.58 **b** 0.64 **c** 0.576 **d** 0.636
**e** 0.5758 **f** 0.6364 **g** 0.37 **h** 0.7748
**i** 0.46 **j** 0.4777 **k** 0.23784 **l** 0.1755

## Review exercise

**1** **a** 0.1 **b** 0.9 **c** 0.01 **d** 0.09
**e** 0.39 **f** 0.001 **g** 0.004 **h** 0.028
**i** 0.298 **j** 0.0001 **k** 0.0009 **l** 0.0019
**m** 0.0241 **n** 0.6939

**2** **a** 0.8 **b** 0.08 **c** 8
**d** 0.008 **e** 0.0008 **f** 80 and 0.0008

**3** **a** 0.4 **b** 0.04 **c** 0.0004 **d** 0.00004

**4** **a** 2.9 **b** 1.91 **c** 3.11
**d** 2.8 **e** 4.999 **f** 5.501
**g** 5.12 **h** 1.231 **i** 4.51

**5** **a** 1.5, 1.4732, 1.28, 1.09 **b** 1.928, 1.36, 1.2849, 1.2
**c** 9.999, 9.909, 9.9, 9.09 **d** 23.5, 23.451, 23.09, 23.0001

**6** **a** 1 **b** 3 **c** 2 **d** 4
**e** 27 **f** 44 **g** 68 **h** 300
**i** 0 **j** 187 **k** 214 **l** 627

**7** **a** $\frac{3}{10}$ **b** $3\frac{9}{10}$ **c** $1\frac{2}{5}$ **d** $\frac{8}{25}$
**e** $\frac{49}{50}$ **f** $\frac{1}{25}$ **g** $\frac{111}{1000}$ **h** $\frac{51}{250}$
**i** $\frac{99}{1000}$ **j** $\frac{3}{4}$ **k** $\frac{1}{8}$ **l** $2\frac{106}{125}$

**8** **a** 0.2 **b** 0.5 **c** 1.02 **d** 0.08
**e** 10.3 **f** 0.08 **g** 4.04 **h** 0.004
**i** 4.7 **j** 10.1 **k** 2.4 **l** 1.0625
**m** 0.024 **n** 0.98 **o** 0.825 **p** 0.795
**q** 0.26 **r** 0.144

**9**

| Decimal | $1.\dot{3}$ | 1.25 | 0.4 | $4.\dot{2}$ | 0.50 | 0.09 | 0.625 |
|---|---|---|---|---|---|---|---|
| Fraction | $1\frac{1}{3}$ | $4\frac{1}{4}$ | $\frac{2}{5}$ | $4\frac{2}{9}$ | $\frac{1}{2}$ | $\frac{9}{100}$ | $\frac{5}{8}$ |

**10** $\frac{147}{100}$

**11** **a** $0.\dot{4}28571\dot{4}$ **b** $0.\dot{1}\dot{8}$ **c** $0.41\dot{6}$
**d** $0.\dot{4}61538\dot{4}$ **e** $0.8\dot{3}$ **f** $0.58\dot{3}$

**12** **a** 0.1, $\frac{1}{5}$, 0.5, $\frac{5}{8}$, $\frac{3}{4}$, 1 **b** $\frac{1}{24}$, 0.24, $\frac{1}{4}$, 0.4, 2.4, 4.1
**c** 0, $\frac{1}{25}$, 0.08, $\frac{1}{5}$, $\frac{5}{8}$, 0.9 **d** 2, 2.07, $\frac{23}{11}$, $2\frac{1}{5}$, 2.7, $\frac{290}{100}$

**13** **a** 3.9 **b** 8.9 **c** 6.95
**d** 8.97 **e** 0.738 **f** 14.4
**g** 23.1 **h** 5.3 **i** 5.9
**j** 1.5 **k** 64.189 **l** 108.5287

**14** **a** 45, 450, 4500 **b** 3, 30, 300
**c** 0.6, 6, 60 **d** 13.96, 139.6, 1396
**e** 1004.85, 10048.5, 100485 **f** 0.002, 0.02, 0.2

**15** **a** 0.45, 0.045, 0.0045 **b** 0.03, 0.003, 0.0003
**c** 0.006, 0.0006, 0.00006 **d** 0.1396, 0.01396, 0.001396
**e** 4.38, 0.438, 0.0438 **f** 10.0485, 1.00485, 0.100485

**16**

| × | 0.007 | 0.07 | 0.7 | 7 | 70 | 700 | 7000 |
|---|---|---|---|---|---|---|---|
| 0.003 | 0.000021 | 0.00021 | 0.0021 | 0.021 | 0.21 | 2.1 | 21 |
| 0.03 | 0.00021 | 0.0021 | 0.021 | 0.21 | 2.1 | 21 | 210 |
| 0.3 | 0.0021 | 0.021 | 0.21 | 2.1 | 21 | 210 | 2100 |
| 3 | 0.021 | 0.21 | 2.1 | 21 | 210 | 2100 | 21000 |
| 30 | 0.21 | 2.1 | 21 | 210 | 2100 | 21000 | 210000 |
| 300 | 2.1 | 21 | 210 | 2100 | 21000 | 210000 | 2100000 |
| 3000 | 21 | 210 | 2100 | 210000 | 210000 | 2100000 | 21000000 |

**17** **a** 42.21 **b** 38.4 **c** 14.76 **d** 17.7 **e** 285.6 **f** 19.888 **g** 4.08 **h** 15.4622

**18**

| × | 0.001 | 0.01 | 0.1 | 1 | 10 | 100 | 1000 |
|---|---|---|---|---|---|---|---|
| 0.052 | 0.000052 | 0.00052 | 0.0052 | 0.052 | 0.52 | 5.2 | 52 |
| 0.52 | 0.00052 | 0.0052 | 0.052 | 0.52 | 5.2 | 52 | 520 |
| 5.2 | 0.0052 | 0.052 | 0.52 | 5.2 | 52 | 520 | 5200 |
| 52 | 0.052 | 0.52 | 5.2 | 52 | 520 | 5200 | 52000 |
| 520 | 0.52 | 5.2 | 52 | 520 | 5200 | 52000 | 520000 |
| 5200 | 5.2 | 52 | 520 | 5200 | 52000 | 520000 | 5200000 |
| 52000 | 52 | 520 | 5200 | 52000 | 520000 | 5200000 | 52000000 |

**19** **a** 3 **b** 5 **c** 60 **d** 0.05 **e** $0.7\dot{5}7142\dot{8}$ **f** $10.21\dot{3}$ **g** 8.32 **h** 3525


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**20** $3.15 **21** 0.8004 kg **22** $23.92

**23** $5.15 **24** $1.87

**25** **a** $28.70 per metre **b** $287

## Challenge exercise

**1** **a** 2.9, 3.0, 3.1, 3.2, 3.3 **b** 4.0, 4.3, 4.6, 4.9, 5.2
**c** 12.2, 12.1, 12.0, 11.9, 11.8 **d** 9.1, 8.0, 6.9, 5.8, 4.7

**2** **a** $0.8 \times 0.765 = 0.612$ **b** $0.85 \times 0.76 = 0.646$

**3** $52.50

**4** **a** $\frac{4}{9}$ **b** $\frac{4}{33}$ **c** $\frac{41}{333}$

**5** $\frac{1}{n}$, for $n$ = 1, 2, 4, 5, 8, 10, 16, 20, 25, 32, 40, 50, 64 and 80

**6** If the denominator is $n$, there are $n - 1$ possible remainders.

# Chapter 9 answers

## Exercise 9A

**1** **a** 4 cm **b** 5 mL **c** 72 kg
**d** 13.3 cm **e** 220 g **f** 650 mL

**2** **a** 60 mm **b** 18 cm **c** 180 mm
**d** 43 mm **e** 2.7 cm **f** 58.3 cm
**g** 300 cm **h** 190 cm **i** 2000 m
**j** 100000 cm **k** 6340 m **l** 0.2 km
**m** 0.09 km **n** 2.901 km **o** 500 m
**p** 25 cm **q** 200 mm **r** 7 mm
**s** 6.25 mm **t** 8.75 mm

**3** **a** 8.7 cm **b** 59 mm **c** 6.7 cm
**d** 365 cm **e** 45 800 m **f** 14.038 km

**4** **a** 0.493 kg **b** 2300 g **c** 3400 kg **d** 250 g
**e** 45 300 g **f** 0.48 g **g** 0.29 g **h** 100 kg

**5** **a** 5023 m **b** 15 020 m **c** 3020 kg **d** 24 015 kg

**6** **a** 0.8 L **b** 1500 mL **c** 4.002 L **d** 3.968 kL

**7** 20 laps **8** 5500 g or 5.5 kg

**9** 51 markers **10** 220 cm

**11** **a** 15.25 kg **b** 13.215 m **c** 185.6 cm
**d** 1984 kg **e** 105.604 t **f** 17 m
**g** 1.38 t **h** 9.044 kg **i** 1.4064 km

**12** **a** $5.94 **b** 42 c **c** $6
**d** $0.28 **e** $6.80 **f** $30
**g** $120 **h** $1.26 **i** $12

## Exercise 9B

**1** 1 000 000 L **2** 10 000 µm
**3** 2 000 000 000 000 L **4** 5 100 000 000 µm
**5** 9 850 000 000 000 000 mg **6** 15.8 dm
**7** 0.89 $dm^3$ **8** 52.596 min

## Exercise 9C

**1** 1125 g **2** $36.44 **3** 0.609 kg **4** 910 km
**5** 560 km **6** 12 oranges for $4.08 is cheaper.
**7** 12 km **8** 6000 m **9** $20

**10** **a** 1200 m **b** 72 km

**11** 2 kg **12** 14.175 days

**13** **a** **i** $8.00 **ii** $800.00 **iii** $9.60
**iv** $960.00 **v** $20.00 **vi** $100.00
**b** **i** $2.80 **ii** $21.00 **iii** $8.40
**iv** $0.84 **v** $2.45 **vi** $71.40

## Exercise 9D

**1** **a** 12 **b** 12 **c** 60
**d** 52 **e** 50 **f** 80

**2** **a** 100 mm **b** 91 mm

**3** **a** 8 cm **b** 10 cm **c** 16 cm

**4** **a** 11.8 m **b** 32.4 m

**5** **a** $a + b + c$ **b** $3x$ **c** $4d$
**d** $4a + 6b$ **e** $8a + 3b$ **f** $14x + 12y$

**6, 7** Various answers are possible.

**8** 40 m

## Exercise 9E

**1** **a** 12 $cm^2$ **b** 24 $cm^2$ **c** 8 $cm^2$ **d** 30 $cm^2$

**2** **a** 9 $m^2$ **b** 63 $cm^2$ **c** 169 $mm^2$ **d** 3.52 $cm^2$

**3** **a** 10 000 $cm^2$ **b** 100 hectares **c** 1 000 000 $m^2$

**4** **a** 4.25 $m^2$ **b** 1.6 $km^2$

**5** 200 hectares **6** 90 000 $m^2$

**7** **a** 42 $cm^3$ **b** 3 cm **c** 5 m **d** 12 km

**8** **a** $9a^2$ **b** $42cd$

**9** $2160 **10** 35 $m^2$ **11** $84.38

**12** $420 **13** 3.8 hectares

**14** 1.69 $m^2$ **15** $4 820 850

**16, 17** Various answers are possible.


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## Exercise 9F

**1** **a** 10 $cm^2$ **b** 23 $cm^2$ **c** 17 $m^2$

**2** **a** 74 $cm^2$ **b** 60 $m^2$ **c** 37 $cm^2$ **d** 60 $m^2$
**e** 32 $cm^2$ **f** 40 $cm^2$ **g** 46 $cm^2$ **h** 32 $m^2$

**3** 54 $m^2$ **4** 32 $m^2$ **5** 16 $m^2$

## Exercise 9G

**1** **a** 24 $cm^2$ **b** 38.5 $m^2$ **c** 26 $cm^2$ **d** 84 $mm^2$

**2** **a** 2 $cm^2$ **b** 4.5 $cm^2$ **c** 3 $cm^2$ **d** 6 $cm^2$
**e** 2 $cm^2$ **f** 3 $cm^2$ **g** 3 $cm^2$ **h** 6 $cm^2$

**3** **a** 21 $cm^2$ **b** 6 cm **c** 10 cm **d** 24 km

**4** **a** 24 $cm^2$ **b** 6 $cm^2$ **c** 25.5 $cm^2$
**d** 6 $cm^2$ **e** 14.5 $cm^2$

**6** An infinite number; fix the base and vary the third vertex so that the height remains the same.

**7** **a** 35 $cm^2$ **b** 36 $km^2$ **c** 102 $cm^2$
**d** 105 $mm^2$ **e** 42 $m^2$ **f** 56 $cm^2$

**8** **a** 120 $m^2$ **b** 18 $m^2$ **c** 17 $m^2$ **d** 125.5 $m^2$

**9** $\Delta FBA$ and $\Delta ECD$ have the same area. Parallelogram $ABCD$ has the same area as rectangle $AFED$.

**10** **a** 42 $cm^2$ **b** 3 cm **c** 5 m **d** 12 km

**11** **a** 104 $cm^2$ **b** 168 $m^2$

**12** 4 cm **13** 2.4 cm

**14** base = 6 cm; height = 3 cm

**15** 24 $cm^2$

## Exercise 9H

**1** 880 $mm^3$

**2** **a** 108 $cm^3$ **b** 160 $cm^3$

**3** **a** 20 **b** 27 **c** 12
**d** 16 **e** 18 **f** 35

**4** **a** 120 $cm^3$ **b** 770 $cm^3$ **c** 6 cm **d** 4 cm

**5** 9 $m^3$ **6** 7.5 $m^3$ **7** 675 $cm^2$

**8** **a** 30000 $mm^3$ **b** 30 $cm^3$

**9** 5 mm × 9 mm × 10 mm; 15 mm × 3 mm × 10 mm; 45 mm × 1 mm × 10 mm

**10** 1 m × 24 m × 10 m; 2 m × 12 m × 10 m; 8 m × 10 m; 4 m × 6 m × 10 m

## Exercise 9I

**1** **a** 1380 seconds **b** 1440 minutes
**c** 408 hours **d** 17 days, 21 hours
**e** 4320 minutes **f** 6 hours, 32 minutes
**g** 1 hour, 26 minutes and 41 seconds

**2** **a** 1430 **b** 0600 **c** 2349 **d** 2035
**e** 0535 **f** 1200 **g** 0000
**h** 1505 **i** 2358

**3** **a** 1 p.m. **b** 6 a.m. **c** 8:20 a.m. **d** 11:30 p.m.

**4** **a** 270 minutes **b** 4 hours 49 minutes
**c** 9 months **d** 168 hours

**5** 12 hours 33 minutes

**6** 71 days 18 hours **7** 9 days 19 hours

**8** **a** 3 hours 45 minutes **b** 2 hours 30 minutes
**c** 6 hours 30 minutes **d** 4 hours 20 minutes
**e** 1 day 45 minutes
**f** 2 days 14 hours and 23 minutes
**g** 4 days 16 hours and 25 minutes

**9** 8 hours

**10** 15 hours 55 minutes

**11** 3 minutes 5 seconds, 18 seconds

**12** 19 minutes

**13** 1:27 a.m. on Wednesday

**14** 1530 seconds

## Exercise 9J

**1** **a** 50 km/h **b** 5 m/min **c** 30 km/h
**d** 120 m **e** 605 km **f** 20 km

**2** 12 km **3** 24 km/h

**4** 1 km **5** 112 km

**6** 730 km/h **7** 48 km

**8** 24 nautical mile/h **9** 630 km

**10** **a** 11 m/s **b** 198 m

**11** **a** 20 km **b** 6 km/h **c** 70 km/h **d** 30 km/h

**12** **a** 1 hr 33 minutes **b** $66\frac{2}{3}$ km/h
**c** $12\frac{1}{2}$ km/h **d** $38\frac{22}{31}$ km/h


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**13** 24 km/h

**14** $16\frac{2}{3}$ m/s, $\frac{5x}{18}$ m/s **15** $28\frac{1}{11}$ km/h

## Review exercise

**1** **a** 1.607 **b** 160.7 **c** 19.7 **d** 0.197
**e** 2300 **f** 2.3 **g** 550 **h** 0.55
**i** 150000 **j** 1500 **k** 830 **l** 0.0083

**2** **a** 3000000 **b** 3000 **c** 0.0048
**d** 4800 **e** 1.029 **f** 0.001 029
**g** 329000 **h** 329000000 **i** 4000
**j** 2300 **k** 2.95 **l** 250000000000

**3** 46 cm **4** 5.35 kg **5** 120.05 m

**6** \$2.70 **7** 1190 g, 210 g

**8** **a** perimeter: 42 m; area: 68 $m^2$
**b** perimeter: 64 km; area: 72 $km^2$

**9** **a** 5:53:59 p.m. **b** 3:06:59 a.m.
**c** 4:50:02 a.m. **d** 9:23:59 p.m.
**e** 7:08:59 p.m.

**10** 1 kg 50 g

**11** from smallest to largest:
$ABC = ACD = ADE$, $ABD = ACE$, $ABE$

**12** 30 $cm^2$ **13** 1000 $m^2$, 375 **14** 0.0065 m

**15** 240 m **16** $7\frac{9}{13}$ m/s **17** $7\frac{1}{3}$ km/h

**18** **a** 18 $m^2$ **b** 40 $m^2$

**19** 49.2 cm **20** 74 km/h **21** $26\frac{2}{3}$ km, 5 km/h

**22** 1.609 344 km **23** 32 $cm^2$

**24** 89 hours, 25 minutes and 59 seconds

**25** 12

**26** **a** 6 $cm^2$ **b** $\frac{12}{7}$ cm

**27** **a** 312 $cm^3$ **b** 60 $cm^2$ **c** 350 $cm^3$

## Challenge exercise

**1** yes, since $6 \times 10^7 > 5.256 \times 10^7$

**4** area = 120 $m^2$, perimeter = 62 m, area of path = 66 $m^2$

**5** Wednesday **6** 65 cards **7** 5 people

**8** 8 times (0:00, 1:11, 2:22, 3:33, 4:44, 5:55, 11:11, 22:22)

**9** 3.5 kg **10** 240 plants **11** 2400 m

**12** Tuesday **13** 4.8 km/h **14** 52 m **15** 48 $cm^2$

**16** 34 cm, 20 cm, 16 cm

**17** smallest possible area is 11 $cm^2$, largest is 36 $cm^2$ (a square)

**18** A square has the smallest perimeter (32 m).

**19** A square has the largest area (729 $m^2$).

# Chapter 10 answers

## 10A Review

### Chapter 1: Whole numbers

**1** **a** 95 **b** 50 **c** 120
**d** 100 **e** 154 **f** 180

**2** **a** 841 **b** 7793 **c** 7746
**d** 943 **e** 30130 **f** 2490

**3** **a** 554 **b** 1551 **c** 102
**d** 38 **e** 25 **f** 8567
**g** 703 **h** 13 **i** 1894

**4** \$680 **5** 229 metres **6** 1500 km

**7** Jacqui is 31 cm taller than David and 36 cm taller than Adrian.

**8** **a** = **b** > **c** = **d** <

**9** **a** 2700 **b** 430 **c** 8200 **d** 750
**e** 966 **f** 1716

**10** **a** 216 **b** 696 **c** 301 **d** 1188
**e** 8280 **f** 67458 **g** 6993 **h** 77208
**i** 9729 **j** 25296 **k** 3172343 **l** 1319396
**m** 358050 **n** 112308 **o** 35819232

**11** 544 **12** 722 **13** 13717 **14** 1377

**15** **a** 222 **b** 211 **c** 323
**d** 71 **e** 31 **f** 81
**g** 66 **h** 22 **i** 63
**j** 302 **k** 361 **l** 759
**m** 108 **n** 105 **o** 308
**p** 1789 remainder 3 **q** 6938 remainder 1
**r** 7914 remainder 3

**16** **a** 22
**b** 24
**c** 23


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## Chapter 2: Factors, multiples, primes and divisibility

**1** **a** 1, 3, 13, 39

**b** 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

**c** 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48,72, 144

**d** 1, 2, 3, 4, 5, 6, 7, 10, 12, 14, 15, 20, 21, 28, 30, 35, 42, 60, 70, 84, 105, 140, 210, 420

**e** 1, 2, 3, 4, 6, 8, 12, 24, 53, 106, 159, 212, 318, 424, 636, 1272

**f** 1, 2, 5, 10, 25, 50, 125, 250, 625, 1250

**2** 24, 144, 232 **3** 76, 95, 152 **4** 2, 3, 6, 62, 93

**5** 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139

**6** **a** $5 + 43 = 48$, $7 + 41 = 48$, $11 + 37 = 48$, $17 + 31 = 48$, $19 + 29 = 48$

**b** $5 + 83 = 88$, $17 + 71 = 88$, $29 + 59 = 88$, $41 + 47 = 88$

**c** $5 + 103 = 108$, $7 + 101 = 108$

**d** $5 + 109 = 114$, $7 + 107 = 114$, $11 + 103 = 114$

**e** $5 + 199 = 204$, $7 + 197 = 204$, $13 + 191 = 204$

**f** $7 + 233 = 240$, $11 + 229 = 240$, $13 + 227 = 240$

**7** **a** $4^1 = 4$, $4^2 = 16$, $4^3 = 64$, $4^4 = 256$, $4^5 = 1024$

**b** $10^1 = 10$, $10^2 = 100$, $10^3 = 1000$, $10^4 = 10000$, $10^5 = 100000$, $10^6 = 1000000$, $10^7 = 10000000$, $10^8 = 100000000$, $10^9 = 1000000000$

**8** **a** $2^3 \times 3$ **b** $2^5 \times 3$ **c** $2 \times 3 \times 5^2$ **d** $2 \times 3^4$

**e** $2^2 \times 3^4$ **f** $2^3 \times 7^2$

**9** **a** 25 **b** 81 **c** 121 **d** 784

**e** 2500 **f** 10000

**10** **a** 6 **b** 20 **c** 14 **d** 21

**e** 27 **f** 1000

**11** **a** $29 = 2^2 + 3^2 + 4^2$ **b** $105 = 1^2 + 2^2 + 10^2$

**c** $70 = 1^2 + 2^2 + 4^2 + 7^2$ **d** $299 = 1^2 + 3^2 + 17^2$

**12** Some possible answers are 2, 3, 8, 14.

**13** **a** 4 **b** 3 **c** 2 **d** 5

**e** 4 **f** 4

**14** **a** 28 **b** 40 **c** 12 **d** 8

**e** 30 **f** 72

**15** Possible answers:

**a** 400030 **b** 400030 **c** 400032

**d** 400030 **e** 400032 **f** 400032

**g** 400230 **h** 400035 **i** 400030

**16** Some possible answers are
$574 = 14 \times 41$, $1300 = 25 \times 52$, $1729 = 19 \times 91$.

## Chapter 3: An introduction to algebra

**1** **a** $6z$ **b** $xy$ **c** $x - 8$ **d** $5m + 3$

**2** $n - 5$ **3** $6x$

**4** **a** $24x^2$ **b** $35x^2$ **c** $3m^2n^3$

**5** **a** $\frac{2x}{y}$ **b** $\frac{3q}{2p}$ **c** $\frac{5w}{x}$ **d** $\frac{x}{3y}$

**6** **a** $z^2$ **b** $5ab$ **c** $12b^2$ **d** $\frac{x}{7}$ **e** $\frac{2p}{q}$

**7** **a** 14 **b** 0 **c** 53 **d** 10 **e** 14

**8** \$500

**9** **a** $9x$ **b** $4a$ **c** $6xy$ **d** $12xy$

**10** **a** $6x - 2$ **b** $5(x - 3)$

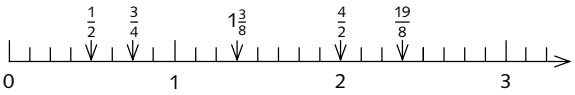
**11** **a** $3a^2$ **b** $(3a)^2$

**12** **a** $9x^2$ **b** $4z^2$ **c** $225x^2$ **d** $49c^2$

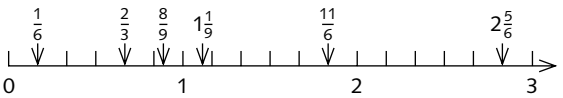
**13** $a + 5$ and $a + 10$

## Chapter 4: Fractions – part 1

**1**




**2**




**3** **a** $\frac{8}{11}$ **b** $\frac{1}{3}$ **c** $\frac{1}{8}$ **d** $\frac{4}{7}$ **e** $\frac{7}{23}$

**4** Various answers are possible.

**5** **a** 30 **b** 5 **c** 50 **d** 96

**e** 20 **f** 7

**6** **a** $\frac{1}{3}$ **b** $\frac{1}{3}$ **c** $\frac{2}{23}$ **d** $\frac{6}{17}$

**e** $\frac{33}{40}$ **f** $\frac{3}{8}$ **g** $\frac{1}{5}$ **h** $\frac{1}{4}$

**i** $\frac{7}{15}$ **j** 4 **k** 7 **l** $1\frac{3}{46}$

**7** **a** $2\frac{2}{3}$ **b** $2\frac{1}{6}$ **c** $3\frac{2}{5}$ **d** $1\frac{1}{8}$

**e** $1\frac{1}{5}$ **f** $3\frac{1}{32}$

**8** **a** $4, 4\frac{4}{5}, 5\frac{3}{5}, 6\frac{2}{5}$ **b** $3\frac{4}{7}, 4\frac{2}{7}, 5, 5\frac{5}{7}$ **c** $\frac{5}{6}, 1, 1\frac{1}{6}, 1\frac{1}{3}$

**9** **a** $\frac{19}{6}$ **b** $\frac{24}{5}$ **c** $\frac{17}{3}$ **d** $\frac{43}{22}$

**e** $\frac{61}{9}$ **f** $\frac{40}{19}$


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**10** **a–f** Various answers are possible.

**11** **a** $\frac{1}{10}, \frac{1}{6}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}$ **b** $\frac{1}{8}, \frac{1}{4}, \frac{3}{8}, \frac{3}{4}, \frac{9}{8}$

**c** $\frac{1}{3}, \frac{4}{9}, \frac{3}{6}, \frac{2}{3}, \frac{5}{6}$ **d** $\frac{1}{18}, \frac{5}{16}, \frac{9}{18}, \frac{17}{19}, \frac{16}{17}$

**e** $\frac{3}{8}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{7}{8}$ **f** $\frac{3}{5}, 1\frac{1}{5}, 1\frac{3}{10}, 2\frac{1}{4}, 2\frac{2}{6}$

**12** **a** $\frac{5}{7}$ **b** $\frac{7}{8}$ **c** $\frac{4}{5}$ **d** 2

**e** $1\frac{1}{3}$ **f** $1\frac{1}{6}$ **g** $\frac{3}{8}$ **h** $\frac{11}{12}$

**i** $1\frac{2}{9}$ **j** $1\frac{1}{20}$ **k** $1\frac{2}{21}$ **l** $1\frac{13}{110}$

**13** **a** $\frac{1}{3}$ **b** $\frac{1}{7}$ **c** $\frac{1}{6}$ **d** $\frac{5}{8}$

**e** $\frac{1}{3}$ **f** $\frac{3}{8}$ **g** $\frac{1}{4}$ **h** $\frac{1}{15}$

**i** $\frac{9}{35}$ **j** $\frac{11}{21}$ **k** $\frac{3}{20}$ **l** $\frac{13}{18}$

**14** Various answers are possible.

**15** $3\frac{1}{12}$ baskets

## Chapter 5: Fractions – part 2

**1** 12 chocolates

**2** **a** $\frac{1}{2}$ **b** $\frac{3}{10}$ **c** $\frac{1}{2}$ **d** $\frac{1}{4}$

**e** $\frac{1}{36}$ **f** $\frac{5}{16}$ **g** $\frac{4}{15}$ **h** $\frac{15}{64}$

**i** $\frac{4}{9}$ **j** $1\frac{1}{3}$ **k** $5\frac{5}{8}$ **l** 18

**3** **a** 2 **b** $3\frac{11}{15}$ **c** $\frac{18}{35}$ **d** $\frac{9}{20}$

**4** **a** 1 **b** 16 **c** 2 **d** $\frac{2}{9}$

**e** 15 **f** $\frac{3}{5}$ **g** $\frac{1}{4}$ **h** 1

**i** $\frac{1}{34}$ **j** $8\frac{1}{2}$ **k** $\frac{1}{2}$ **l** $\frac{8}{9}$

**m** $\frac{2}{9}$ **n** 2 **o** $\frac{4}{5}$ **p** $\frac{20}{49}$

**5** **a** $1\frac{1}{3}$ **b** $1\frac{1}{4}$ **c** $\frac{3}{4}$ **d** 1

**e** $1\frac{1}{6}$ **f** $\frac{4}{9}$ **g** $\frac{2}{9}$ **h** $\frac{392}{729}$

**6** **a** $8\frac{1}{2}$ **b** $\frac{1}{5}$ **c** $1\frac{1}{3}$

**7** 1788 L **8** 72 km **9** $\frac{2}{5}$

**10** 210 seconds

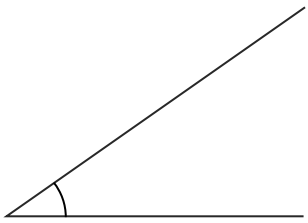
## Chapter 6: An introduction to geometry

**1** **a i** 70° **ii** 45° **iii** 8°
**iv** 72° **v** 14°
**b i** 40° **ii** 48° **iii** 95°
**iv** 112° **v** 14°

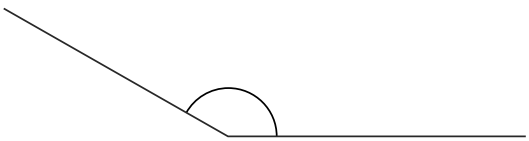
**2** **a** 90° **b** 90° **c** 120° **d** 90°
**e** 120° **f** 90° **g** 30° **h** 150°

**3** **a i** 50° **ii** 166°
**b i** 310° **ii** 194°

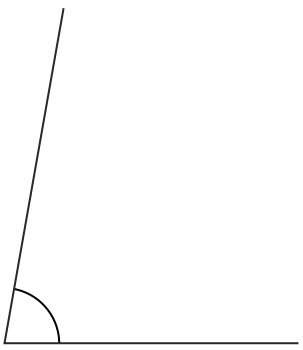
**4** **a**



**b**



**c**



**5** **a** $\theta = 60°$ (corresponding angles, $AB \| CD$)
**b** $\theta = 60°$ (co-interior angles, $DG \| EF$)
**c** $\alpha = 38°$ (alternate angles, $RS \| TU$)
**d** $\beta = 72°$ (alternate angles, $BD \| CA$)
**e** $\gamma = 130°$ (co-interior angles, $BY \| AX$)


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**6** **a** $\angle OAB = \angle ACD$ (corresponding angles are equal), $\beta = 38°$
**b** $\angle VAB = \angle VDC$ (alternate angles are equal), $\gamma = 30°$
**c** $\angle VAB = \angle VDC$ (alternate angles are equal), $\theta = 30°$
**d** $\angle BAC = \angle DCA$ (co-interior angles are supplementary), $\alpha = 105°$

## Chapter 7: Algebra with fractions

**1** **a** $\frac{x}{5} + 6$ **b** $\frac{x+5}{3}$ **c** $\frac{x}{8} - 5$
**d** $\frac{x+4}{11}$ **e** $\frac{x-12}{12}$ **f** $7\left(\frac{x}{10} - 4\right)$
**g** $\frac{5x}{8} + 7$ **h** $\frac{7x}{8} + 12$ **i** $9\left(\frac{x}{8} - 5\right)$

**2** **a** 10 **b** 7 **c** $7\frac{1}{3}$
**d** $4\frac{1}{2}$ **e** 30 **f** 16
**g** 0 **h** 48 **i** 3

**3** **a** $(n + 70)$ L **b** $\frac{n+70}{150}$ L

**4** **a** $\frac{a}{12} - 5$ **b** $2(a + 7)$ **c** $\frac{b-11}{5}$ **d** $\frac{10-b}{15}$

**5** **a** $\frac{4}{5}$ **b** $1\frac{1}{4}$ **c** 9 **d** $\frac{4}{5}$
**e** 14 **f** $4\frac{1}{4}$ **g** 12 **h** 4

**6** **a** $\frac{7x}{2}$ **b** $\frac{11x}{3}$ **c** $\frac{x}{5}$
**d** $\frac{3y}{4}$ **e** $13\left(\frac{y}{7} + 11\right)$

**7** **a** $23\frac{1}{3}$ **b** 15 **c** $16\frac{4}{5}$

**8** **a** $\frac{x}{7}$ m **b** **i** 3 m **ii** 6 m **iii** $14\frac{2}{7}$ m

## Chapter 8: Decimals

**1** **a** 6 **b** 600 **c** 0.6
**d** 0.0006 **e** 0.6 **f** 0.006
**g** 0.06 **h** 0.0006

**2** **a** 0.8 **b** 0.96 **c** 3.5
**d** 2.1 **e** 9.8 **f** 10.45
**g** 1.03 **h** 4.92 **i** 0.39

**3** **a** 3.009, 3.059, 3.061, 3.1, 3.45
**b** 4.0002, 4.203, 4.222 22, 4.3, 4.32

**4** **a** $\frac{3}{10}$ **b** $\frac{1}{4}$ **c** $\frac{3}{5}$
**d** $1\frac{87}{100}$ **e** $4\frac{216}{625}$ **f** $2\frac{1}{20}$
**g** $7\frac{1}{2}$ **h** $10\frac{153}{500}$ **i** $\frac{1}{2500}$
**j** $4\frac{7}{1000}$ **k** $9\frac{9029}{10000}$ **l** $10\frac{3}{8}$

**5** **a** 0.75 **b** 0.5 **c** 0.1
**d** 0.45 **e** 0.06 **f** 0.234
**g** 0.4 **h** 3.2 **i** 1.4
**j** $0.\dot{6}$ **k** $1.\dot{6}$ **l** $0.\dot{7}14\,285\dot{7}$

**6** **a** 8.788 **b** 5.842 **c** 86.0872
**d** 342 **e** 35.668 **f** 156.9
**g** 520.5 **h** 411.3518 **i** $91.8\dot{3}$
**j** 13.299 173 **k** 156.558 **l** 7239.785

**7** **a** $\frac{3}{8}$ **b** 0.4 **c** $\frac{3}{4}$
**d** 2.75 **e** $3\frac{1}{8}$

**8** **a** $0.1\dot{6}$ **b** $0.\dot{2}85\,714\dot{2}$ **c** $0.\dot{2}\dot{7}$
**d** $0.8\dot{3}$ **e** $0.\dot{8}57\,142\dot{8}$

## Chapter 9: Measurement

**1** **a** 120 mm **b** 250 mL **c** 4 mL **d** −3°C

**2** **a** 1.2 **b** 14.3, 0.143 **c** 230, 2300
**d** 800 **e** 4 **f** 3000
**g** 324 000, 0.324 **h** 25 000 **i** 2.305
**j** 3200 **k** 40

**3** 3.035 kg or 3035 g **4** 6607 g

**5** $7.11 **6** 17.5 days **7** 8 apples

**8** 20 cm, 15 $cm^2$

**9** **a** 260 cm, 2016 $cm^2$ **b** 26 m, 40 $m^2$
**c** 232 mm, 2788 $mm^2$ **d** $10x + 6y$, $15xy$

**10** **a** 36 hours 25 minutes **b** 6 hours 43 minutes
**c** 51 minutes 35 seconds

**11** **a** 5:54:30 p.m.
**b** 6:57 a.m. Saturday
**c** 2:46 p.m.

## 10B Problem-solving

### Evens and odds

**1** sometimes true and sometimes false

**2** always true

**3** always false

**4** sometimes true and sometimes false

**5** sometimes true and sometimes false


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## The Collatz conjecture

| Number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Steps | 1 | 2 | 8 | 3 | 6 | 9 | 17 | 4 | 20 | 7 |
| Largest | 1 | 2 | 16 | 4 | 16 | 16 | 52 | 8 | 52 | 16 |

| Number | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|
| Steps | 15 | 10 | 10 | 18 | 18 | 5 | 13 | 21 | 21 | 8 |
| Largest | 52 | 16 | 40 | 52 | 160 | 16 | 52 | 52 | 88 | 20 |

| Number | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|
| Steps | 8 | 16 | 16 | 11 | 24 | 11 | 112 | 19 | 19 | 19 |
| Largest | 64 | 52 | 160 | 24 | 88 | 40 | 9232 | 52 | 88 | 160 |

## Totient numbers

**1** **7**: $\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$; totient number of 6

**8**: $\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}$; totient number of 4

**9**: $\frac{1}{9}, \frac{2}{9}, \frac{4}{9}, \frac{5}{9}, \frac{7}{9}, \frac{8}{9}$; totient number of 6

**10**: $\frac{1}{10}, \frac{3}{10}, \frac{7}{10}, \frac{9}{10}$; totient number of 4

**11**: $\frac{1}{11}, \frac{2}{11}, \frac{3}{11}, \frac{4}{11}, \frac{5}{11}, \frac{6}{11}, \frac{7}{11}, \frac{8}{11}, \frac{9}{11}, \frac{10}{11}$; totient number of 10

**12**: $\frac{1}{12}, \frac{5}{12}, \frac{7}{12}, \frac{11}{12}$; totient number of 4

**2** **15**: $\frac{1}{15}, \frac{2}{15}, \frac{4}{15}, \frac{7}{15}, \frac{8}{15}, \frac{11}{15}, \frac{13}{15}, \frac{14}{15}$; totient number of 8

**18**: $\frac{1}{18}, \frac{5}{18}, \frac{7}{18}, \frac{11}{18}, \frac{13}{18}, \frac{17}{18}$; totient number of 6

**20**: $\frac{1}{20}, \frac{3}{20}, \frac{7}{20}, \frac{9}{20}, \frac{11}{20}, \frac{13}{20}, \frac{17}{20}, \frac{19}{20}$; totient number of 8

**24**: $\frac{1}{24}, \frac{5}{24}, \frac{7}{24}, \frac{11}{24}, \frac{13}{24}, \frac{17}{24}, \frac{19}{24}, \frac{13}{24}$; totient number of 8

**3** **72**: $\frac{1}{72}, \frac{5}{72}, \frac{7}{72}, \frac{11}{72}, \frac{13}{72}, \frac{17}{72}, \frac{19}{72}, \frac{23}{72}, \frac{25}{72}, \frac{29}{72}, \frac{31}{72}, \frac{35}{72}, \frac{37}{72}, \frac{41}{72}, \frac{43}{72}, \frac{47}{72}, \frac{49}{72}, \frac{53}{72}, \frac{55}{72}, \frac{59}{72}, \frac{61}{72}, \frac{65}{72}, \frac{67}{72}, \frac{71}{72}$; totient number of 24

**81**: $\frac{1}{81}, \frac{2}{81}, \frac{4}{81}, \frac{5}{81}, \frac{7}{81}, \frac{8}{81}, \frac{10}{81}, \frac{11}{81}, \frac{13}{81}, \frac{14}{81}, \frac{16}{81}, \frac{17}{81}, \frac{19}{81}, \frac{20}{81}, \frac{22}{81}, \frac{23}{81}, \frac{25}{81}, \frac{26}{81}, \frac{28}{81}, \frac{29}{81}, \frac{31}{81}, \frac{32}{81}, \frac{34}{81}, \frac{35}{81}, \frac{37}{81}, \frac{38}{81}, \frac{40}{81}, \frac{41}{81}, \frac{43}{81}, \frac{44}{81}, \frac{46}{81}, \frac{47}{81}, \frac{49}{81}, \frac{50}{81}, \frac{52}{81}, \frac{53}{81}, \frac{55}{81}, \frac{56}{81}, \frac{58}{81}, \frac{59}{81}, \frac{61}{81}, \frac{62}{81}, \frac{64}{81}, \frac{65}{81}, \frac{67}{81}, \frac{68}{81}, \frac{70}{81}, \frac{71}{81}, \frac{73}{81}, \frac{74}{81}, \frac{76}{81}, \frac{77}{81}, \frac{79}{81}, \frac{80}{81}$; totient number of 54

**98**: $\frac{1}{98}, \frac{3}{98}, \frac{5}{98}, \frac{9}{98}, \frac{11}{98}, \frac{13}{98}, \frac{15}{98}, \frac{17}{98}, \frac{19}{98}, \frac{23}{98}, \frac{25}{98}, \frac{27}{98}, \frac{29}{98}, \frac{31}{98}, \frac{33}{98}, \frac{37}{98}, \frac{39}{98}, \frac{41}{98}, \frac{43}{98}, \frac{45}{98}, \frac{47}{98}, \frac{51}{98}, \frac{53}{98}, \frac{55}{98}, \frac{57}{98}, \frac{59}{98}, \frac{61}{98}, \frac{65}{98}, \frac{67}{98}, \frac{69}{98}, \frac{71}{98}, \frac{73}{98}, \frac{75}{98}, \frac{79}{98}, \frac{81}{98}, \frac{83}{98}, \frac{85}{98}, \frac{87}{98}, \frac{89}{98}, \frac{93}{98}, \frac{95}{98}, \frac{97}{98}$; totient number of 42

**100**: $\frac{1}{100}, \frac{3}{100}, \frac{7}{100}, \frac{9}{100}, \frac{11}{100}, \frac{13}{100}, \frac{17}{100}, \frac{19}{100}, \frac{21}{100}, \frac{23}{100}, \frac{27}{100}, \frac{29}{100}, \frac{31}{100}, \frac{33}{100}, \frac{37}{100}, \frac{39}{100}, \frac{41}{100}, \frac{43}{100}, \frac{47}{100}, \frac{49}{100}, \frac{51}{100}, \frac{53}{100}, \frac{57}{100}, \frac{59}{100}, \frac{61}{100}, \frac{63}{100}, \frac{67}{100}, \frac{69}{100}, \frac{71}{100}, \frac{73}{100}, \frac{77}{100}, \frac{79}{100}, \frac{81}{100}, \frac{83}{100}, \frac{87}{100}, \frac{89}{100}, \frac{91}{100}, \frac{93}{100}, \frac{97}{100}, \frac{99}{100}$; totient number of 40

## Last digits of powers

### *Square numbers*

Square numbers never end in 2, 3, 7 or 8.

The possible pairs of digits that odd squares may end with are:

- 01, 21, 41, 61 and 81 if the end digit of the number is 1 or 9
- 09, 29, 49, 69, and 89 if the end digit of the number is 3 or 7
- 25 if the end digit is 5.

In a square number, the last digit can only be 0, 1, 4, 5, 6 or 9. The second-last digit of an odd square is always even.

### *Cube numbers*

For cube numbers the complete table is:

| If a number ends in | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| Its cube will end in | 0 | 1 | 8 | 7 | 4 | 5 | 6 | 3 | 2 | 9 |

### *Fifth powers*

The fifth power of a number always ends in the same final digit as the original number.

## Obstinate numbers

**a** $45 = 43 + 2 = 41 + 4 = 37 + 8 = 29 + 16 = 13 + 32$ … five pairs.

**b** $29 = 13 + 16$ … only one pair as 27, 25 and 21 are all composite.

**c** $59 = 43 + 16$ … only one pair as 57, 55, 51 and 27 are all composite.

**d** $95 = 79 + 16 = 31 + 64$ … two pairs as 93, 91, 87 and 63 are all composite.

The two smallest obstinate numbers are 127 ($125 = 5 \times 25$, $123 = 3 \times 41$, $119 = 7 \times 17$, $111 = 3 \times 37$, $95 = 5 \times 19$, $63 = 7 \times 9$ … all residuals are composite) and 149 ($147 = 3 \times 49$, $145 = 5 \times 29$, $141 = 3 \times 47$, $133 = 7 \times 19$, $117 = 9 \times 13$, $85 = 5 \times 17$, $21 = 3 \times 7$ … all residuals are composite).

## Areas

**1** **a** area of $BWA = 3$ cm$^2$, area of $CWX = 0.75$ cm$^2$

**b** area of $BAC = 4.5$ cm$^2$, area of $BWC = 1.5$ cm$^2$

**c** 3.75 cm$^2$

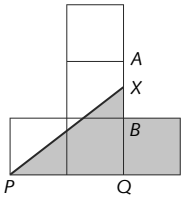
**2** **a** 144 cm$^2$

**b** area of $PAX = 64$ cm$^2$, area of $PBX = 32$ cm$^2$

**c** $QB = 3$ cm


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**3**




$BX = \frac{1}{2} \times AB$

## The grid method of multiplying decimals

**1** 0.217728 **2** 37.9974 **3** 39.8978

### 10C Number bases

**1** **a** $202_4$ **b** $320_4$ **c** $1032_4$
**d** $1210_4$ **e** $2002_4$

**2** **a** $24_8$ **b** $42_8$ **c** $106_8$
**d** $144_8$ **e** $1000_8$ **f** $1130_8$

**3** **a** $279_{10}$ **b** $59_{10}$ **c** $19_{10}$ **d** $289_{10}$

### 10D Binary numbers

**1** **a** $101_2$ **b** $1010_2$ **c** $1100_2$
**d** $10000_2$ **e** $101101_2$

**2** **a** $9_{10}$ **b** $7_{10}$ **c** $11_{10}$ **d** $12_{10}$ **e** $73_{10}$

**3** **a** $1000_2$ **b** $10100_2$ **c** $1101_2$ **d** $10100_2$

**4** **a** $11000_2$ **b** $1111_2$ **c** $100_2$
**d** $10_2$ **e** $100100_2$

**5** **a** $10101_2$ **b** $11110_2$ **c** $11110_2$

**6** $101011_2$ **7** $1_2$

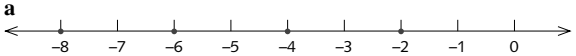
# Chapter 11 answers

### Exercise 11A

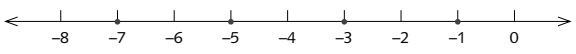
**1** **a** −4, −3, −2, −1, 0, 1, 2 **b** −7, −6, −5, −4, −3, −2
**c** −9, −8, −7, −6, −5
**d** −131, −130, −129, −128, −127, −126, −125, −124

**2** **a** −1000, −100, −10, −6, 0, 5, 10, 100
**b** 55, 45, 26, −26, −30, −31, −45, −550

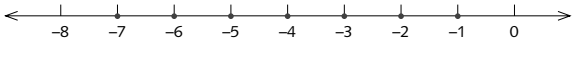
**3** **a**



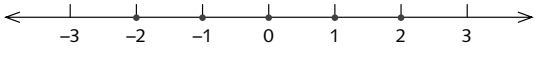

**b**



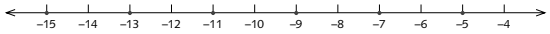

**c**



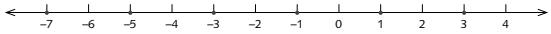

**d**



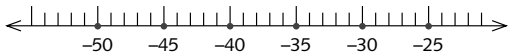

**4** −9, −7, −5




**5** −3, −5, −7




**6** −35, −30, −25




**7** **a** −5 **b** 4 **c** 10 **d** 12
**e** −7 **f** 8 **g** 4 **h** 3

**8** **a** 2 **b** 7 **c** 20
**d** −10 **e** −30 **f** 40

**9** **a** $3 < 5$ **b** $3 > -5$
**c** $-7 < -4$ **d** $2 < -(-3)$

**10** **a** −15°C **b** −30°C **c** −18°C **d** −40°C

### Exercise 11B

**1** **a** 2 **b** 1 **c** 5 **d** 3
**e** 4 **f** −3 **g** 6 **h** 5
**i** −7 **j** −8 **k** 2 **l** 11

**2** **a** −1 **b** −6 **c** −15 **d** −21
**e** −23 **f** −7 **g** −8 **h** −5
**i** −15 **j** −17 **k** −19 **l** −13

**3** **a** 1 **b** −11 **c** −29 **d** −21
**e** −43 **f** −7 **g** −28 **h** 15
**i** −45 **j** 3 **k** −60 **l** −6

**4** **a** −34 **b** −14 **c** 0
**d** 1 **e** −20

**5** **a** \$1600 **b** \$1214 **c** \$240

**6** **a** −280 m **b** −9°C

**7** **a** 1 − 2 + 3 − 4 **b** 3 − 10 + 9 + 5
**c** 1 + 2 − 3 + 4 − 5

### Exercise 11C

**1** **a** 3 **b** −4 **c** 5 **d** −15
**e** 4 **f** −7 **g** −8 **h** −15
**i** 7 **j** −8 **k** −50 **l** −10

**2** **a** 11 **b** 18 **c** 5 **d** 15
**e** 4 **f** −3 **g** −4 **h** 15
**i** −15 **j** −8 **k** 5 **l** 70

**3** **a** −36 **b** −14 **c** 14 **d** −1
**e** 2 **f** 9 **g** −27 **h** 71
**i** 16 **j** 44 **k** 200 **l** 1550

**4** **a** 43 **b** −22 **c** 13


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**5** **a** −5 **b** 35 **c** 65 **d** 12
**e** 22 **f** −23 **g** 42 **h** −17
**i** −19 **j** 41 **k** 116 **l** −25

**6** 17°C rise **7** 10°C rise

**8**

| Temperature at 5 a.m. (°C) | Temperature at 2 p.m. (°C) | Rise in temperature (°C) |
|---|---|---|
| −10 | 5 | 15 |
| −15 | 5 | 20 |
| −25 | −3 | 22 |
| −20 | −15 | 5 |
| −7 | −5 | 2 |
| −11 | 2 | 13 |
| −13 | 5 | 18 |

**9** 6 minutes **10** −14°C

**11**

| Day | Temperature inside (°C) | Temperature outside (°C) | Inside − outside (°C) |
|---|---|---|---|
| M | 20 | 25 | −5 |
| T | 13 | 18 | −5 |
| W | 24 | 20 | 4 |
| T | 10 | −5 | 15 |
| F | −5 | −10 | 5 |
| S | 3 | −6 | 9 |

A negative number means it is warmer outside than inside.

**12** Jane will have $70 left. **13** −50

## Exercise 11D

**1** **a** −5 **b** −6 **c** 6 **d** −16
**e** 0 **f** 6 **g** −18 **h** −24
**i** −56 **j** 0 **k** 10 **l** −24

**2** **a** −10 **b** −12 **c** −5 **d** −44
**e** −192 **f** −10 **g** −12 **h** −50
**i** −44 **j** −48 **k** 120 **l** −48
**m** 126 **n** 169 **o** −152 **p** −60
**q** 153 **r** 102 **s** −280 **t** 180

**3** **a** 36 **b** −168 **c** 2400
**d** 6300 **e** 2700 **f** −204
**g** −12 **h** 18 **i** −6

**4** **a** −15 **b** −13 **c** −6 **d** −7
**e** −6 **f** −5 **g** 12 **h** 8
**i** 0 **j** 12 **k** −27 **l** −16

## Exercise 11E

**1** **a** −5 **b** −13 **c** −5 **d** −7
**e** −40 **f** −5 **g** −18 **h** −9
**i** −3 **j** −456 **k** 17 **l** 6
**m** 2 **n** −11 **o** 16 **p** 24
**q** 3 **r** 50 **s** −27 **t** −36

**2** **a** −5 **b** 5 **c** −3 **d** −2
**e** 1 **f** −1 **g** 50 **h** −2
**i** −5 **j** −4 **k** 3 **l** −1

**3** **a** 4 **b** −4 **c** −5 **d** 14
**e** −33 **f** −120 **g** −15 **h** −73
**i** −8 **j** −29 **k** −14 **l** 10
**m** −24 **n** −64 **o** 14 **p** 18

**4** **a** −5 **b** −5 **c** 104
**d** 169 **e** −400 **f** 0
**g** 12 **h** −450 **i** −11

**5** **a** −150 **b** −204 **c** −12
**d** 40 **e** 10 **f** 50

**6** **a** $8 \div 4 \times (-2) \times 3 = -12$
**b** $3 \times 4 \times (-5) \div 6 = -10$

## Exercise 11F

**1** **a** −1 **b** 26 **c** −9
**d** 22 **e** −16 **f** 149
**g** −42 **h** −120 **i** 65

**2** **a** 56 **b** 48 **c** −78 **d** 30
**e** −45 **f** −18 **g** 34 **h** −110

**3** **a** 14 **b** 27 **c** 0 **d** −48
**e** −18 **f** 93 **g** 126 **h** −140
**i** 49 **j** −532 **k** −350

**4** **a** 64 **b** −121 **c** 32 **d** −81
**e** −900 **f** 144 **g** −125 **h** 16
**i** −32 **j** 64 **k** −1 **l** 1

**5** **a** 128 **b** −96 **c** −256 **d** 20
**e** 48 **f** −2 **g** −108 **h** −7

**6** **a** 24 **b** −10 **c** −25 **d** −60
**e** −40 **f** −189 **g** 9 **h** −170

**7** **a** −1 **b** −24 **c** 10 **d** −12

**8** **a** 80 **b** −100000 **c** −1900 **d** 2000

**9** **a** −8 **b** −2 **c** −8 **d** −1
**e** 23 **f** −29 **g** 7 **h** 15


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**10** \$3200, −\$200, a loss of \$200

**11** **a** \$15 000 **b** −\$5000

**12** **a** \$340 **b** \$430 **c** \$20 loss

**13** **a** \$140 loss **b** \$220 profit **c** \$40 profit

## Review exercise

**1** **a** 23 **b** −14 **c** 15 **d** −95
**e** −16 **f** −68 **g** −180 **h** −60
**i** 70 **j** −80 **k** 32 **l** −50
**m** −25 **n** −135 **o** −6 **p** −85
**q** −190 **r** −65 **s** 70 **t** −82
**u** −547

**2** −10

**3** **a** 25°C **b** −25°C

**4** **a** 25°C **b** −25°C

**5** **a** −250 **b** −396 **c** −1750 **d** 2040
**e** −48 **f** 1150 **g** 3200 **h** 500
**i** −440 **j** −2400 **k** 100 **l** 100

**6** **a** −25 **b** −4 **c** −7 **d** 17
**e** 4 **f** 5 **g** 8 **h** 150
**i** $-2\frac{3}{4}$ **j** −6 **k** 14 **l** −25

**7** **a** 4 **b** −63 **c** −60
**d** 60 **e** −168 **f** −16
**g** 16 **h** 49 **i** −90

**8** −56

**9** −6

**10** **a** 64 **b** −64 **c** −12 **d** −22
**e** −26 **f** −1000 **g** $-1\frac{1}{2}$ **h** 8

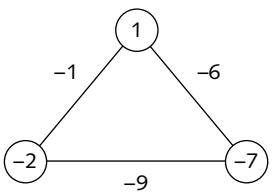
**11** **a** 55 **b** −93 **c** −13 **d** −64
**e** −37 **f** 30 **g** −119 **h** 51

## Challenge exercise

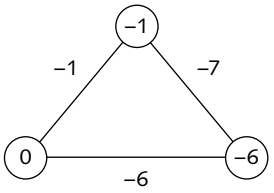
**1** −32

**2** **a** 1 **b** −1

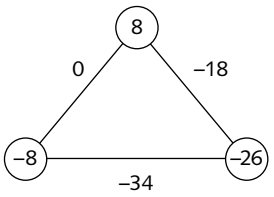
**3** **a**



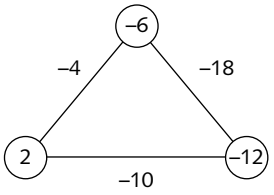

**b**




**c**




**d**




**4** **a** 4 + (−7) − (−2) **b** 4 + (−2) − (−7)
**c** −7 + (−2) − 4

**5** **a** 5 + (−4) − (−5) **b** −5 + (−4) − 5
**c** 5 + (−5) − (−4)

**6** −2

**7** **a** (5 + (−3)) × (3 + 4) = 14
**b** 5 + (−3) × 3 + 4 × 2 = 4
**c** (5 − 5) × 6 + 7 × 6 − 5 = 37

**8**

| | | |
|---|---|---|
| 2 | −5 | 0 |
| −3 | −1 | 1 |
| −2 | 3 | −4 |

**9** **a** **i** −6 **ii** −6 **b** −50

**10** 100 **11** −6

**12** **a** −10 **b** −52

# Chapter 12 answers

## Exercise 12A

**1** **a** −4 **b** 2 **c** 0 **d** −5
**e** −1 **f** −8 **g** 8 **h** 4
**i** 5 **j** 7 **k** 1 **l** 12

**2** **a** −57 **b** 36 **c** −64 **d** 35
**e** 66 **f** 125 **g** 15 **h** −17

**3** **a** −1 **b** −28 **c** 20 **d** 96
**e** −72 **f** 6 **g** 288 **h** −8

**4** **a** −6 **b** 14 **c** 10
**d** −10 **e** −20 **f** 100


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**5** **a** $-1$ **b** 15 **c** $-3$
**d** 7 **e** 27 **f** 27
**g** 36 **h** 216 **i** $-25$

**6** **a** 9 **b** 81 **c** 11
**d** $-216$ **e** 27 **f** $-18$

**7** **a** $-30$ **b** 30 **c** $-60$ **d** 200
**e** $-1000$ **f** $-\frac{2}{5}$ **g** $-\frac{5}{2}$ **h** 10 000

**8** **a** $-400$ **b** $-10\,000$ **c** $-10$
**d** $-8000$ **e** $-900$ **f** 0

**9** **a** 40 **b** $-16$ **c** 14
**d** 64 **e** 12 **f** $-48$

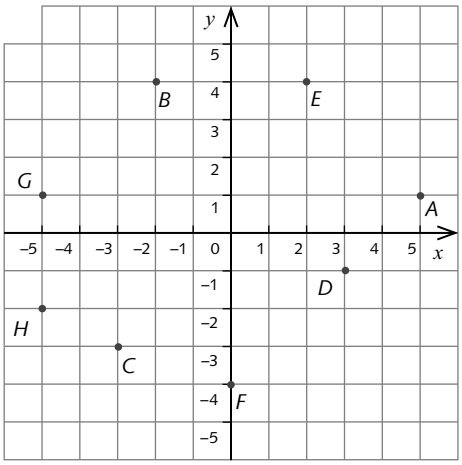
**10** **a** **i** $\$(1000 - x)$ **ii** $\$(1000 - 5x)$
**b** **i** \$500 **ii** \$0 **iii** $\$ - 250$

**11** **a** **i** $(25 - x)°C$ **ii** $(25 - 6x)°C$
**b** $-5°C$

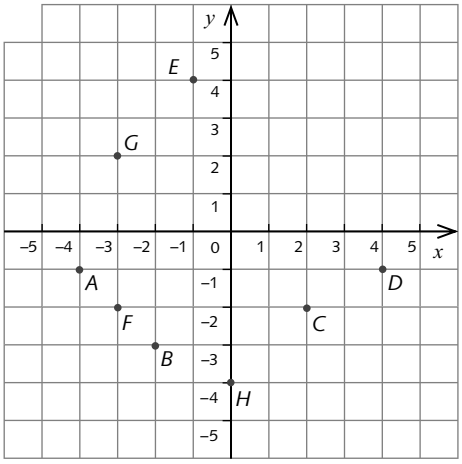
## Exercise 12B

**1** $A(4, 4)$, $B(3, -1)$, $C(0, 4)$, $D(-5, 0)$, $E(-3, -3)$, $F(-5, 5)$, $G(5, -4)$

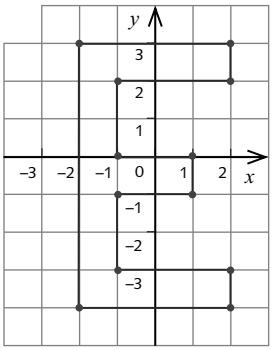
**2**




**3**

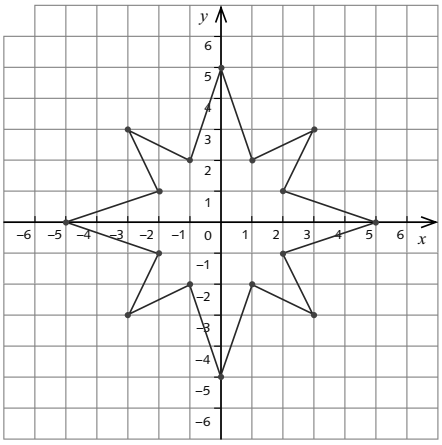



**4** **a**



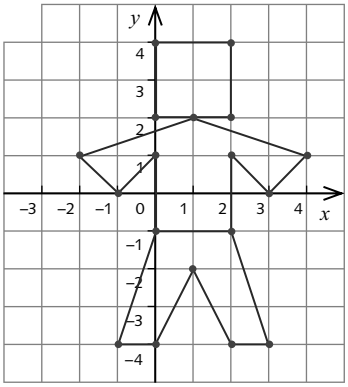

The letter E

**b**




An eight-pointed star

**c**

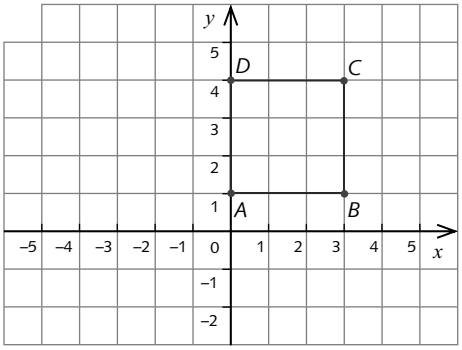



Ned Kelly

**5** **a** $A(3, 2)$, $B(4, 0)$, $C(3, -3)$, $D(-1, -1)$, $E(-3, 4)$, $F(-5, 3)$, $G(0, 2)$

**b** $A(1, 2)$, $B(3, 3)$, $C(3, -1)$, $D(2, -1)$, $E(0, 0)$, $F(-3, 0)$, $G(-2, 3)$

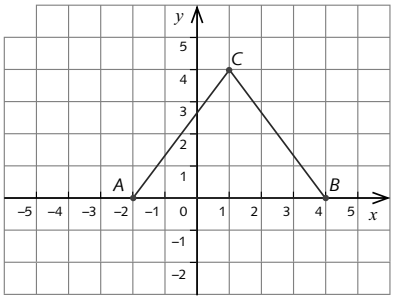
**6** **a**




square; area = 9 cm$^2$

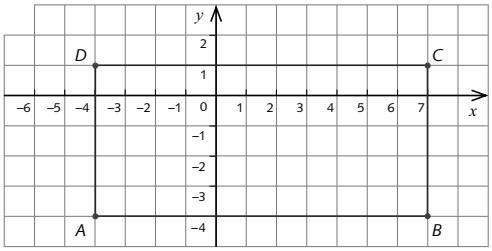

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**b**



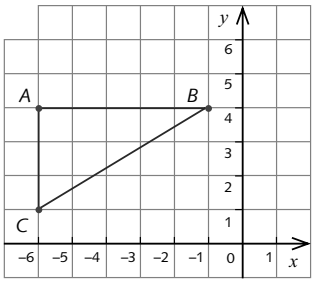

triangle; area = 12 cm$^2$

**c**



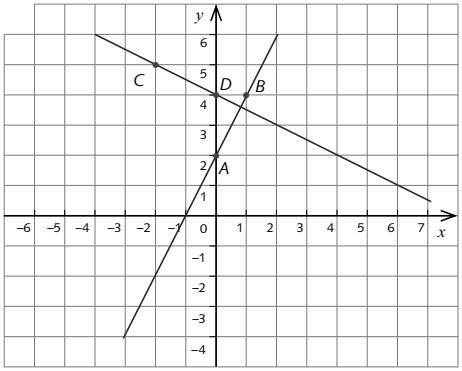

rectangle; area = 55 cm$^2$

**d**



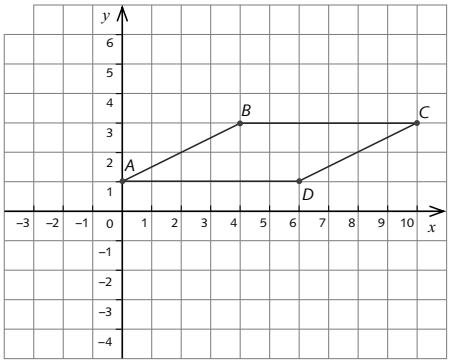

right-angled triangle; area = 7.5 cm$^2$

**e**



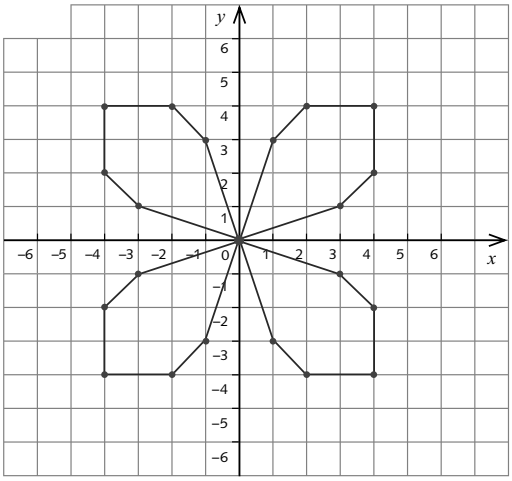

The lines meet at right angles.

**f**




parallelogram; area = 12 cm$^2$

**7** **a, b, c**




**b** (0, 0), (3, −1), (4, −2), (4, −4), (2, −4), (1, −3), (0, 0)

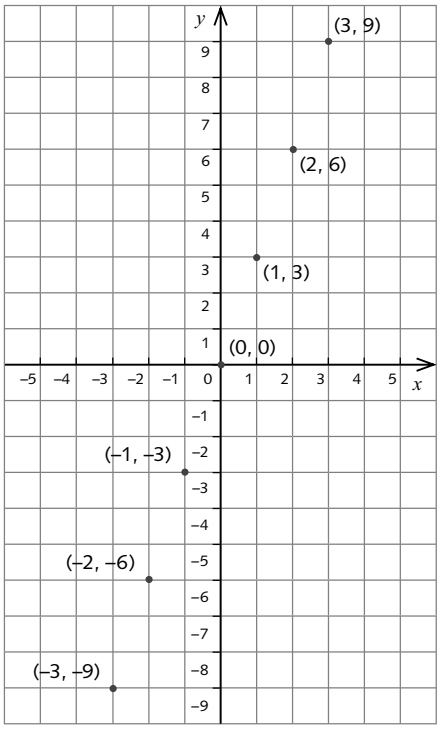
**d** (0, 0), (−3, −1), (−4, −2), (−4, −4), (−2, −4), (−1, −3), (0, 0)

(0, 0), (3, −1), (−4, 2), (−4, 4), (−2, 4), (−1, 3), (0, 0)

## Exercise 12C

**1** **a** $y = 3x$

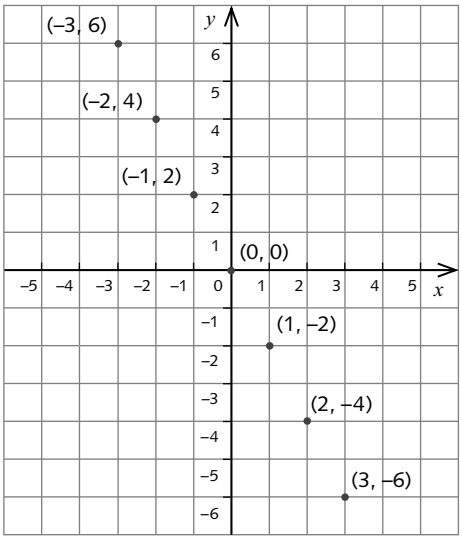
| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | −9 | −6 | −3 | 0 | 3 | 6 | 9 |




**b** $y = -2x$

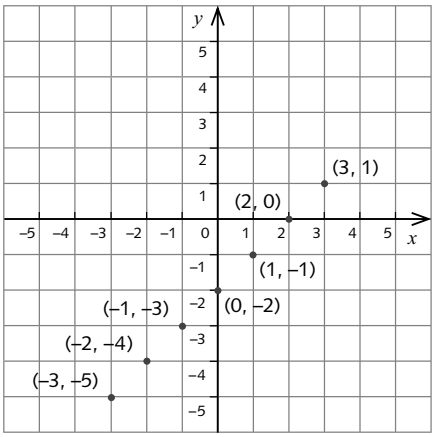
| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | 6 | 4 | 2 | 0 | −2 | −4 | −6 |


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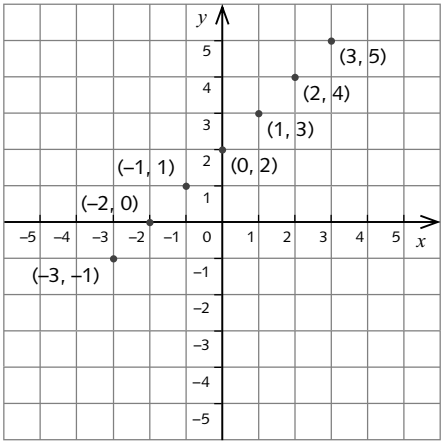

**c** $y = x - 2$

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | –5 | –4 | –3 | –2 | –1 | 0 | 1 |



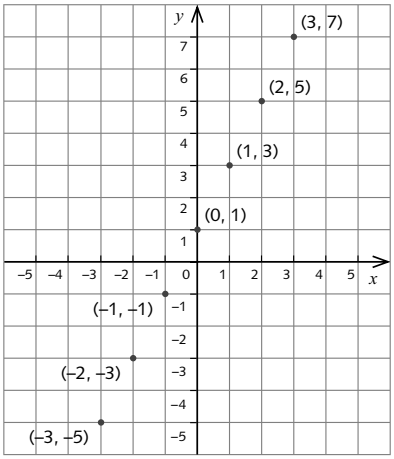

**d** $y = x + 2$

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | –1 | 0 | 1 | 2 | 3 | 4 | 5 |



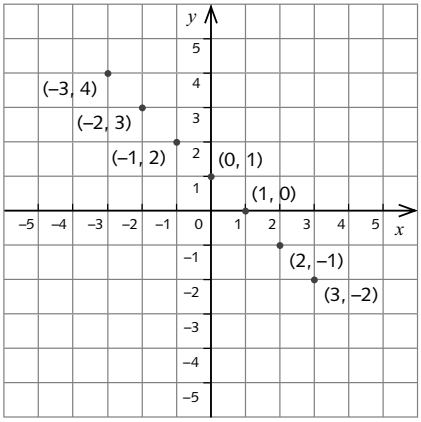

**e** $y = 2x + 1$

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | –5 | –3 | –1 | 1 | 3 | 5 | 7 |



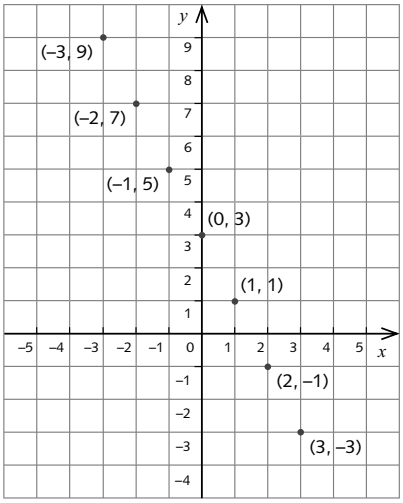

**f** $y = 1 - x$

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | 4 | 3 | 2 | 1 | 0 | –1 | –2 |




**g** $y = 3 - 2x$

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | 9 | 7 | 5 | 3 | 1 | –1 | –3 |

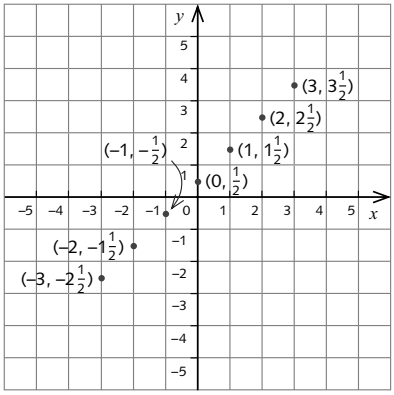




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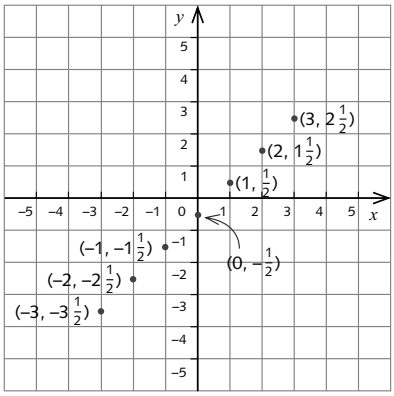
**2** **a** $y = x + \frac{1}{2}$

| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|---|
| $y$ | $-2\frac{1}{2}$ | $-1\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $1\frac{1}{2}$ | $2\frac{1}{2}$ | $3\frac{1}{2}$ |



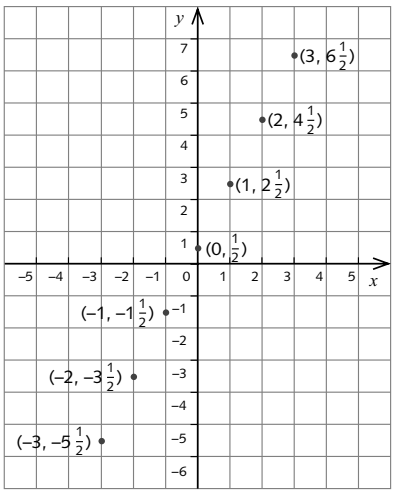

**b** $y = x - \frac{1}{2}$

| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|---|
| $y$ | $-3\frac{1}{2}$ | $-2\frac{1}{2}$ | $-1\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $1\frac{1}{2}$ | $2\frac{1}{2}$ |



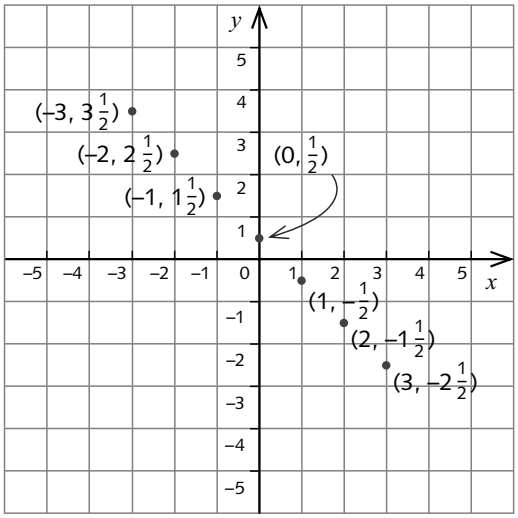

**c** $y = 2x + \frac{1}{2}$

| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|---|
| $y$ | $-5\frac{1}{2}$ | $-3\frac{1}{2}$ | $-1\frac{1}{2}$ | $\frac{1}{2}$ | $2\frac{1}{2}$ | $4\frac{1}{2}$ | $6\frac{1}{2}$ |




**d** $y = -x + \frac{1}{2}$

| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|---|
| $y$ | $3\frac{1}{2}$ | $2\frac{1}{2}$ | $1\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $-1\frac{1}{2}$ | $-2\frac{1}{2}$ |




**3** **a**

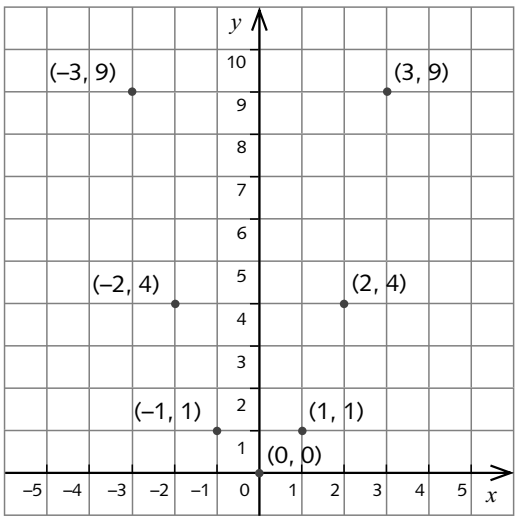
| $x$ | $0$ | $1$ | $2$ | $3$ | $4$ | $5$ | $6$ |
|---|---|---|---|---|---|---|---|
| $y$ | $-7$ | $-2$ | $3$ | $8$ | $13$ | $18$ | $23$ |

**b**

| $x$ | $-4$ | $-7$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ | $4$ |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | $25$ | $21$ | $17$ | $13$ | $9$ | $5$ | $1$ | $-3$ | $-7$ |

**4** **a** $y = x^2$

| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|---|
| $y$ | $9$ | $4$ | $1$ | $0$ | $1$ | $4$ | $9$ |

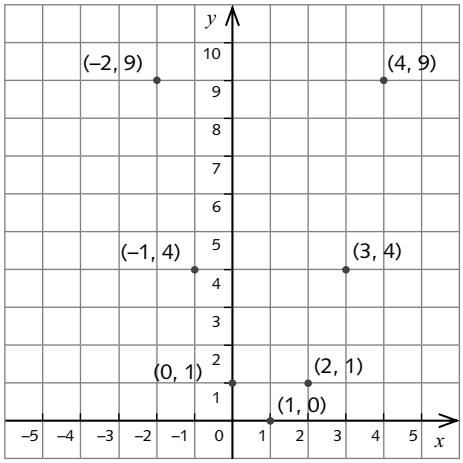




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**b** $y = (1 - x)^2$

| $x$ | –2 | –1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|
| $y$ | 9 | 4 | 1 | 0 | 1 | 4 | 9 |




## Exercise 12D

**1** **a** $y = 1 \times x + 1$
$= x + 1$

**b** $d = 3 \times t + 6$
$= 3t + 6$

**c** $y = 2 \times x + 2$
$= 2x + 2$

**d** $d = -3 \times t + 6$
$= -3t + 6$

**e** $n = 3 \times m + 6$
$= 3m + 6$

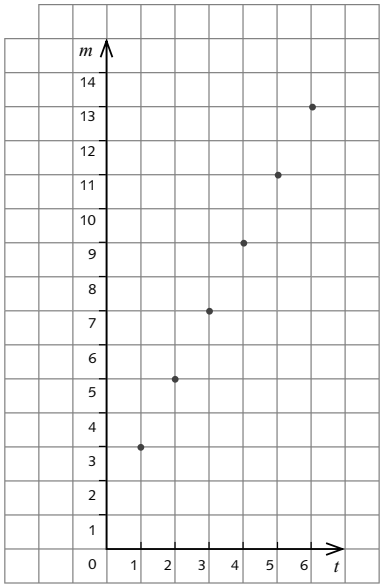
**f** $y = -2 \times x - 1$
$= -2x - 1$

**2** **a**

| 4 | 5 | 6 |
|---|---|---|
| 9 | 11 | 13 |

**b** 2

**c**



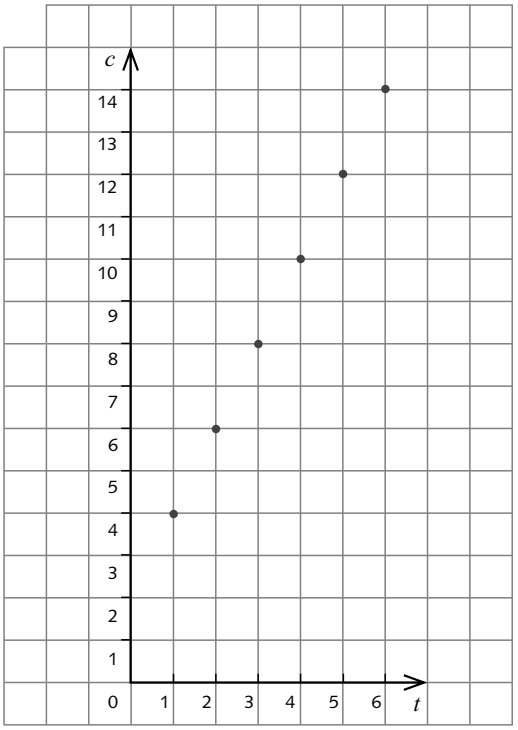

**d** $m = 2t + 1$

**3** **a**

| 4 | 5 | 6 |
|---|---|---|
| 10 | 12 | 14 |

**b** 2

**c**



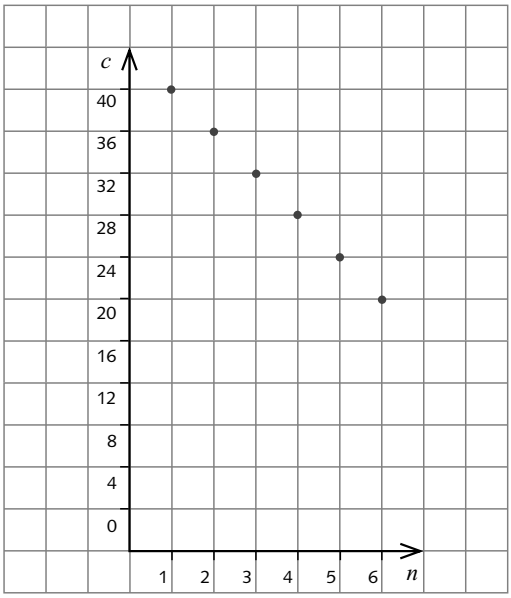

**d** $c = 2t + 2$

**4** **a** 40

**b**

| 4 | 5 | 6 |
|---|---|---|
| 28 | 24 | 20 |

**c**



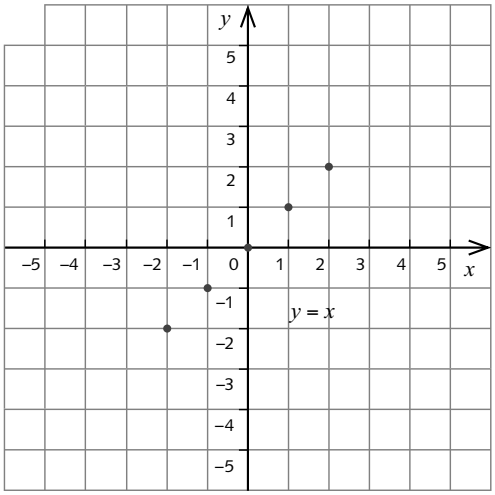


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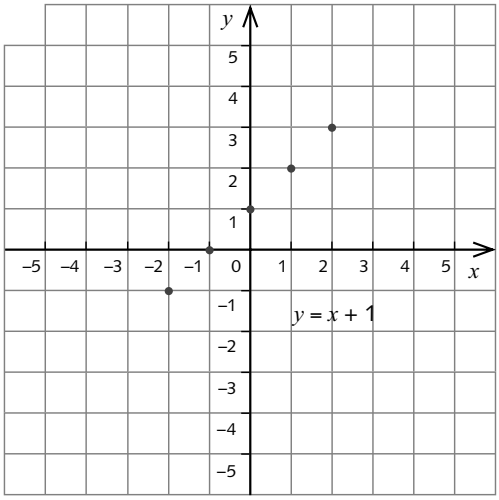
**d** $c = -4n + 44$

**e** 6

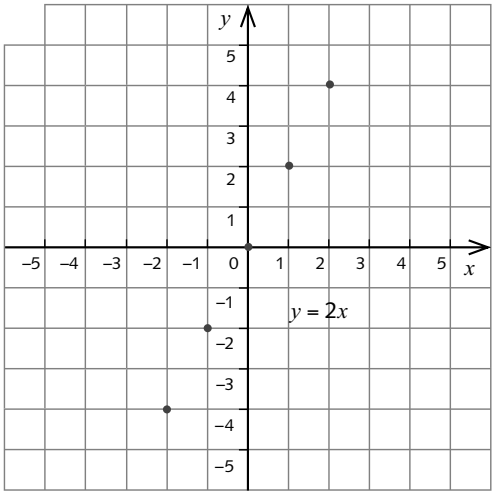
**5** **a**



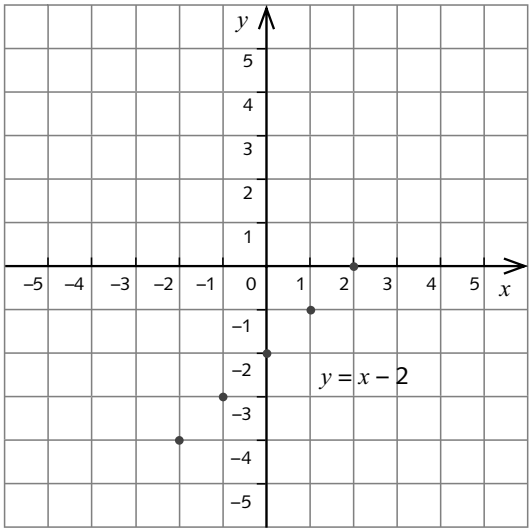

**b**




**c**




**d**




**6** **a**

| | | | | |
|---|---|---|---|---|
| 2 | 3 | 4 | 5 | 6 |
| 960 | 940 | 920 | 900 | 880 |

**b** $d = 1000 - 20m$

**c** \$800 **d** 50 months

**7** **a**

| | | |
|---|---|---|
| 4 | 5 | 6 |
| 20 | 25 | 30 |

**b** $h = 5w$

**c** 60 **d** 9

**8** **a** $y = x + 3$ **b** $y = -2x$ **c** $y = -2x + 1$

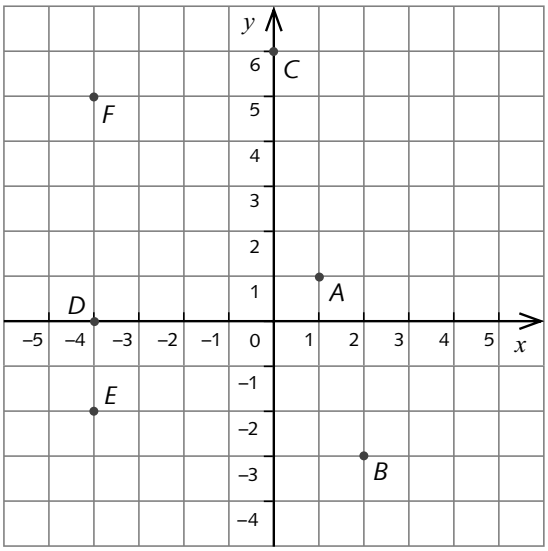
**d** $y = x - 4$ **e** $y = 3x$ **f** $y = -2x - 1$

## Review exercise

**1** **a** 1 **b** −7 **c** 5

**d** 6 **e** −12 **f** 6

**2**

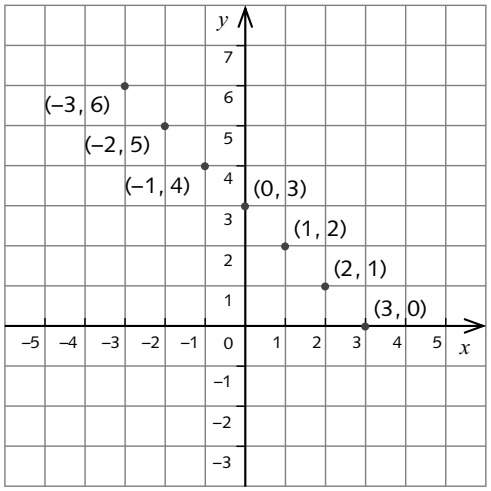



**3** $A(4, 4)$, $B(-4, 3)$, $C(-3, -2)$, $D(3, -3)$, $E(1, 1)$, $F(0, -4)$, $G(-2, 0)$


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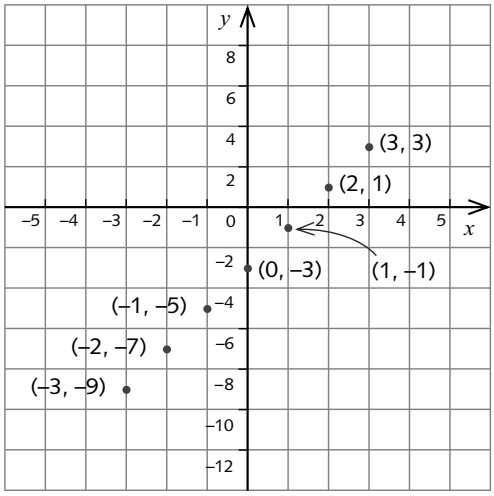
**4** **a** $y = 3 - x$

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | 6 | 5 | 4 | 3 | 2 | 1 | 0 |




**b** $y = 2x - 3$

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | –9 | –7 | –5 | –3 | –1 | 1 | 3 |




**5** **a** 13 **b** 7 **c** 9 **d** –27
**e** –9 **f** 225 **g** –225 **h** 20

**6** **a i** $\$(600 - x)$ **ii** $\$(600 - 5x)$
**b i** \$100 **ii** –\$400

**7** **a i** $(20 - 2x)°C$ **ii** $(20 - 10x)°C$
**b** 12°C **c** 0°C **d** 11 p.m.

**8** $D(6, 0)$ **9** $D(7, -6)$

**10** **a** $y = x + 2$ **b** $d = 2t + 8$ **c** $n = 4m - 4$
**d** $q = 2p - 1$ **e** $d = -3t + 15$ **f** $y = -3x - 7$

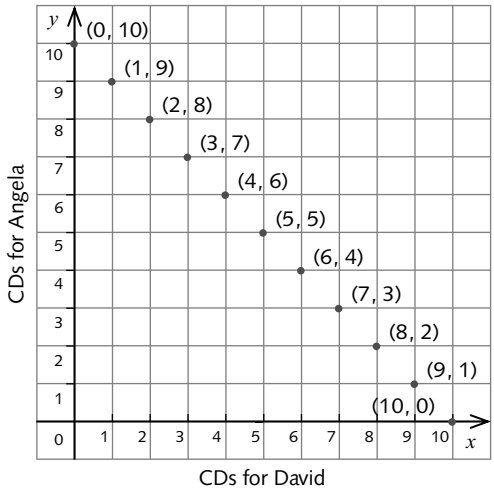
## Challenge exercise

**1** **a**

| CDs for David | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| CDs for Angela | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |

**b** (0, 10), (1, 9), (2, 8), (3, 7), (4, 6), (5,5), (6, 4), (7, 3), (8, 2), (9, 1), (10, 0)

**c**



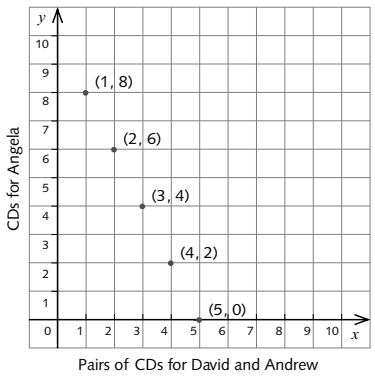

**d** $y = 10 - x$

**2** **a**

| Pairs of CDs for David and Andrew | 5 | 4 | 3 | 2 | 1 |
|---|---|---|---|---|---|
| Single CDs for Angela | 0 | 2 | 4 | 6 | 8 |

**b** (5, 0), (4, 2), (3, 4), (2, 6), (1, 8)

**c**




**d** $y = 10 - 2x$

**3** **a** $C(8, 9)$
**b** $Y(5, 5), Z(5, 0)$ or $Y(-5, 5), Z(-5, 0)$
**c** $C(0, 8), D(-4, 4)$ or $C(8, 0), D(4, -4)$

**4** $C$ can be any point with $y$-coordinate 8 or –8.

**5** $C(6, 10), D(6, 4)$ or $C(-6, 10), D(-6, 4)$

**6** $C(7, 10), D(7, 4)$ or $C(-7, 10), D(-7, 4)$

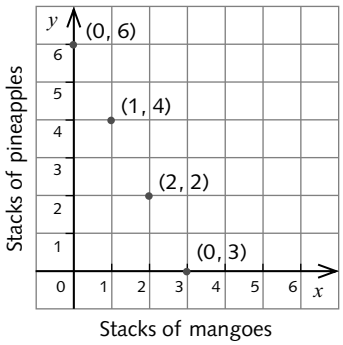

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**7** **a**

| Stacks of mangoes | Stacks of pineapples |
|---|---|
| 3 | 0 |
| 2 | 2 |
| 1 | 4 |
| 0 | 6 |

**b** (0, 6), (1, 4), (2, 2), (3, 0)

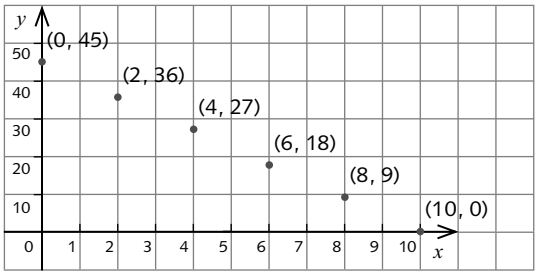
**c**




**d** $2x + y = 6$

**8** **a** (0, 45), (2, 36), (4, 27), (6, 18), (8, 9), (10, 0)

**b**




**c** **i** 2 adults and 36 children

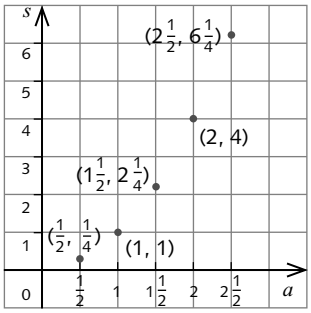
**iv** 4 adults and 27 children

**9** **a** $S = a^2$

**b**

| $a$ | $\frac{1}{2}$ | 1 | $1\frac{1}{2}$ | 2 | $2\frac{1}{2}$ |
|---|---|---|---|---|---|
| $s$ | $\frac{1}{4}$ | 1 | $2\frac{1}{4}$ | 4 | $6\frac{1}{4}$ |

**c**




# Chapter 13 answers

## Exercise 13A

**2** **a** $\alpha = 80°$ **b** $\beta = 140°$

**c** $\gamma = 55°$ **d** $\theta = 14°$

**e** $\alpha = 73°$ **f** $\beta = 43°$

**g** $\gamma = 60°$ **h** $\theta = 90°$

**i** $\alpha = 60°$ (alternate angles, $RS \parallel PQ$),
$\beta = 70°$ (alternate angles, $PQ \parallel RS$),
$\gamma = 50°$ (straight angle at $B$)

**3** **a** $\alpha = 120°$ **b** $\beta = 44°$ **c** $\gamma = 31°$

**d** $\theta = 27°$ **e** $\alpha = 53°$ **f** $\beta = 152°$

**g** $\gamma = 117°$ **h** $\theta = 123°$

**4** **a** No, because two obtuse angles add to greater than 180°.

**b** No, because two right angles add to 180°.

**c** 2, because of parts **a** and **b**.

**d** Yes, if the interior angle is obtuse.

**e** No, because 2 obtuse interior angles would be required, which is impossible.

**5** **a** $\alpha = 72°$ (straight angle at $N$),
$\alpha + \beta = 112°$ (exterior angle of $\triangle$), $\beta = 40°$

**b** $\theta = 115°$ (straight angle at $H$),
$\gamma = 105°$ (exterior angle of $\triangle$)

**c** $\alpha = 45°$ (straight angle at $M$),
$\beta = 135°$ (exterior angle of $\triangle$)

**d** $\gamma = 110°$ (straight angles at $W$ and $A$; exterior angle of $\triangle$)

**e** $\theta = 90°$ (straight angles at $A$ and $B$; exterior angle of $\triangle$)

**f** $\alpha = 93°$ (exterior angle of $\triangle FLS$),
$\beta = 117°$ (exterior angle of $\triangle ALS$)

**g** $\alpha = 85°$ (straight angle at $L$; exterior angle of $\triangle GIL$),
$\beta = 115°$ (exterior angle of $\triangle AGI$)

**h** $125° = \alpha + 20°$ (exterior angle of $\triangle EGH$), $\alpha = 105°$
$\alpha = \beta + 32°$ (vertically opposite angles at $E$; exterior angle of $\triangle EFG$), $\beta = 73°$

**6** **a** $\alpha = 30°$ **b** $\beta = 30°$ **c** $\gamma = 60°$

**d** $\theta = 50°$ **e** $\alpha = 40°$ **f** $\beta = 70°$

**g** $\theta = 30°$ **h** $\gamma = 12°$

**7** **a** $\theta = 35°$ (alternate angles, $HO \parallel ER$),
$\alpha = 75°$ (angle sum of $\triangle ORS$)

**b** $115° = 75° + \theta$ (exterior angles of $\triangle$), $\theta = 40°$,
$\beta = 40°$ (alternate angles, $UL \parallel RI$)

**c** $\alpha = 55°$ (alternate angles, $RA \parallel DC$),
$\beta = 25°$ (alternate angles, $CA \parallel DR$),
$\gamma = 100°$ (angle sum of $\triangle ADR$)

**d** $\alpha = 50°$ (alternate angles, $PL \parallel YA$),
$\alpha + \beta = 110°$ (co-interior angles, $PY \parallel LA$),
$\beta = 60°$, $\gamma = 60°$ (alternate angles, $PY \parallel LA$)

**e** $\alpha = 32°$ (alternate angles, $PA \parallel TR$),
$\beta = 58°$ (angle sum of $\triangle TRP$),
$\gamma = 100°$ (angle sum of $\triangle ARP$)

**f** $\alpha = 45°$(angle sum of $\triangle FGH$),
$\beta = 45°$ (alternate angles, $IF \parallel HG$),
$\gamma = 20°$ (angle sum of $\triangle FHI$)

**g** $95° = \alpha + 40°$ (exterior angle of $FGW$),
$\alpha = 55°$, $\beta = 55°$ (alternate angles, $FG \parallel TI$),
$\gamma = 40°$ (alternate angles, $FG \parallel TI$)


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**h** $\alpha = 150°$ (straight angle at $A$),
$\beta = 65°$ (alternate angles, $MS \parallel TR$),
$\alpha = \beta + \gamma$ (exterior angle of $\triangle ART$), $\gamma = 85°$

**8** **a** $\angle AVB = 25°$(angle sum of $\triangle AVB$; corresponding angles, $AB \parallel PQ$)

**b** $\angle AVB = 86°$(angle sum of $\triangle AVB$; corresponding angles, $VB \parallel PQ$)

**c** $\angle VAP = 90°$(angle sum of $\triangle RAP$),
$\angle AVB = 63°$ (angle sum of $\triangle VAB$)

**d** $\angle APV = 72°$ (exterior angle of $\triangle BOP$),
$\angle AVB = 97°$ (exterior angle of $\triangle APV$)

**e** $\angle APV = 102°$ (co-interior angles, $AP \parallel QB$),
$\angle AVB = 130°$ (exterior angle of $\triangle APV$)

**f** $\angle VAB = 115°$ (co-interior angles, $AV \parallel PQ$),
$\angle AVB = 20°$ (exterior angle of $\triangle ABV$)

**g** $AP \parallel BR$ (alternate angles at $P$ and $Q$ are equal),
$\angle AVB = 70°$ (alternate angles, $AP \parallel BR$)

**h** $PA \parallel QV$ (co-interior angles are supplementary),
$\angle VAP = 45°$ (co-interior angles, $PA \parallel QV$),
$45° = 25° + \angle AVB$ (exterior angle of $\triangle ABV$),
$\angle AVB = 20°$

**i** $\angle BCV = 70°$ (corresponding angles, $AB \parallel VC$),
$110° = 40° + \angle AVB$ (co-interior angles, $PQ \parallel RS$),
$\angle AVB = 70°$

## Exercise 13B

**1** **a** centre **b** radius **c** diameter
**d** chord **e** minor arc **f** major arc

**2** **a** $O$ **b** $RT$, $SU$ **c** $RS$, $ST$
**d** $OR$, $OU$, $OT$, $OS$
**e** $SRT$, $RUS$, $UTR$, $TSU$
**f** the curves $RS$, $RU$, $UT$, $TS$

**5** **e** $\angle A \approx 31.6°$, $\angle B \approx 38.9°$, $\angle C \approx 109.5°$

**6** **b** $BC \approx 9.2$ cm

**7** **b** Angle sum of a triangle is 180°.
**e** $AC = 4.3$ cm, $BC = 5.5$ cm

## Exercise 13C

**1** **a** $P$ **b** $PQ$ and $PR$, 3 cm
**c** $QR$ **d** $\angle PQR = \angle PRQ \approx 67°$
**e** Base angles are equal.

**2** **a** Yes, base angles are equal.
**b** Yes, $\angle B = \angle C$.
**c** No, all angles are different.
**d** No, all angles are different.
**e** Yes $\angle B = \angle A$.
**f** No, all angles are different.

**3** **a** $\angle ABC = 70°$ **b** $\angle OPQ = 62°$

**4** **a** $\alpha = 43°$ **b** $\alpha = 45°$, $x = 5$
**c** $\alpha = 75°$, $\beta = 90°$ **d** $\beta = 45°$, $y = 6$
**e** $\alpha = 30°$, $x = 5$ **f** $\alpha = \gamma = 60$, $\beta = 120°$, $x = 6$
**g** $\alpha = 90°$, $\beta = 60°$ **h** $\alpha = 45°$, $x = 4$
**i** $\alpha = 60°$, $x = 5$

**5** **a** $\alpha = 75°$ **b** $\beta = 36°$
**c** $\alpha = 110°$ **d** $\beta = 31°$
**e** $\alpha = 46°$, $\beta = 67°$ **f** $\alpha = 90°$

## Exercise 13D

**1–3** Refer to teacher

**4** **a** Construct an equilateral triangle on $AB$ and then bisect angle at $A$ twice.
**b** Construct a right angle at $A$ and bisect angle twice.
**c** Construct 30° as in part **a** and take the supplement.
**d** Construct 45° as in part **b** and construct the supplement.

**5–9** Refer to teacher

## Exercise 13E

**1** **a** $\alpha = 85°$ **b** $\alpha = 60°$ **c** $\alpha = 114°$
**d** $\alpha = 65°$ **e** $\alpha = 70°$ **f** $\alpha = 99°$

**2** **a** 60° **b** 90° **c** 85°
**d** 135° **e** 330° **f** 110°

**3** **a** $\alpha = 90°$, $\beta = 115°$ **b** $\alpha = 112.5°$
**c** $\alpha = \gamma = 118°$, $\beta = 62°$ **d** $\alpha = 90°$

**4** **a** $\alpha = 160°$
**b** $\alpha = 50°$, $\beta = 130°$, $\gamma = 120°$, $\theta = 40°$
**c** $\alpha = 110°$, $\beta = \gamma = 35°$ **d** $\alpha = 52°$, $\beta = 48°$
**e** $\alpha = 101°$ **f** $\beta = 46.5°$

**5** **a** $\alpha = 13°$ **b** $\alpha = 135°$
**c** $\beta = 62°$ **d** $\alpha = 72°$
**e** $\alpha = 40°$ **f** $\alpha = 78°$, $\beta = 143°$

**6** **a** 90°, 45° and 45° **b** 135°, 22.5° and 22.5°

## Review exercise

**1** **a** $\alpha = 30°$ (vertically opposite angles at $A$),
$\beta = 90°$ (alternate angles, $CB \parallel QP$),
$\gamma = 60°$ (angle sum of $\triangle ABC$)

**b** $\alpha = \beta = 65°$ (co-interior angles, $AB \parallel DC$ and $AD \parallel BC$),
$\gamma = 115°$ (co-interior angles)

**c** $\beta = 90°$ (co-interior angles, $FI \parallel GJ$),
$\alpha = 110°$ (corresponding angles)

**d** $\theta = 70°$ (construct $MX$ parallel to $KL$ and $ON$; alternate angles)

**2** **a** $\theta = 110°$ (exterior angle of $\triangle RST$)

**b** $\alpha = 40°$ (angle sum of $\triangle UVW$, vertically opposite angles at $W$)

**c** $\beta = 22.5°$ (angle sum of $\triangle BRT$)

**d** $125° = \gamma + 90°$ (exterior angle of $\triangle DEF$), $\gamma = 35°$

**e** $\theta = 70°$ (angle sum of $\triangle ABC$; corresponding angles, $BC \parallel PQ$)

**f** $5\theta = 140°$ (exterior angle of $\triangle JKM$), $\theta = 28°$


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**3** $\angle XAB = \beta$ and $\angle YAC = \gamma$ (alternate angles, $XY \parallel BC$), so $\alpha + \beta + \gamma = 180°$ (straight angle at $A$).

**4** **a** $\alpha = 25°$, $\beta = 130°$ (base angles of isosceles triangle are equal)

**b** $\gamma = 78°$ (base angles of isosceles triangle are equal)

**c** $\theta = 35°$ (base angles of isosceles triangle are equal)

**d** $\alpha = \beta = 20°$ (base angles of isosceles triangle are equal; alternate angles, $PQ \parallel RS$)

**e** $\theta = 100°$ (angle sum in quadrilateral)

**f** $\theta = 65°$ (angle sum in quadrilateral; vertically opposite angles at $A$, $B$, $C$ and $D$)

**5** **a** 10 cm (equal angles) **b** 8 cm ($\angle S = \angle R$)

**c** 5 cm ($\angle M = 40°$ due to alternate angles, $PQ \parallel GF$, so $\angle P = 70° = \angle F$)

**6** **a** $AB \parallel DC$ (co-interior angles are supplementary)

**b** $MN \parallel PQ$ (alternate angles $\angle N$ and $\angle P$ are equal)

**c** $PQ \parallel AC$, $AP \parallel BQ$ (alternate angles $\angle APB$ and $\angle PBQ$ are equal), $BP \parallel CQ$ (alternate angles $\angle PBQ$ and $\angle BQC$ are equal)

**9** **a** Construct an equilateral triangle.

**b** Bisect the 60° angle. **c** Bisect the 30° angle.

**d** Construct a right angle. **e** Bisect the 90° angle.

**f** Bisect the 45° angle.

**12** **a** $\alpha = 32°$, $\beta = 68°$ **b** $\alpha = 108°$

**c** $\alpha = 94°$ **d** $\alpha = 90°$

**e** $\beta = 65°$, $\alpha = 45°$, $\gamma = 110°$ **f** $\alpha = 79°$

**g** $\alpha = 90°$ **h** $\alpha = 64°$

## Challenge exercise

**1** **a** $\beta = \alpha + \angle DCB$, so $\beta > \alpha$.

**b** $\angle ACD = \alpha$ (base angles of $\Delta ABC$ are equal)

**c** It is impossible that $\beta > \alpha$ and $\beta < \alpha$ at the same time, so $AB$ and $AC$ must have equal lengths.

**2** **b** Draw in radius $OA$, and construct two 30° angles starting from $A$ on either side of $OA$, cutting the circle at $B$ and $C$.

**3** **c** $\angle POB = 2\angle APO$ and $\angle POA = 2\angle BPO$ (exterior angles of $\Delta BOP$ and $\Delta AOP$), so

$$\angle APB = \angle APO + \angle BPO = \frac{1}{2}(\angle POA + \angle POB) = \frac{1}{2} \times \angle AOB = 90°$$

**5** **d** Proof follows from the result in question **3**.

**6** **b** Draw in two perpendicular diameters, $AOB$ and $COD$.

**7** **b** Construct right angles at $A$ and $B$, and their bisectors. Let the bisectors meet at $C$.

**8** **c** Let the base angles of the four isosceles triangles be $\alpha$, $\beta$, $\gamma$ and $\theta$. Then the angle sum of $ABCD$ is $2(\alpha + \beta + \gamma + \theta) = 360°$, so $\alpha + \beta + \gamma + \theta = 180°$.

Observing that $\angle A + \angle C = \alpha + \beta + \gamma + \theta = \angle B + \angle D$ completes the proof.

**d** Follow the same procedure as in part **c**, except now let the flattest triangle have base angle $\theta$. Then the angle sum of $ABCD$ is $2(\alpha + \beta + \gamma - \theta) = 360°$, so $\alpha + \beta + \gamma - \theta = 180°$. In this case, $\angle A + \angle C = \alpha + \beta + \gamma - \theta = \angle B + \angle D$, and the result is proved.

**9** Each reflex angle is more than 180°. If there is more than one such angle, then the angle sum would exceed $2 \times 180° = 360°$, which is impossible.

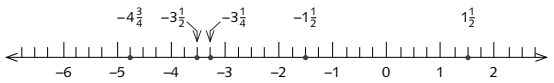
# Chapter 14 answers

## Exercise 14A

**1** **a** $-2, -\frac{7}{5}, -1, \frac{1}{4}, \frac{1}{2}, 1$ **b** $-\frac{11}{5}, -\frac{7}{4}, \frac{1}{4}, \frac{7}{2}$

**c** $-\frac{11}{12}, -\frac{4}{5}, -\frac{2}{3}, \frac{12}{13}$ **d** $-2, -1\frac{11}{13}, -\frac{15}{13}, -1$

**2**




**3** **a** $\frac{1}{2}$ **b** $2\frac{1}{2}$ **c** $-\frac{1}{5}$ **d** $\frac{3}{7}$

**e** $-\frac{1}{2}$ **f** $-2\frac{1}{2}$ **g** $-\frac{1}{3}$ **h** $\frac{2}{5}$

**4** **a** $\frac{1}{4}$ **b** $\frac{1}{10}$ **c** $-\frac{7}{12}$ **d** $\frac{4}{21}$

**e** $-\frac{3}{10}$ **f** $-1\frac{5}{21}$ **g** $-\frac{1}{15}$ **h** $-1\frac{1}{45}$

**i** $-\frac{1}{10}$ **j** $-\frac{7}{12}$ **k** $\frac{1}{15}$ **l** $\frac{1}{15}$

**5** **a** $-\frac{3}{4}$ **b** $-\frac{13}{10}$ **c** $-\frac{11}{12}$ **d** $-\frac{10}{21}$

**e** $\frac{3}{10}$ **f** $-\frac{1}{15}$ **g** $-\frac{3}{20}$ **h** $-\frac{27}{40}$

**i** $-1\frac{1}{10}$ **j** $-\frac{13}{30}$ **k** $\frac{4}{15}$ **l** $-\frac{11}{15}$

**6** **a** $2\frac{1}{2}$ **b** $2\frac{1}{2}$ **c** $-1$ **d** $-\frac{1}{3}$

**e** $-1\frac{1}{2}$ **f** $\frac{2}{15}$ **g** $-\frac{1}{2}$ **h** $-\frac{3}{5}$

**i** $2\frac{1}{4}$ **j** $-3\frac{3}{5}$ **k** $-2\frac{5}{18}$ **l** $-2\frac{1}{14}$

**7** **a** $2\frac{19}{44}$ **b** $4\frac{19}{20}$ **c** $-4\frac{19}{20}$ **d** $-5\frac{31}{40}$

**e** $-\frac{3}{4}$ **f** $-5\frac{3}{10}$ **g** $\frac{5}{12}$ **h** $-6\frac{10}{21}$

**i** $-5\frac{7}{10}$ **j** $3\frac{14}{15}$ **k** $-5\frac{13}{20}$ **l** $\frac{13}{40}$

**8** **a** $-\frac{1}{12}$ **b** $\frac{7}{12}$ **c** $-\frac{5}{12}$

**d** $-\frac{7}{12}$ **e** $-\frac{5}{12}$ **f** $-1\frac{11}{12}$


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## Exercise 14B

**1** **a** $-\frac{1}{10}$ **b** $-\frac{2}{35}$ **c** $-\frac{9}{20}$ **d** $-\frac{15}{77}$
**e** $-\frac{2}{7}$ **f** $-\frac{1}{6}$ **g** $-\frac{6}{55}$ **h** $-\frac{35}{78}$
**i** $-\frac{2}{21}$ **j** $-\frac{4}{45}$ **k** $\frac{3}{10}$ **l** $\frac{1}{5}$

**2** **a** $-12$ **b** $-11\frac{2}{3}$ **c** $-\frac{7}{10}$
**d** $-\frac{3}{7}$ **e** 10 **f** $\frac{3}{19}$

**3** **a** $-4$ **b** $-3$ **c** 9 **d** $-\frac{3}{4}$
**e** $4\frac{1}{2}$ **f** $-\frac{1}{3}$ **g** $\frac{3}{8}$ **h** $-7\frac{3}{7}$
**i** $-6\frac{4}{5}$ **j** $65\frac{1}{7}$ **k** $-17\frac{1}{4}$ **l** $-12\frac{3}{7}$

**4** **a** $4\frac{31}{44}$ **b** $-2\frac{14}{15}$ **c** $-1\frac{5}{13}$
**d** $1\frac{8}{27}$ **e** $-5\frac{5}{24}$ **f** $-2\frac{7}{40}$

**5** **a** $-9$ **b** $1\frac{10}{11}$ **c** $-1\frac{83}{192}$
**d** $1\frac{2}{25}$ **e** $3\frac{4}{5}$ **f** $\frac{21}{44}$

**6** **a** $\frac{1}{9}$ **b** $-\frac{1}{27}$ **c** $\frac{4}{9}$ **d** $2\frac{1}{4}$
**e** $-3\frac{3}{8}$ **f** $-\frac{8}{343}$ **g** $\frac{1}{12}$ **h** $\frac{4}{81}$

## Exercise 14C

**1** **a** 2.6 **b** 5.25 **c** 1.2 **d** $-1.2$
**e** 3.5 **f** 0.7 **g** $-2.5$ **h** $-1.55$

**2** **a** $-0.7$ **b** $-8.2$ **c** 6.12 **d** $-0.4$
**e** $-5.8$ **f** $-10.3$ **g** 4.93 **h** 9.47

**3** **a** $-0.24$ **b** 0.28 **c** 0.24 **d** 0.182
**e** $-0.75$ **f** 0.002 **g** 0.024 **h** 0.63

**4** **a** $-5$ **b** 41 **c** 41 **d** $-25$
**e** $-0.3$ **f** 2.1 **g** 1.1 **h** 310

**5** **a** $-0.027$ **b** $-0.064$ **c** $-0.091$
**d** 0.009 **e** $-0.008\,01$ **f** $-0.0064$

**6** **a** $-5.\dot{0}\dot{9}$ **b** $-30.4$ **c** $-16.35$
**d** $-7.76$ **e** $-3.03$ **f** $-14.67$
**g** $-26.26$ **h** $-6.\dot{0}\dot{9}$ **i** 4.5

**7** **a** 0.021 **b** 0.0075 **c** 0.21
**d** $-0.026\,25$ **e** 0.3125 **f** $-0.005\,75$

**8** **a** $-1.4$ **b** 0.3 **c** $-0.4$
**d** $-1.6$ **e** $-1.1$ **f** $-0.1$
**g** $-0.054$ **h** 0.054 **i** 0.033

## Exercise 14D

**1** **a** $-1$ **b** $\frac{1}{2}$ **c** $1\frac{1}{2}$ **d** $-3\frac{1}{2}$
**e** 2 **f** $-\frac{1}{8}$ **g** $\frac{1}{8}$ **h** $\frac{1}{4}$
**i** $3\frac{1}{2}$ **j** 4 **k** 4 **l** 2

**2** **a** $-3\frac{2}{3}$ **b** $-4\frac{1}{2}$ **c** $-3\frac{1}{2}$ **d** 2
**e** $-\frac{1}{6}$ **f** $\frac{1}{8}$ **g** $\frac{2}{3}$ **h** $-1\frac{1}{2}$

**3** **a** 3.5 **b** 4.5 **c** 19.5
**d** $-19.5$ **e** $-0.05$ **f** 0.25

**4** **a** 4 **b** 8 **c** 28
**d** $-28$ **e** $-\frac{4}{5}$ **f** $-12\frac{1}{2}$

**5** **a** $6\frac{3}{4}$ **b** $5\frac{1}{4}$ **c** $-\frac{27}{64}$
**d** $-\frac{243}{1024}$ **e** $-\frac{9}{16}$ **f** $2\frac{1}{4}$
**g** $-1\frac{1}{8}$ **h** $6\frac{1}{2}$

**6** **a** $-1$ **b** $-5.1$ **c** 4.1
**d** 4.5 **e** 0.5 **f** 0.02

**7** **a** $1\frac{1}{6}$ **b** $-5\frac{2}{3}$ **c** $-2\frac{2}{3}$
**d** $6\frac{1}{4}$ **e** $-8$ **f** $\frac{9}{25}$
**g** $33\frac{1}{3}$ **h** $-\frac{9}{32}$

**8** **a** 11.1 **b** 8.9 **c** $-1.331$ **d** $-1.610\,51$
**e** $-1.21$ **f** 19.36 **g** $-6.05$ **h** 12.2

**9** **a** $-4.55$ **b** 3.75 **c** $-3.45$
**d** $-0.24$ **e** $-0.015$ **f** $-1.5$

**10** **a** 0.159 **b** $-0.161$ **c** 0.007
**d** 2.8 **e** 7000 **f** $-20.14$

## Review exercise

**1** **a** $-3, -1\frac{3}{4}, -1\frac{1}{2}, \frac{3}{4}, 2, 2\frac{3}{4}$ **b** $-6\frac{3}{7}, -\frac{11}{3}, -\frac{7}{3}, \frac{4}{7}, \frac{5}{7}, \frac{19}{3}$

**2** **a** $1\frac{1}{3}$ **b** $\frac{1}{2}$ **c** $1\frac{1}{5}$
**d** $\frac{4}{35}$ **e** $\frac{7}{8}$ **f** $-1\frac{17}{20}$
**g** $-3\frac{17}{26}$ **h** $-\frac{29}{36}$

**3** **a** $-11\frac{2}{3}$ **b** $\frac{6}{25}$ **c** $\frac{28}{15}$
**d** $-17\frac{1}{2}$ **e** $-\frac{152}{215}$ **f** $7\frac{1}{3}$
**g** $-\frac{1}{5}$ **h** $-1\frac{1}{12}$


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**4** **a** $-1.75$ **b** $-16.42$ **c** $-26.6$ **d** $-1.2$
**e** $-6.44$ **f** $-17.27$ **g** $-17.4$ **h** $-7$
**i** $26.4$ **j** $5.2$ **k** $-1.5$

**5** **a** $0.49$ **b** $0.035$ **c** $0.5$
**d** $0.5$ **e** $4.74$ **f** $-5.5$

**6** **a** $-1\frac{2}{3}$ **b** $1\frac{1}{3}$ **c** $\frac{1}{9}$ **d** $1\frac{1}{3}$

**7** **a** $0.4$ **b** $5$ **c** $-9.8$ **d** $0.96$

**8** **a** $0.1$ **b** $-1.9$ **c** $-0.2$ **d** $-4.6$
**e** $0.0125$ **f** $0.35$ **g** $0.2$ **h** $3.9$

### Challenge exercise

**1**

| | | |
|---|---|---|
| $-3$ | $-2\frac{2}{3}$ | $\frac{2}{3}$ |
| $2$ | $-1\frac{2}{3}$ | $-5\frac{1}{3}$ |
| $-4$ | $-\frac{2}{3}$ | $-\frac{1}{3}$ |

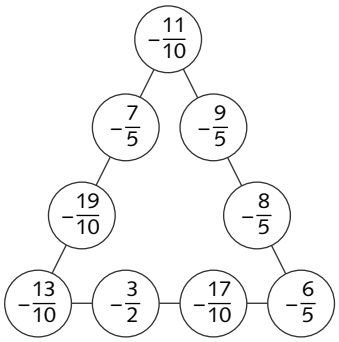
**2** For example, $-\frac{5}{12}$, $-\frac{9}{24}$

**3** **a** $-3.63$, $-3.6\dot{3}$, $-3\frac{7}{11}$, $-3.\dot{6}$

**b** $-1.4\dot{3}\dot{5}$, $-1.\dot{4}3\dot{5}$, $-1.43\dot{5}$, $-1\frac{4}{9}$

**4** $54\frac{2}{3}$

**5** There are 16 solutions. Find them.




**6** $-\frac{83}{30}$, $-\frac{13}{30}$

## Chapter 15 answers

### Exercise 15A

**1** **a** 100% **b** 66% **c** 40%
**d** 50% **e** 37.5%

**2** **a** $\frac{2}{25}$ **b** $\frac{3}{20}$ **c** $\frac{3}{4}$ **d** $\frac{3}{5}$
**e** $\frac{3}{10}$ **f** $\frac{4}{5}$ **g** $1\frac{1}{5}$ **h** $1\frac{13}{20}$
**i** $2\frac{1}{10}$ **j** $4\frac{1}{2}$ **k** $1\frac{1}{4}$ **l** $4\frac{12}{25}$

**3** **a** 0.07 **b** 0.25 **c** 0.75
**d** 0.35 **e** 0.9 **f** 1.27
**g** 4.1 **h** 4.6 **i** 1.7
**j** 0.125 **k** 0.062 **l** 0.375

**4** **a** 27% **b** 97% **c** 13% **d** 229%
**e** 70% **f** 90% **g** 0.7% **h** 52.7%

**5** **a** 60% **b** 85% **c** 44% **d** 46%
**e** 76% **f** 95% **g** 106% **h** 208%
**i** 435% **j** 7.5% **k** 17.5% **l** 42.5%

**6** **a** 35% **b** 27% **c** 73% **d** 130%
**e** 560% **f** 129% **g** 12.5% **h** 37.5%

**7** **a** $\frac{1}{40}$ **b** $\frac{11}{200}$ **c** $\frac{1}{16}$ **d** $\frac{7}{8}$

**8**

| Decimal | 0.5 | 0.25 | 0.75 | 0.4 | 1 | 0.457 | 0.23 | 0.403 | 0.625 |
|---|---|---|---|---|---|---|---|---|---|
| Fraction | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{3}{4}$ | $\frac{2}{5}$ | 1 | $\frac{457}{1000}$ | $\frac{23}{100}$ | $\frac{403}{1000}$ | $\frac{5}{8}$ |
| Percentage | 50% | 25% | 75% | 40% | 100% | 45.7% | 23% | 40.3% | 62.5% |

**9** **a** $16\frac{2}{3}\%$ **b** $12\frac{1}{2}\%$
**c** $37\frac{1}{2}\%$ **d** $28\frac{4}{7}\%$
**e** $83\frac{1}{3}\%$ **f** $22\frac{2}{9}\%$
**g** $45\frac{5}{11}\%$ **h** $58\frac{1}{3}\%$
**i** $71\frac{3}{7}\%$ **j** $91\frac{2}{3}\%$

### Exercise 15B

**1** **a** 80% **b** 62.5% **c** 28%
**d** 85% **e** 30% **f** 37.5%
**g** 125% **h** 93.75% **i** 200%
**j** 87.5% **k** 20% **l** 312.5%

**2** **a** 60% **b** 75% **c** 8.5%
**d** 24% **e** 46% **f** 15%
**g** 24% **h** 45% **i** 62.5%
**j** 0.4% **k** 23.5% **l** 57%

**3** **a** 72% **b** 80% **c** 12%
**d** 67.5% **e** 62.5%

### Exercise 15C

**1** \$222

**2** **a** 20 **b** 40 **c** 200 **d** 150
**e** 15 **f** 1.5 **g** 150 **h** 750
**i** 45 **j** 200 **k** 236 **l** 7


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**3** **a** 184 **b** 36 **c** 41
**d** 207 **e** 150 **f** 102

**4** \$26 **5** \$60

**6** **a** 3.36 hours **b** 8.4 hours **c** 35.5%

## Exercise 15D

**1** **a** 7 : 11 **b** 11 : 7 **c** 11 : 18

**2** **a** 1 : 1 **b** 3 : 7 **c** 1 : 1 **d** 5 : 3

**3** **a** 1 : 2 **b** 2 : 3 **c** 4 : 3 **d** 2 : 3
**e** 4 : 1 **f** 3 : 2 **g** 6 : 7 **h** 11 : 12

**4** **a** $\frac{2}{5}$ **b** $\frac{3}{5}$

**5** 1 : 3 **6** 3 : 4 **7** 1 : 2

**8** **a** 25 : 26 **b** 1 : 25

**9** **a** 5 : 6 **b** 1 : 4 **c** 1 : 2 **d** 5 : 24

## Exercise 15E

**1** **a** 20, 25 **b** 54, 42
**c** 27, 45 **d** 60, 84
**e** 60, 40 **f** 108, 24
**g** 96, 160 **h** 27, 72

**2** 230 pansies **3** 40 blue marbles

**4** 7 green balls **5** 15 girls

**6** 160 cm, 96 cm **7** 370 cm

**8** $AC = 6$ cm and $CB = 4$ cm

**9** length = 20 cm and width = 8 cm

**10** 35 cm **11** 368 cm **12** 135 cm$^2$

## Exercise 15F

**1** 83 cents per mango **2** 13 cents, 12 cents

**3** **a** \$2.50 **b** \$2.40

**4** \$1.875, \$1.60

**5** The price per sheet of the first is 1.4 cents and the price per sheet of the second is 1.2 cents. The second is better.

**6** **a** 23 cents, 27 cents **b** 45 cents, 53 cents

**7** Trade 15 blue marbles with Alice to get 25 red marbles. Exchange 24 red marbles with Carol to get 15 blue marbles. We end up with 15 blue marbles and one extra red marble.

## Review exercise

**1** **a** $\frac{1}{4}$ **b** $\frac{14}{25}$ **c** $\frac{3}{100}$ **d** $1\frac{2}{5}$
**e** $1\frac{2}{25}$ **f** $9\frac{99}{100}$ **g** $\frac{1}{40}$ **h** $\frac{1}{8}$

**2** **a** 0.34 **b** 0.99 **c** 0.23 **d** 0.02
**e** 0.09 **f** 1.23 **g** 2.5 **h** 3.83

**3** **a** 20% **b** 45% **c** 98% **d** 20%
**e** 230% **f** 1000% **g** 180% **h** 80%
**i** 370% **j** 40% **k** 3% **l** 5%
**m** 25% **n** 225% **o** 0.04%

**4** **a** 33 **b** 89 **c** 48 **d** 18
**e** 18 **f** 9 **g** 73.2 **h** 111.15
**i** 29.58 **j** 236 **k** 30 **l** 1500

**5** **a** 48% **b** $5\frac{5}{9}$%
**c** 11.839% **d** 8.625%

**6** 92% **7** 129 **8** $26\frac{2}{3}$%

**9** **a** 11 : 8 **b** 8 : 11 **c** 11 : 19

**10** 1 : 5

**11** **a** 1 : 2 **b** 3 : 4 **c** 12 : 7 **d** 3 : 4
**e** 8 : 1 **f** 5 : 3 **g** 8 : 7 **h** 7 : 12

**12** **a** 48, 60 **b** 48, 42 **c** 48, 96 **d** 60, 84

**13** 207 carrots **14** 96 blue marbles

**15** 427 green balls **16** 28 girls

**17** 75 cents **18** 30 cents and 33 cents

## Challenge exercise

**1** \$800 **2** 68%

**3** **a** 6 **b** 5 **c** 3

**4** \$144 **5** \$824.32 **6** 625 000

**7** $9\frac{1}{11}$ mL **8** 264 **9** 41:99 **10** 31:74

# Chapter 16 answers

## Exercise 16A

**1** **a** $3x + 4 = 31$ **b** $x = 9$
**c** There are 9 pencils in each pencil case.

**2** **a** $x = 7$ **b** $x = 15$ **c** $x = 15$
**d** $x = 5$ **e** $x = 21$ **f** $x = 21$
**g** $x = 4$ **h** $x = 7$ **i** $x = 4\frac{1}{2}$
**j** $x = 9$ **k** $x = 24$ **l** $x = 100$


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**3** **a** $a = 8$ **b** $a = 30$ **c** $a = 15$
**d** $a = 6$ **e** $a = 35$ **f** $a = 3$
**g** $a = 7$ **h** $a = 10$

**4** **a** $23a + 17 = 707$ **b** $a = 30$

**5** **a** $10x + 23 = 523$ **b** $x = 50$

**6** **a** $6n = 72$ **b** $n = 12$

**7** **a** $6c + 7 = 127$ **b** $c = 20$

**8** **a** $19 - x = 12$ **b** $x = 7$

**9** $x = 11$ **10** $x = 5$ **11** $x = 24$

## Exercise 16B

**1** **a** $m = 3$ **b** $m = 11$ **c** $m = 2$
**d** $m = 16$ **e** $m = 17$ **f** $m = 16$

**2** **a** $n = 3$ **b** $n = 3$ **c** $n = 5$
**d** $n = 5\frac{1}{3}$ **e** $n = 8\frac{1}{3}$ **f** $n = 2\frac{5}{9}$
**g** $n = 3\frac{2}{5}$ **h** $n = 8\frac{1}{2}$ **i** $n = 5\frac{2}{3}$

**3** **a** $x = 36$ **b** $x = 30$ **c** $x = 80$
**d** $x = 20$ **e** $x = 18$ **f** $x = 32$

**4** **a** $x = 4$ **b** $x = 5\frac{2}{5}$ **c** $x = 20$
**d** $x = 9$ **e** $x = 12$ **f** $x = 4$
**g** $x = 9$ **h** $x = 6$ **i** $x = 10$
**j** $x = 14$ **k** $x = 7$ **l** $x = 2\frac{1}{3}$
**m** $x = 2\frac{1}{4}$ **n** $x = 2\frac{1}{7}$ **o** $x = 2\frac{5}{6}$

**5** **a** $x + 5 = 21, x = 16$ **b** $7x = 35, x = 5$
**c** $5x = 37, x = 7\frac{2}{5}$ **d** $\frac{x}{3} = 23, x = 69$
**e** $x - 15 = 37, x = 52$

**6** **a** $z + 7 = 12, z = 5$ **b** $z + 12 = 19, z = 7$
**c** $z - 6 = 14, z = 20$ **d** $9 - z = 6, z = 3$
**e** $3z = 5, z = 1\frac{2}{3}$ **f** $5z = 45, z = 9$
**g** $\frac{z}{6} = 7, z = 42$

## Exercise 16C

**1** **a** $x = 3$ **b** $x = 2\frac{2}{5}$ **c** $x = 2$
**d** $x = 4$ **e** $x = 4$ **f** $x = 4\frac{3}{4}$
**g** $x = 4\frac{1}{5}$ **h** $x = 8\frac{1}{5}$ **i** $x = 7\frac{7}{10}$

**2** **a** $z = 24$ **b** $z = 45$ **c** $z = 60$
**d** $z = 187$ **e** $z = 72$ **f** $z = 160$
**g** $z = 27$ **h** $z = 175$ **i** $z = 132$

**3** **a** $x = 4\frac{1}{2}$ **b** $a = 14$ **c** $b = 10\frac{7}{11}$
**d** $x = 3$ **e** $b = 9$ **f** $a = 11\frac{2}{3}$
**g** $z = 2\frac{1}{2}$ **h** $k = 1\frac{10}{11}$ **i** $a = 2\frac{3}{5}$
**j** $x = 204$ **k** $x = 240$ **l** $z = 84$

**4** **a** $7x + 6 = 20, x = 2$ **b** $\frac{x}{4} + 11 = 20, x = 36$
**c** $\frac{x}{3} - 4 = 23, x = 81$ **d** $4x + 6 = 30, x = 6$
**e** $\frac{m}{7} - 11 = 20, m = 217$ **f** $6m - 14 = 10, m = 4$

## Exercise 16D

**1** **a** $x = -9$ **b** $x = -6$ **c** $x = -17$
**d** $x = 5$ **e** $x = 0$ **f** $x = 5$
**g** $x = -4$ **h** $x = 0$ **i** $x = -6$
**j** $x = -7$ **k** $x = -9$ **l** $m = 0$

**2** **a** $x = -3$ **b** $x = -5$ **c** $x = 3$
**d** $x = -12$ **e** $x = -50$ **f** $x = -28$
**g** $x = 3$ **h** $m = 9$ **i** $m = -14$
**j** $z = -100$ **k** $a = -72$ **l** $p = -99$

**3** **a** $x = -7$ **b** $x = -3$ **c** $x = 1$
**d** $x = -1\frac{1}{2}$ **e** $x = -2\frac{1}{2}$ **f** $x = -9\frac{4}{5}$
**g** $x = 22\frac{1}{2}$ **h** $x = 7$ **i** $x = 5$
**j** $x = -2$ **k** $m = -1\frac{1}{5}$ **l** $p = -2\frac{1}{3}$

**4** **a** $x = -12$ **b** $x = -12$ **c** $x = -3$
**d** $x = -224$ **e** $x = -35$ **f** $x = -36$
**g** $m = -180$ **h** $m = -30$ **i** $m = -192$

## Exercise 16E

**1** **a** $4x + 20$ **b** $3x - 12$ **c** $12x - 24$
**d** $15x - 25$ **e** $14a - 28$ **f** $24 + 18x$
**g** $54x - 99$ **h** $x^2 - 3x$ **i** $x^2 + 4x$
**j** $2x^2 - x$ **k** $3x - x^2$ **l** $2x^2 + x$

**2** **a** $z = 1\frac{2}{3}$ **b** $z = 10\frac{1}{5}$ **c** $z = 7\frac{1}{6}$
**d** $z = 3\frac{3}{4}$ **e** $z = -4\frac{1}{7}$ **f** $z = 2\frac{1}{6}$

**3** **a** $x = 5\frac{7}{10}$ **b** $m = 1\frac{13}{18}$ **c** $m = 2\frac{6}{35}$
**d** $x = 9$ **e** $m = \frac{25}{36}$ **f** $n = 2\frac{25}{28}$
**g** $m = 7\frac{9}{14}$ **h** $n = \frac{43}{49}$ **i** $m = \frac{8}{55}$

**4** **a** $3(x + 4) = 32, x = 6\frac{2}{3}$
**b** $5(x - 2) = 42, x = 10\frac{2}{5}$
**c** $2(x - 3) = 15, x = 10\frac{1}{2}$
**d** $6(z + 5) = 42, z = 2$
**e** $12(n + 11) = 150, n = 1\frac{1}{2}$
**f** $6(z - 8) = 100, z = 24\frac{2}{3}$

**5** 21


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## Exercise 16F

**1** **a** $14x$ **b** $16x$ **c** $13x$
**d** $7x$ **e** $16x$ **f** $7 + 9x$
**g** $10x + 2$ **h** $14x + 7y$ **i** $7x + 2y$
**j** $8x + 6y$

**2** **a** $7x + 6$ **b** $24x - 6$ **c** $27x + 22$
**d** 30 **e** $46x + 14$ **f** $11x - 12$
**g** $13x - 8$ **h** $19x + 34$

**3** **a** $x = 6$ **b** $x = 3$ **c** $x = 5\frac{2}{7}$
**d** $x = 2$ **e** $x = -4\frac{4}{5}$ **f** $x = 2$
**g** $x = 3$ **h** $x = 2$

**4** **a** $z = 1\frac{1}{6}$ **b** $z = 3\frac{1}{3}$ **c** $z = 5$ **d** $z = 1\frac{2}{5}$

**5** **a** $m = 3$ **b** $m = 1\frac{4}{9}$ **c** $x = -1\frac{6}{19}$
**d** $m = \frac{20}{23}$ **e** $x = 9$ **f** $n = 1\frac{15}{23}$

**6** 5 cents

## Exercise 16G

**1** **a** $x = 3$ **b** $x = 3$ **c** $x = \frac{3}{4}$
**d** $x = 5$ **e** $x = 2$ **f** $x = 1\frac{1}{4}$
**g** $x = 1$ **h** $x = 3$ **i** $x = 20$

**2** **a** $m = -5$ **b** $z = 1$ **c** $n = -\frac{1}{2}$
**d** $m = -\frac{4}{13}$ **e** $z = -\frac{1}{2}$ **f** $m = 1\frac{1}{5}$

**3** **a** $x = 16\frac{2}{3}$ **b** $x = 20$ **c** $x = 2\frac{6}{7}$
**d** $x = 5$ **e** $x = 5$ **f** $x = 1\frac{8}{17}$

## Exercise 16H

**1** **a** 8 **b** 34 **c** 98 **d** 12
**e** 6 **f** $4\frac{1}{7}$ **g** 7 **h** 11

**2** 21 cm **3** 12 cm; 36 cm **4** 15 and 18

**5** 54 and 56 **6** 5 and $4\frac{1}{4}$ **7** 32 cm

**8** $27x + 21 = 2181$, $x = 80$. There are 80 bananas in each case.

**9** **a** $4x - 3 = 6x - 7$, $x = 2$
**b** $11x - 14 = 9x + 6$, $x = 10$
**c** $3x - 8 = 12 - 2x$, $x = 4$

**10** $x = 5$ **11** $x = 3\frac{1}{3}$ **12** $x = 140$

## Review exercise

**1** **a** $12x + 3 = 363$ **b** $x = 30$

**2** **a** $x = 12$ **b** $a = 5\frac{2}{5}$ **c** $z = 41\frac{1}{3}$
**d** $b = 9$ **e** $x = 9\frac{3}{5}$ **f** $x = 70$
**g** $n = 14$ **h** $p = 3$ **i** $y = 4\frac{1}{2}$
**j** $x = 108$ **k** $x = -20$ **l** $x = 160$
**m** $x = 15$ **n** $m = 14$ **o** $x = 5\frac{1}{2}$
**p** $x = 2\frac{4}{5}$

**3** **a** $x + 11 = 23$, $x = 12$ **b** $7x = 25$, $x = 3\frac{4}{7}$
**c** $\frac{x}{7} - 3 = 6$, $x = 63$

**4** length is 48 cm, width is 12 cm

**5** 28, 16

**6** **a** $x = -16$ **b** $x = -5$ **c** $x = -4$
**d** $x = 3$ **e** $x = -1$ **f** $x = -28$
**g** $x = 5\frac{1}{3}$ **h** $x = -2\frac{1}{4}$ **i** $x = 11$

**7** **a** $x = 1$ **b** $x = 2$ **c** $x = -\frac{1}{4}$
**d** $x = -1$ **e** $x = 1\frac{1}{3}$ **f** $x = \frac{3}{11}$

**8** **a** $x = 2\frac{4}{13}$ **b** $x = 3\frac{9}{11}$
**c** $x = 7\frac{1}{2}$ **d** $x = 8\frac{9}{20}$

**9** $\frac{x}{5} = 4$; $x = 20$

**10** $x + 29 = 65$; the man is 36 years old.

**11** $x + 163 = 390$; 227 pages

**12** $\frac{2x}{3} = 34$; 51 cakes **13** $6x - 6 = 3x - 4$; $x = \frac{2}{3}$

**14** Let $x$ be Brett's amount, $x + x + 16 + x + 24 = 400$, $3x + 40 = 400$; Brett: \$120, Jennifer: \$136, David: \$144

**15** $2x - 5 = 55$; 30 cm

**16** Suppose Anne runs $x$ km, $x - 12 = \frac{1}{2}x$; Anne: 24 km, Jennifer: 12 km, Carl: 12 km

**17** $6x - 10 = 7x + 6$, $x = -16$

**18** $x = 50$

**19** Let Jason receive $x$ dollars; $x + 3x + 2x = 450$; Jason: \$75 David: \$225, Isabel: \$150.

**20** \$4

## Challenge exercise

**1** **a** $10x$ cm **b** $x = 6\frac{2}{5}$

**2** **a** $2(20 - 10(x + 6)) = -80 - 20x$
**b** $x = -3\frac{7}{11}$ **c** $x = 43\frac{17}{21}$

**3** 5:12 p.m., 192 km from Tantown

**4** student: 11; sister: 14; father: 36

**5** 4 **6** \$52


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# Chapter 17 answers

## Exercise 17A

**1** **a** 1 **b** 0 **c** $\frac{1}{2}, \frac{1}{2}$ **d** 1 **e** 0

## Exercise 17B

**1** **a** 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
**b** 1, 3, 5, 7, 9

**2** **a** 2, 4, 6 **b** 1, 3, 5
**c** 3, 6 **d** 2, 3, 4, 5, 6

**3** **a** 1, 2, 3, 4, 5 **b** 1, 2, 3

**4** **a** $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{1}{3}$ **d** $\frac{5}{6}$

**5** **a** $\frac{1}{10}$ **b** $\frac{1}{2}$ **c** $\frac{4}{5}$
**d** $\frac{3}{10}$ **e** $\frac{9}{10}$

**6** **a** $\frac{1}{5}$ **b** $\frac{3}{5}$

**7** **a** $\frac{1}{9}$ **b** $\frac{2}{9}$ **c** $\frac{4}{9}$ **d** $\frac{5}{9}$

**8** **a** $\frac{1}{6}$ **b** $\frac{5}{6}$ **c** $\frac{1}{3}$ **d** $\frac{2}{3}$

**9** **a** $\frac{1}{10}$ **b** $\frac{1}{2}$ **c** $\frac{1}{5}$
**d** $\frac{1}{2}$ **e** $\frac{2}{5}$ **f** $\frac{3}{10}$

**10** **a** $\frac{1}{26}$ **b** $\frac{11}{13}$ **c** $\frac{5}{26}$
**d** $\frac{5}{26}$ **e** $\frac{3}{13}$

**11** $\frac{3}{5}$

**12** **a** $\frac{1}{6}$ **b** $\frac{1}{2}$ **c** $\frac{2}{3}$
**d** 0 **e** $\frac{19}{30}$

**13** $\frac{1}{5}$

**14** **a** $\frac{5}{18}$ **b** $\frac{17}{36}$ **c** $\frac{5}{18}$
**d** $\frac{1}{12}$ **e** $\frac{19}{36}$

**15** **a** $\frac{4}{11}$ **b** $\frac{7}{11}$ **c** $\frac{10}{11}$ **d** $\frac{1}{33}$

**16** **a** $\frac{1}{114}$ **b** $\frac{7}{19}$ **c** $\frac{211}{228}$
**d** $\frac{77}{114}$ **e** $\frac{113}{114}$ **f** $\frac{17}{228}$

## Review exercise

**1** **a** $\frac{1}{8}$ **b** $\frac{1}{2}$ **c** $\frac{5}{8}$ **d** $\frac{1}{4}$

**2** **a** $\frac{7}{12}$ **b** $\frac{1}{2}$ **c** $\frac{5}{12}$ **d** $\frac{5}{12}$ **e** 0

**3** **a** $\frac{1}{11}$ **b** $\frac{2}{11}$ **c** $\frac{4}{11}$ **d** $\frac{7}{11}$

**4** **a** $\frac{7}{13}$ **b** $\frac{6}{13}$ **c** $\frac{6}{13}$ **d** $\frac{4}{13}$
**e** $\frac{3}{13}$ **f** $\frac{3}{13}$ **g** $\frac{5}{13}$

**5** **a** $\frac{25}{86}$ **b** $\frac{7}{43}$ **c** $\frac{21}{43}$ **d** $\frac{22}{43}$

**6** **a** $\frac{1}{11}$ **b** $\frac{2}{11}$ **c** $\frac{2}{11}$ **d** $\frac{4}{11}$

**7** **a** $\frac{99}{500}$ **b** $\frac{1}{10}$ **c** $\frac{1}{5}$

## Challenge exercise

**1** **a** $\frac{1}{50}$ **b** $\frac{49}{50}$ **c** $\frac{1}{2}$ **d** $\frac{2}{5}$
**e** $\frac{9}{50}$ **f** $\frac{1}{10}$ **g** $\frac{9}{10}$ **h** $\frac{7}{25}$
**i** $\frac{13}{50}$ **j** $\frac{18}{25}$

**2** **a** $\frac{1}{5}$ **b** $\frac{1}{2}$ **c** $\frac{7}{10}$
**d** 0 **e** $\frac{1}{10}$ **f** $\frac{3}{50}$

**3** **a**

| | | |
|---|---|---|
| Thomas | Leslie | Anthony |
| Thomas | Leslie | Tracie |
| Thomas | Leslie | Kim |
| Thomas | Anthony | Tracie |
| Thomas | Anthony | Kim |
| Thomas | Tracie | Kim |
| Leslie | Anthony | Tracie |
| Leslie | Anthony | Kim |
| Leslie | Tracey | Kim |
| Anthony | Tracie | Kim |

**b** $\frac{3}{5}$ **c** $\frac{2}{5}$

**4** **a** $\frac{1}{3}$ **b** $\frac{32}{225}$ **5** 11 red balls

**6** HH = item 1, HT = item 2, TH = item 3, TT = repeat trial

**7** $\frac{1}{3}$


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# Chapter 18 answers

## Exercise 18A

**1** **a** 3 units right and 5 units up

**b** 2 units right and 1 unit up

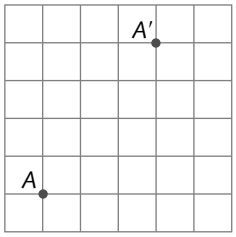
**c** 3 units right and 2 units down

**d** 3 units right and 2 units down

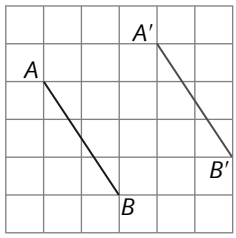
**e** 3 units right

**f** 2 units right and 2 units down

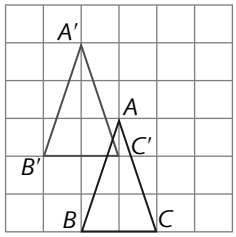
**2** **a**



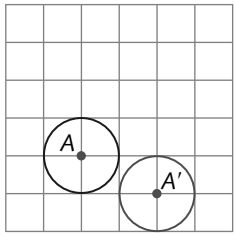

**b**




**c**




**d**




**3** **a** They are the same.

**b** **i** $\angle X'Z'Y'$ **ii** $\angle Z'X'Y'$

**c** They are the same.

**d** They are both 12.5 square units.

**4** **a** $L'$ is the midpoint of $B'C'$.

**b** 2 down, 10 right

**c** yes

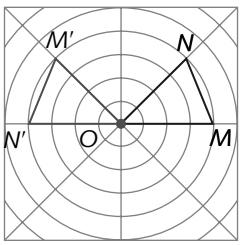
**d** **i** $A'C'$ **ii** $A'L'$

**e** **i** $B'L'$ **ii** $C'A'$

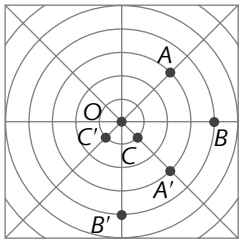
**f** no

## Exercise 18B

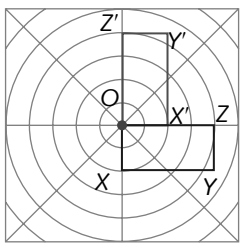
**1** **a**



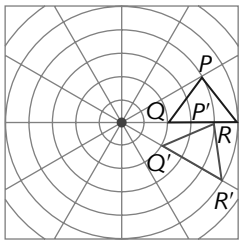

**b**



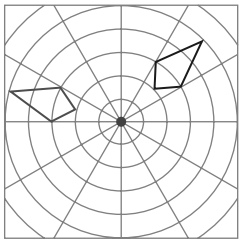

**c**



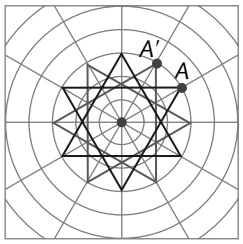

**d**



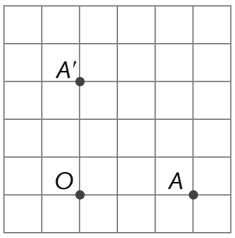

**e**



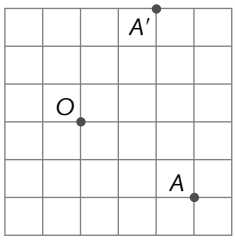
**f**



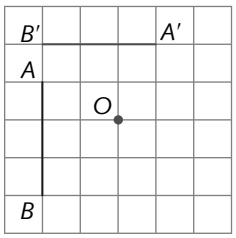

**2** **a**



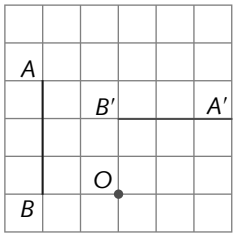

**b**



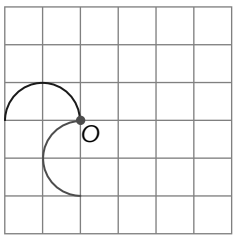

**c**



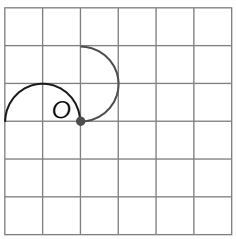

**d**



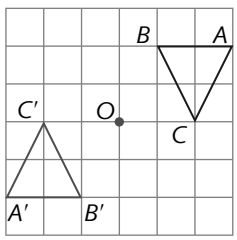

**e**



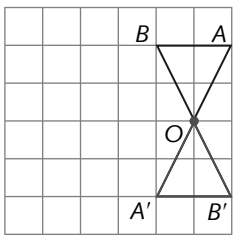

**f**



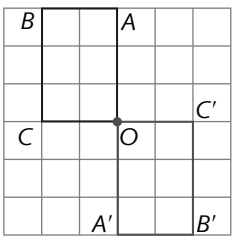

**g**



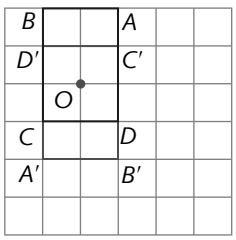

**h**



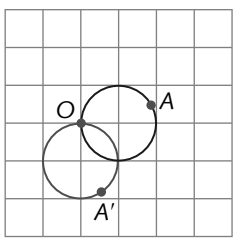

**i**



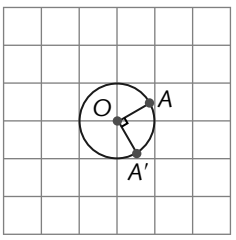

**j**




**k**




**l**

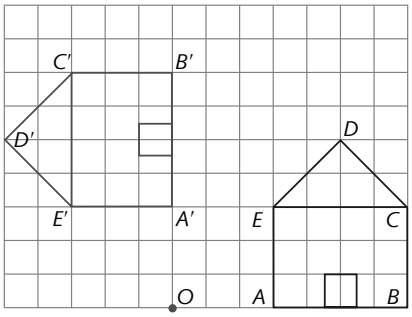





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**3** Only the centre of rotation is fixed.

**4** **a**

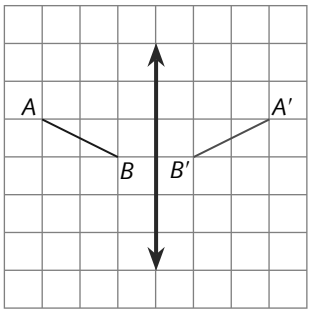



**b** parallel

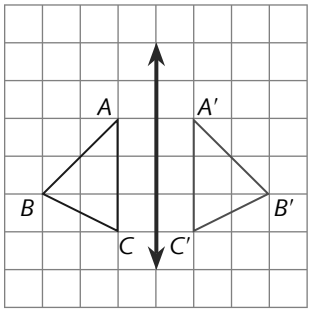
**c** They are both 12 square units.

## Exercise 18C

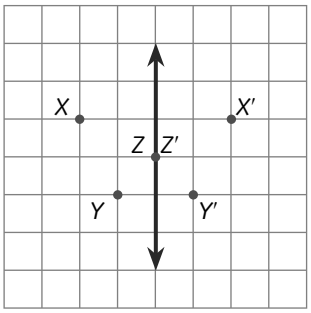
**1** **a**



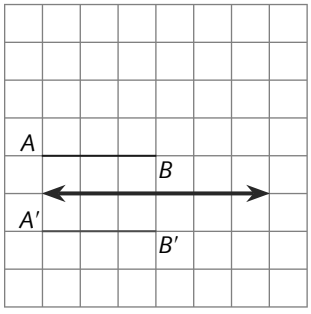

**b**



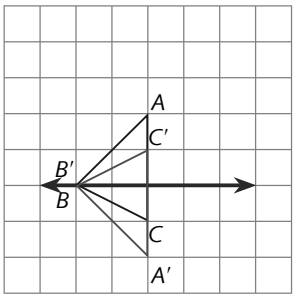

**c**



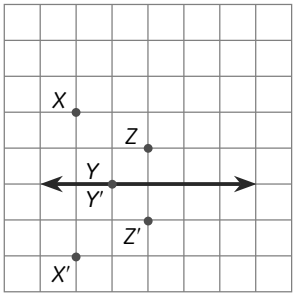

**d**



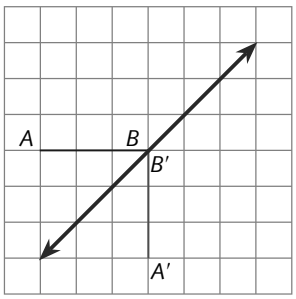

**e**



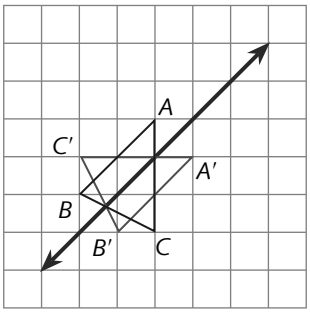

**f**



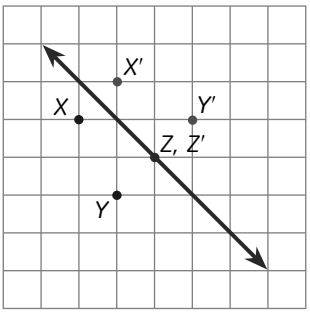

**g**



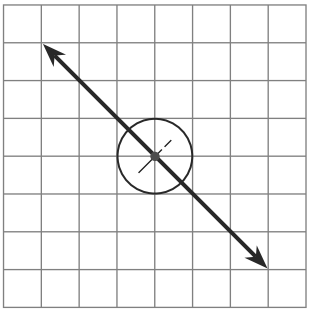

**h**




**i**

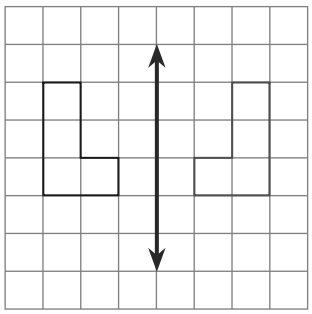



**j**

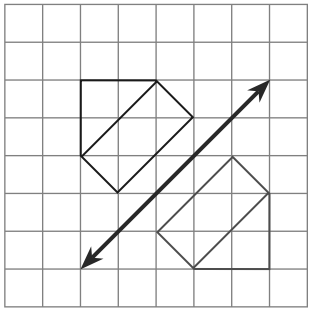



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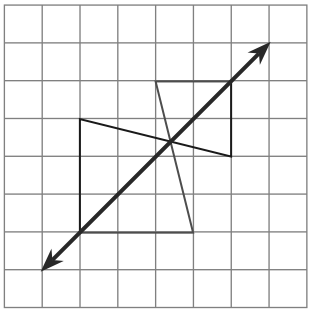
**k**



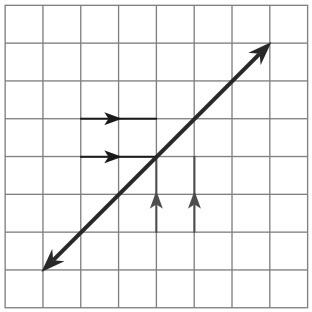
**l**



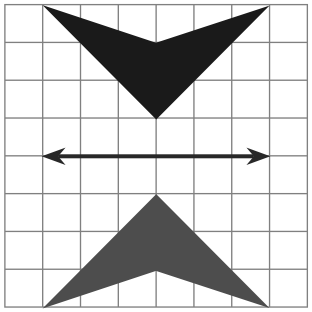
**m**



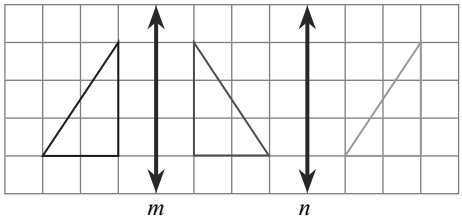
**n**



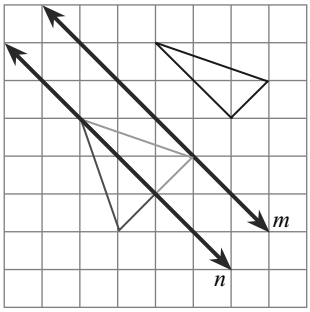
**o**



**2** **a**

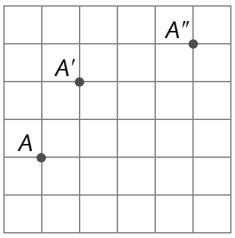



**b**



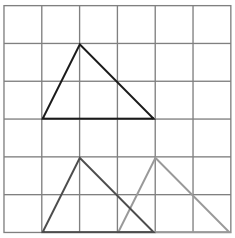

## Exercise 18D

**1** **a**



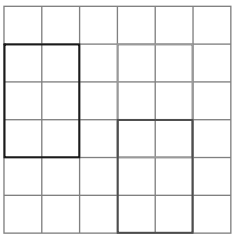

4 right, 3 up

**b**



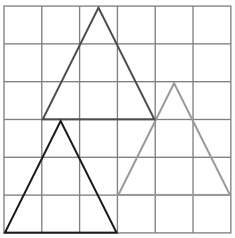
3 down, 2 right

**c**



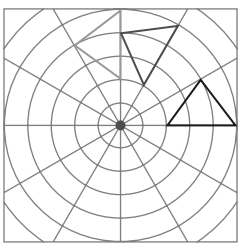
3 right

**d**



3 right, 1 up

**2** **a**

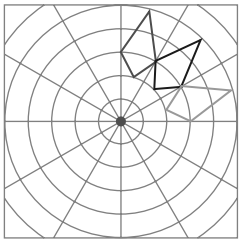


90° anticlockwise



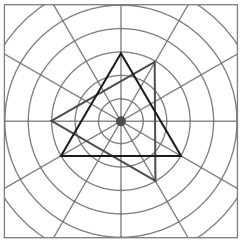
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**b**



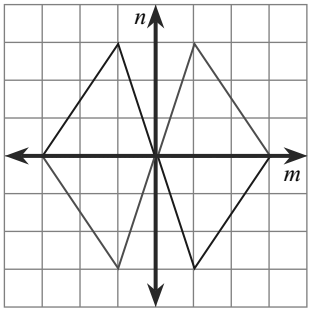
30° clockwise

**c**

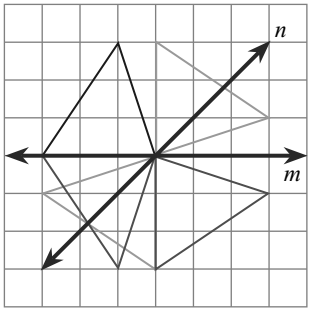


210° anticlockwise

**3** **a** **i-iii** **iv** yes




**b** **i, ii, iv** **iv** no




**4** Many answers are possible. For example:

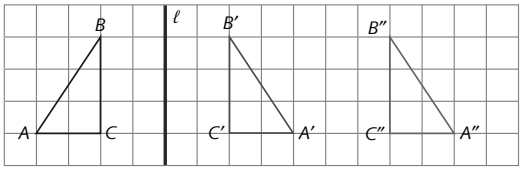
**a** Rotate 90° clockwise about $B$; translate 3 right, 4 up.

**b** Reflect in $AC$; translate 1 up.

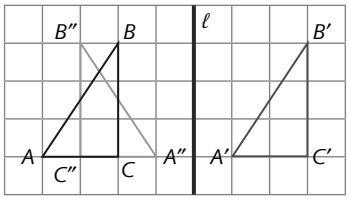
**c** Reflect in $BC$; translate 2 right, 3 up.

**d** Rotate 90° clockwise about $C$; translate 1 down.

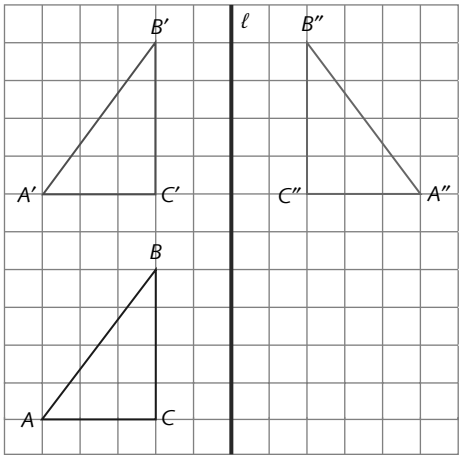
**5** **a**



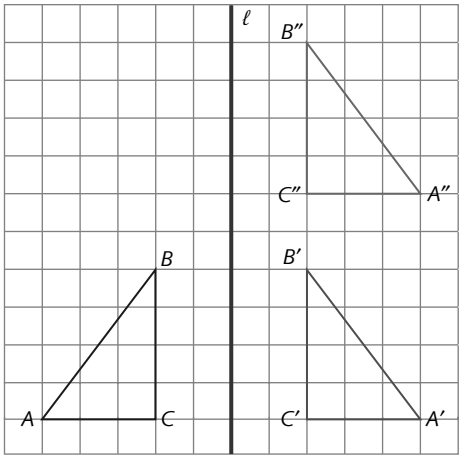

**b**



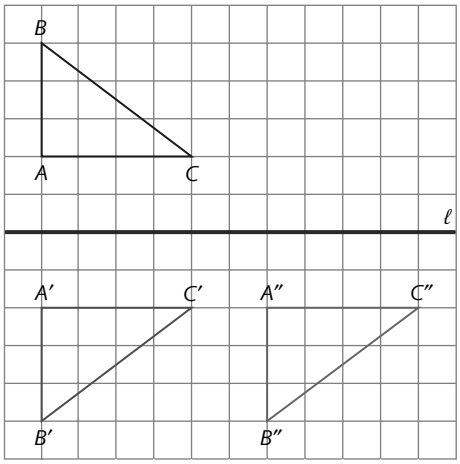

**c**



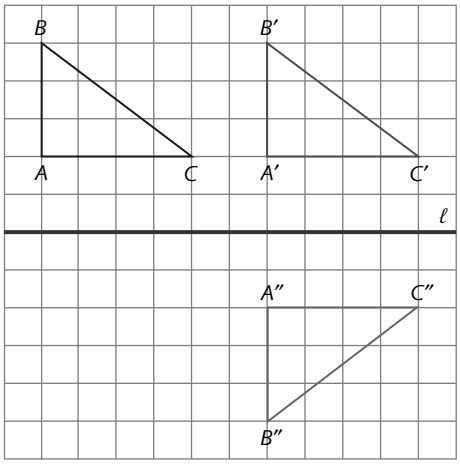

**d**




**6** **a**



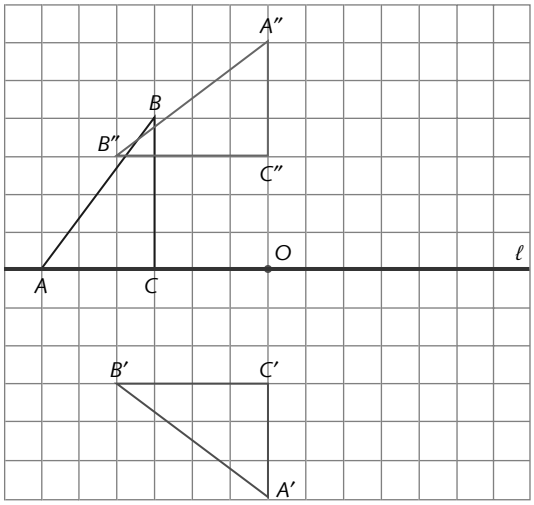

**b**



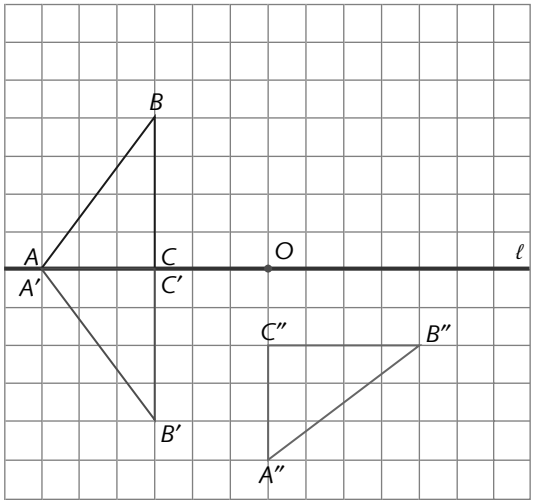


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**7** **a**

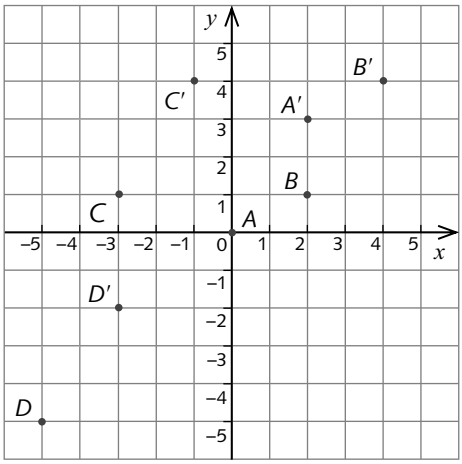



**b**



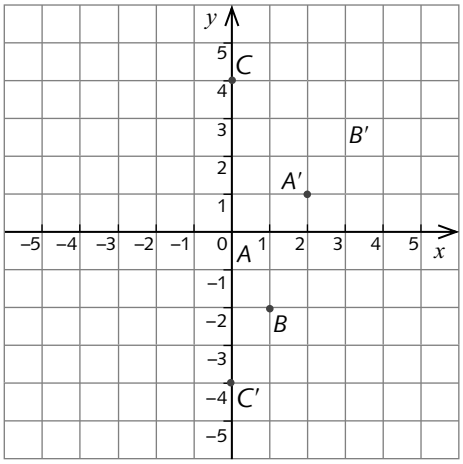

## Exercise 18E

**1**




**a** $A'(2,\ 3)$ **b** $B'(4,\ 4)$ **c** $C'(-1,\ 4)$ **d** $D'(-3,\ -2)$

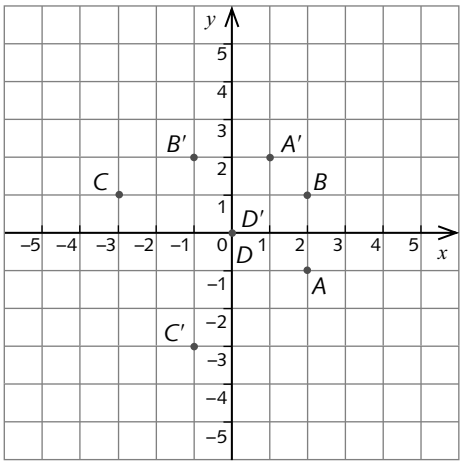
**2**




**a** 1 up and 2 right **b** 2 right and 5 up

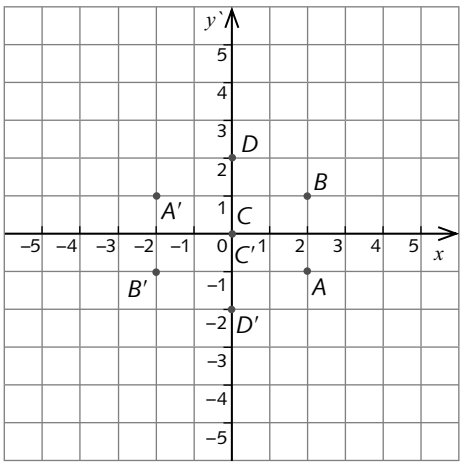
**c** 8 down

**3**




**a** $A'(1,\ 2)$ **b** $B'(-1,\ 2)$ **c** $C'(-1,\ -3)$ **d** $D'(0,\ 0)$

**4**

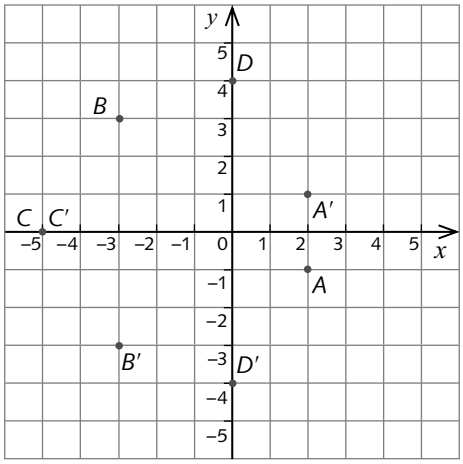



**a** $A'(-2,\ 1)$ **b** $B'(-2,\ -1)$ **c** $C'(0,\ 0)$ **d** $D'(0,\ -2)$



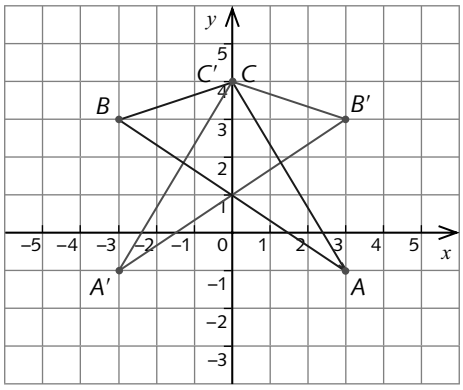
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**5**




**a** $A'(2, 1)$ **b** $B'(-3, -3)$ **c** $C'(-5, 0)$ **d** $D'(0, -4)$

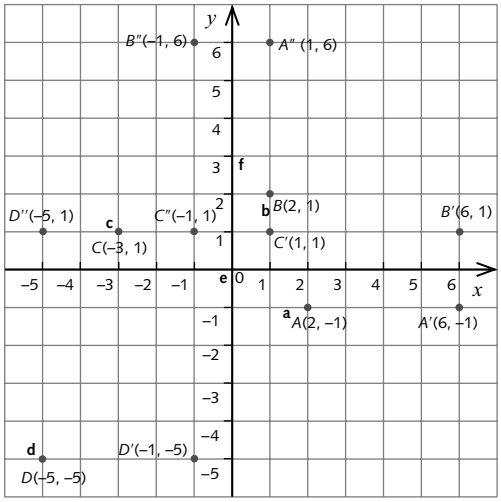
**6**




**a i** $A'(-3, -1)$ **ii** $B'(3, 3)$ **iii** $C'(0, 4)$

**b** $\triangle A'B'C'$

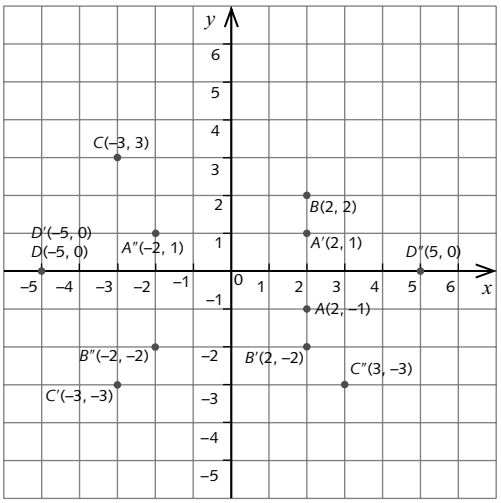
**7**




**a** $A'(6, -1), A''(1, 6)$ **b** $B'(6, 1), B''(-1, 6)$

**c** $C'(1, 1), C''(-1, 1)$ **d** $D'(-1, -5), D''(-5, 1)$

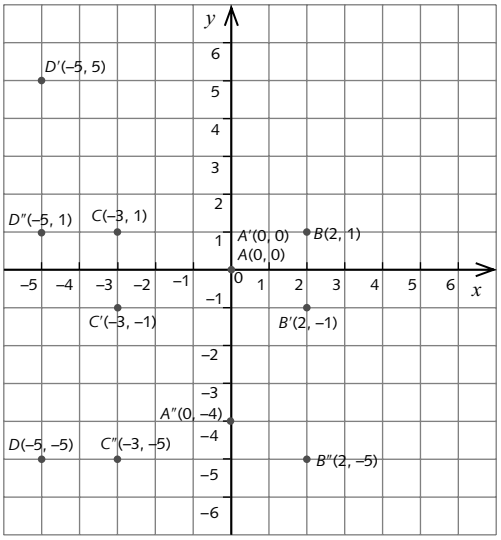
**8**




**a** $A'(2, 1), A''(-2, 1)$ **b** $B'(2, -2), B''(-2, -2)$

**c** $C'(-3, -3), C''(3, -3)$ **d** $D'(-5, 0), D''(5, 0)$

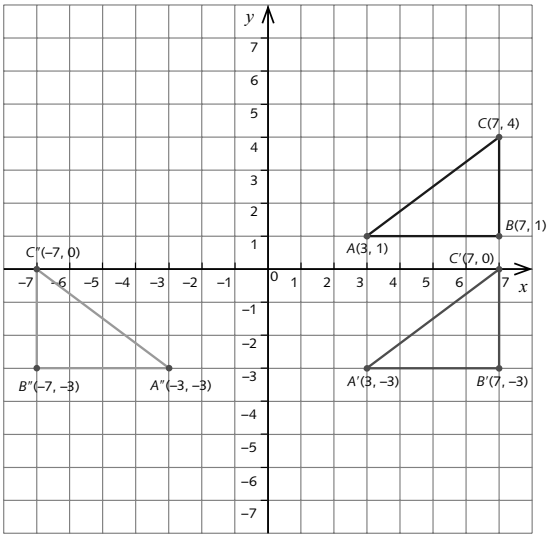
**9**




**a** $A'(0, 0), A''(0, -4)$ **b** $B'(2, -1), B''(2, -5)$

**c** $C'(-3, -1), C''(-3, -5)$ **d** $D'(-5, 5), D''(-5, 1)$

**10**




$A(3, 1) \to A'(3, -3), A''(-3, -3)$

$B(7, 1) \to B'(7, -3), B''(-7, -3)$

$C(7, 4) \to C'(-7, 0), C''(-7, 0)$

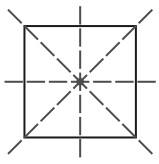
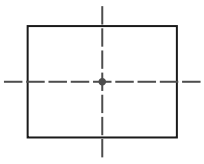
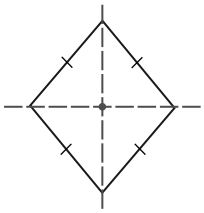
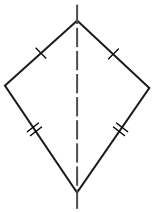
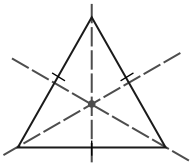
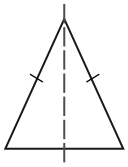
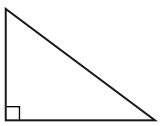
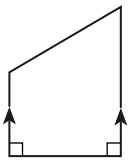
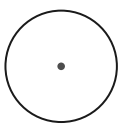
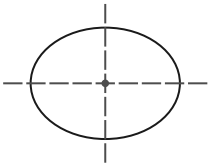
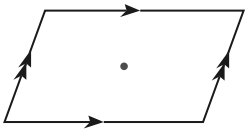

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## Exercise 18F

**1**

| | | | | | | |
|---|---|---|---|---|---|---|
| | A | B | C | D | E | F |
| Order: | 1 | 1 | 1 | 1 | 1 | 1 |
| | G | H | I | J | K | L |
| Order: | 1 | 2 | 2 | 1 | 1 | 1 |
| | M | N | O | P | Q | R |
| Order: | 1 | 2 | 2 | 1 | 1 | 1 |
| | S | T | U | V | W | X |
| Order: | 2 | 1 | 1 | 1 | 1 | 2 |
| | Y | Z | = | + | $ | % |
| Order: | 1 | 2 | 2 | 4 | 2 | 2 |

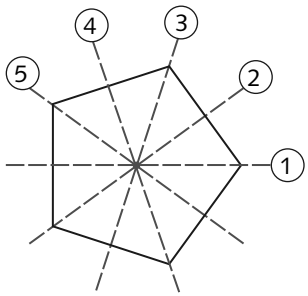
**2**

| | | | | |
|---|---|---|---|---|
| |  square |  rectangle |  rhombus |  kite |
| Order: | 4 | 2 | 2 | 1 |
| |  equilateral triangle |  isosceles triangle |  right triangle |  trapezium |
| Order: | 3 | 1 | 1 | 1 |
| |  any diameter is an axis of reflection circle |  ellipse |  parallelogram | |
| Order: | infinite | 2 | 2 | |

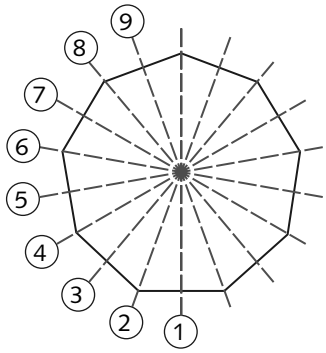

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## Exercise 18G

**1** **a, b**




**2**




**a** $\frac{360°}{9} = 40°$ **c** 9

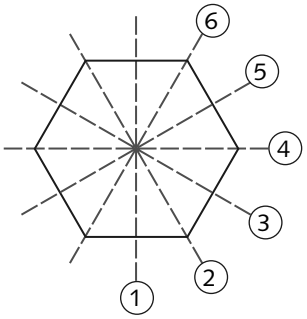
**3** **a** 8 **c** 45°

**d** The diameters shown are the axes of symmetry, together with the lines joining opposite midpoints; 8.

**e** the diameters shown, 4

**4** **b** All acute triangles thus constructed are equilateral, and hence have angles of 60°, which is the angle required for the hexagon $\left(\frac{360°}{6} = 60\right)$.

**c** order = 6



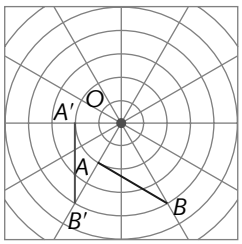

**5** **b** a clock face **c** 12, 12

**6** 36 and 37, respectively

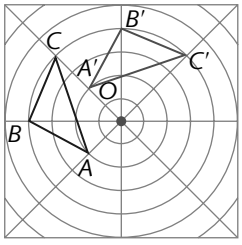
## Review exercise

**1** **a** 3 right, 1 up **b** 2 down **c** 2 right, 2 down

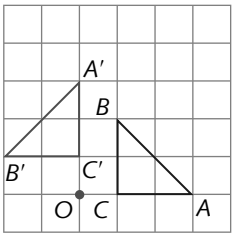
**2**



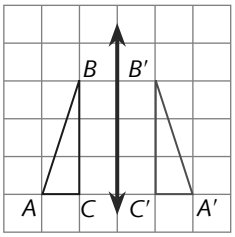

**3**



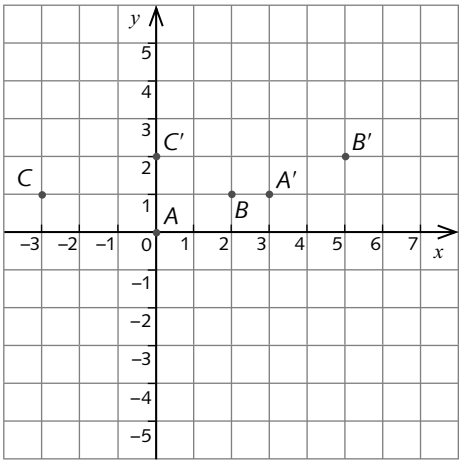

**4**




**5**

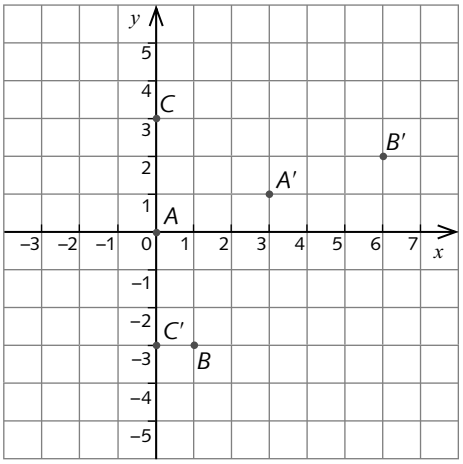



**6**



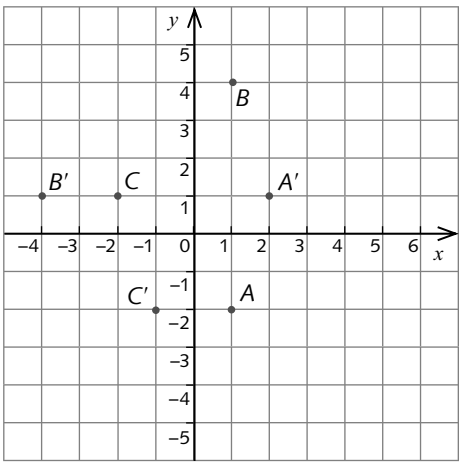

**a** $A'(3, 1)$ **b** $B'(5, 2)$ **c** $C'(0, 2)$

**7**




**a** 3 right, 1 up **b** 5 right, 5 up **c** 6 down

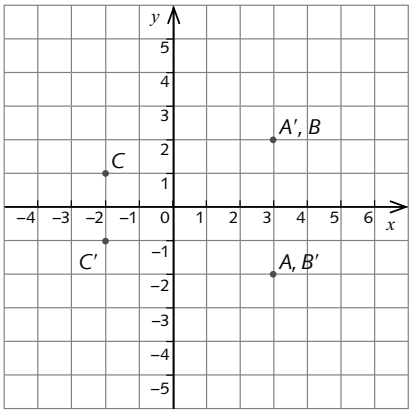
**8**




**a** $A'(2, 1)$ **b** $B'(-4, 1)$ **c** $C'(-1, -2)$


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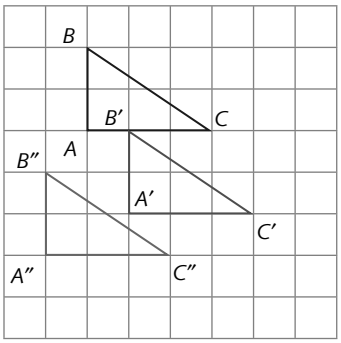
**9**




**a** $A'(3, 2)$ **b** $B'(3, -2)$ **c** $C'(-2, -1)$

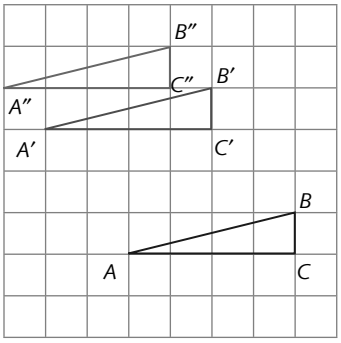
**10** **a** 2 **b** 4 **c** 6 **d** 2

**11** **a**



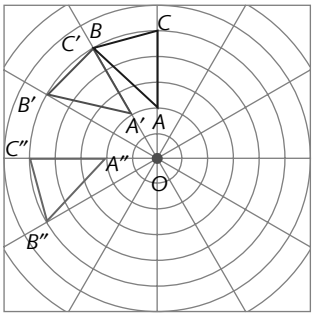

1 left and 3 down

**b**



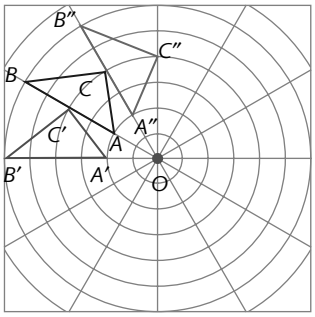

3 left and 4 up

**12** **a**



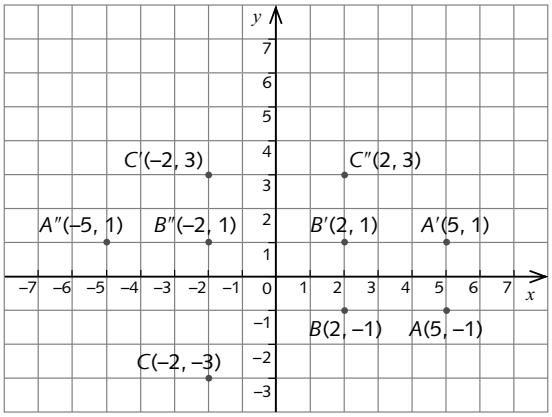

90° anticlockwise

**b**



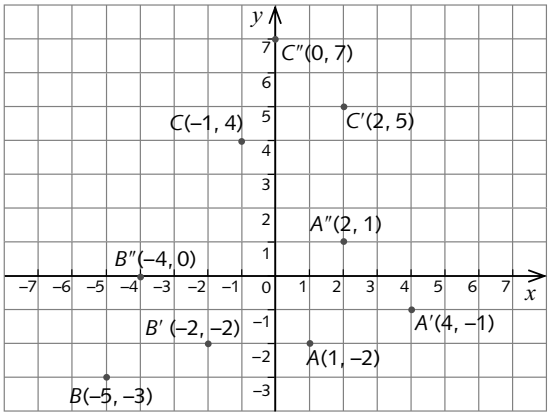

30° clockwise

**13**




**a** $A''(-5, 1)$ **b** $B''(-2, 1)$ **c** $C''(2, 3)$

**14**




**a** $A''(2, 1)$ **b** $B''(-4, 0)$ **c** $C''(0, 7)$

**15** Many answers are possible. For example:

**a** Rotate 90° anticlockwise about $B$; translate 3 left, 4 down.

**b** Reflect in $AB$; translate 5 left, 3 down.

**c** Reflect in $AC$; translate 1 down.

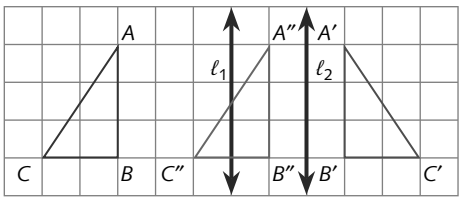
**d** Reflect in $AB$; translate 3 left, 6 down.



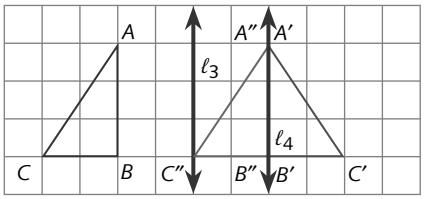
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## Challenge exercise

**1** **a**

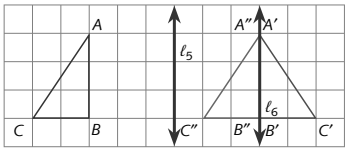



**b**




**c** Both are 4 units right.

**2** **a**

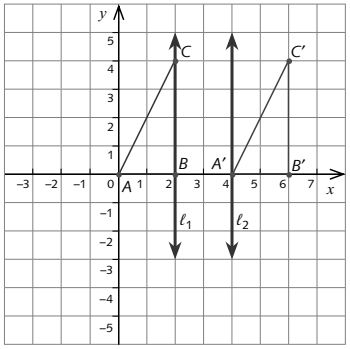



**b** 6 units right

**c** 8 units in a direction perpendicular to the parallel line (can be left or right)

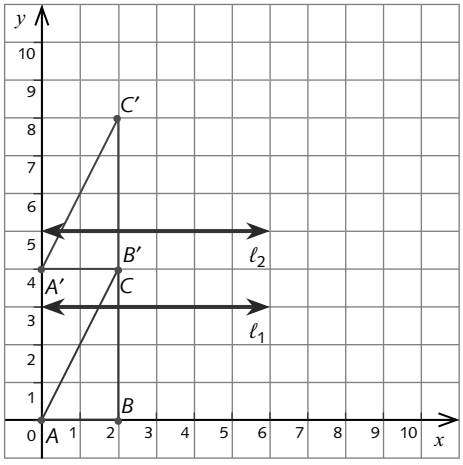
**d** Translation will be $2a$ units perpendicular to the lines.

**3**



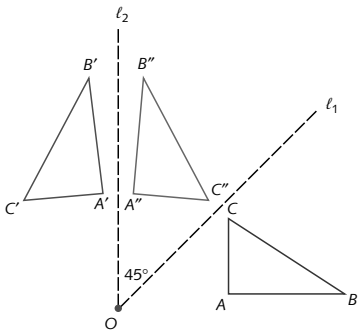

No, there are many ways to do this; the distance between the lines is 2 units.

**4**




No, there are many ways to do this; the distance between the lines is 2 units.

**5** **a**




**b** 45°

**c** No, but the angle between $l_1$ and $l_2$ must be 45°. There are many possibilities.

# Chapter 19 answers

## Exercise 19A

**1** **a** 41 089

**b** 57%

**c** 25%

**d** There is no entry for the 0–5 years age group.

**2** **a** 230 000 **b** 330 000 **c** 80%

**3** **a** 146 **b** 22

**c** 980 **d** 13 : 146

**4**

| | Matches | | Sets | |
|---|---|---|---|---|
| | Won | Lost | Won | Lost |
| Rishi | 2 | 1 | 6 | 7 |
| Bryce | 3 | 0 | 9 | 1 |
| Gerald | 1 | 2 | 6 | 8 |
| John | 0 | 3 | 4 | 9 |

**a** Bryce **b** Gerald **c** $33\frac{1}{3}\%$

## Exercise 19B

**1** **a** The pictogram is a poor representation of the amount of time spent star-gazing. The picture size doesn't clearly show the number of hours.

**b** **Time spent star-gazing**

Stuart ★ ★ ★ ★ = 1 hour of star-gazing

Loys ★

Jyothi ★ ★ ★

Warwick ★ ★ ★ ★ ★ ★ ★ ★

Durham ★ ★ ★ ★ ★ ★ ★ ★ ★ ★

**2** **a** 4500

**b** 18 000

**c** 25%

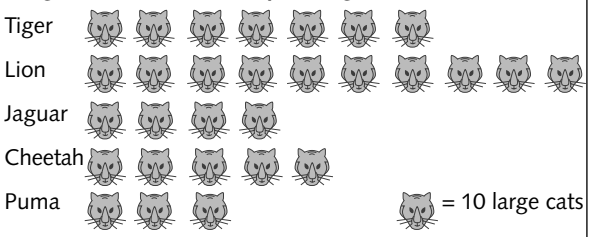
**d** 31%


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**3** **Basketball club singlet sizes**




**4** **Large cat numbers at an open range zoo**



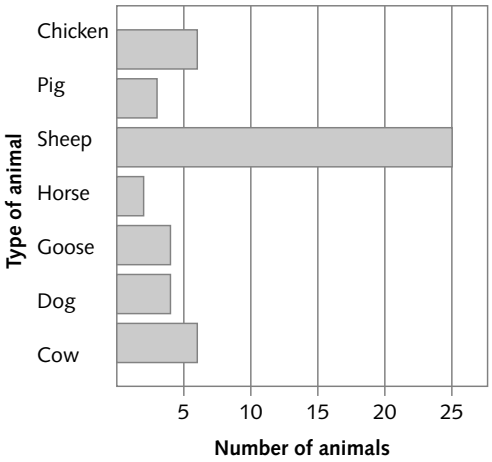

## Exercise 19C

**1** **a** 5 **b** netball
**c** 25 **d** 8%

**2** **a** 60 **b** 2000 and 2002
**c** 500 **d** 28%

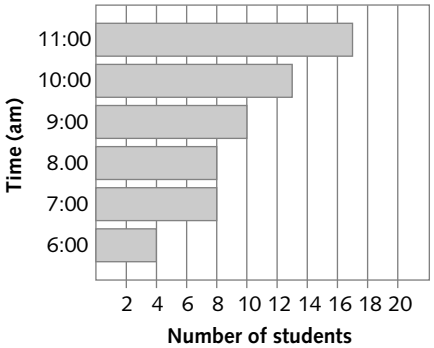
**3** **a** **i** 2001 **ii** 2002
**b** 2001 **c** 2 **d** 7000
**e** 12000 **f** 2006 or 2007

**4** **a** **Hobby farm animal numbers**




**b** 20% **c** 50%

**5** **a** **Saturday waking times of Year 10 boys**

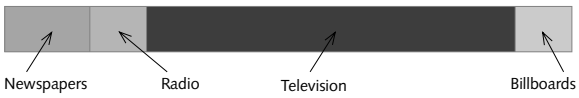



**b** $33\frac{1}{3}\%$

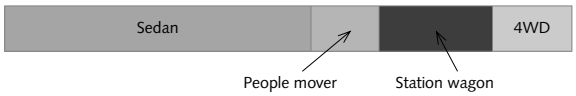
**c** Most of the boys sleep in on Saturday morning.

## Exercise 19D

**1** **Sporting event advertising expenditure**



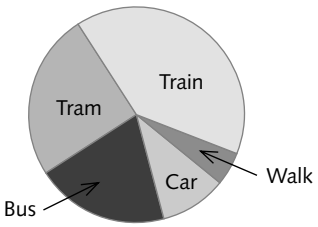

**2** **Car sales over 12 months**




**3** **a** staff meetings, 40°; client meetings, 160°; paperwork, 60°; telephone calls, 80°; other, 20°

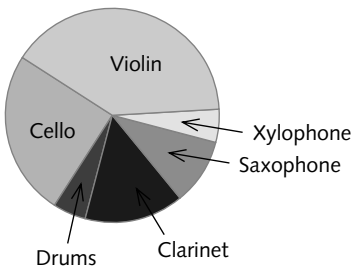
**b** 2 hours **c** $\frac{1}{9}$ **d** 44% **e** 15°

**4** **a** **Mode of transport**




**b** 20%

**5** **School band**




**6** **a** Michelle \$108; Sarah \$100.

**b** Michelle \$108; Sarah \$110.

**c** Although Michelle spends a higher percentage on clothes, she does not necessarily spend more than Sarah. It depends on how much they each earn in a given month.

**7** **a** food **b** 126° **c** 35%

**8** **a** 390000

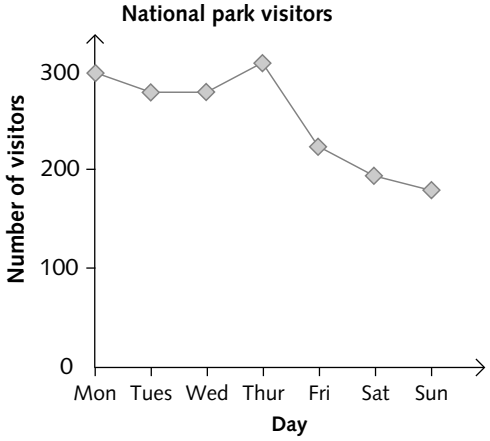
**b** 4000000



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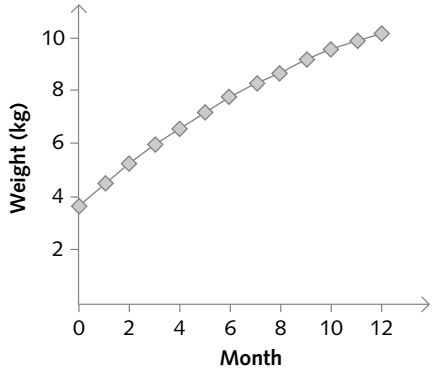
## Exercise 19E

**1**




**2** **a** 0.8 kg **b** 6.5 kg

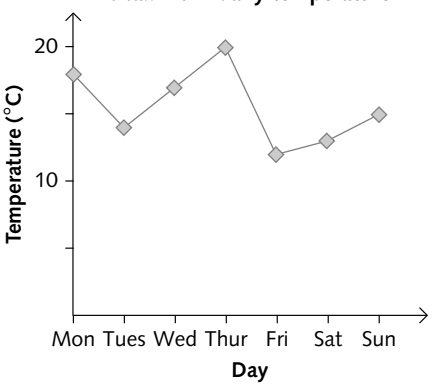
**c**




**3** **a** \$100 **b** 15
**c** 3 **d** more money

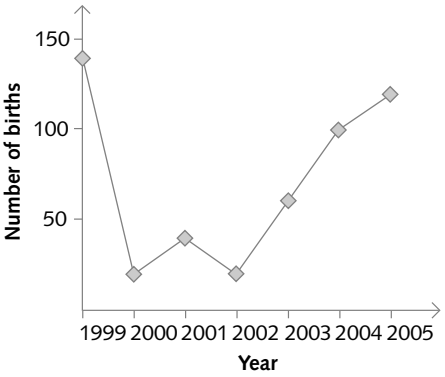
**4** **a** Singapore **b** Niger **c** 1996
**d** Oman **e** Vietnam

**5**




**6** **a** 2004

**b**




**c** The answer is subjective. They are quite similar. The lines connecting the points in the line graph possibly make the increasing trend over the last three years more apparent, while the column graph gives a better impression of the numbers of births.

## Exercise 19F

**1** **a** **i** £4 **ii** £8 **iii** £16
**b** **i** A\$25 **ii** A\$50 **iii** A\$80

**2** **a** 6 hours
**b** **i** 4 km **ii** 6 km **iii** 2 km
**c** 1400, for 1 hour **d** 6 km
**e** 12 km **f** 4 km/h
**g** 3 km/h **h** 2 km/h

**3** **a** 40 km **b** 65 km **c** 2 p.m.
**d** 25 minutes **e** 60 km/h
**f** **i** $53\frac{1}{3}$ km/h **ii** 120 km/h **iii** 80 km/h

**4** **a** 1 hour **b** 6 km **c** 5 hours
**d** $2\frac{1}{2}$ km **e** $2\frac{1}{2}$ km/h

## Review exercise

**1** **a** January and February **b** June and July
**c** February **d** June, July and August
**e** January and November **f** May

**2** **a** October **b** January and July
**c** approximately 650 mm **d** spring
**e** winter


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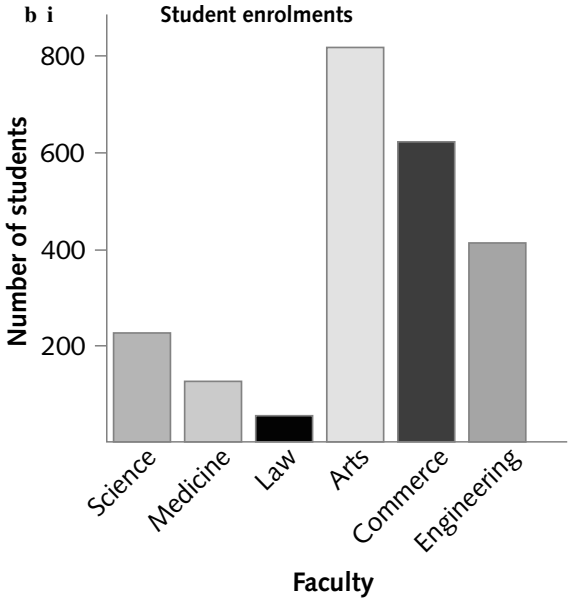
**3** **a**

| Predictions | 1st | 2nd | 3rd | 4th | 5th |
|---|---|---|---|---|---|
| Adrian | Adrian | Jonathon | | | |
| Jonathon | | | Rory | Adrian | |
| Khan | | | Khan | | Jonathon |
| Lars | Lars | Khan | | | Rory |
| Rory | | | | Lars | |

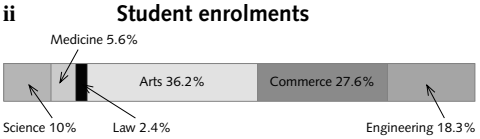
**b** Adrian 1st, Khan 2nd, Rory 3rd, Lars 4th, Jonathon 5th

**4** **a i** 1435 **ii** 10%

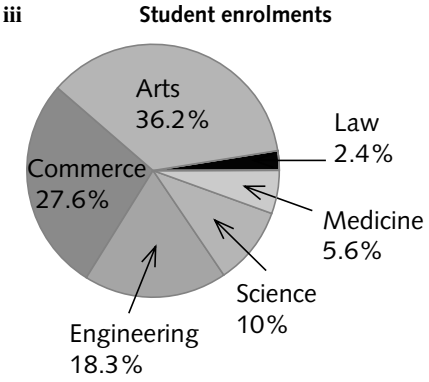
**b i**




**ii**




**iii**




**5** **a i** 60 km/h **ii** 90 km/h

**b** 20 km from Walton for 10 minutes

**c** 100 km/h

**d** 20 km from Walton 22 minutes after Bertie started his journey.

**6** **a** 80 km **b** 60 km/h

**c** 60 minutes **d** 80 km/h

**e** 10 minutes

**f** $26\frac{2}{3}$ km from Smolyvar after $13\frac{1}{3}$ minutes

# Chapter 20

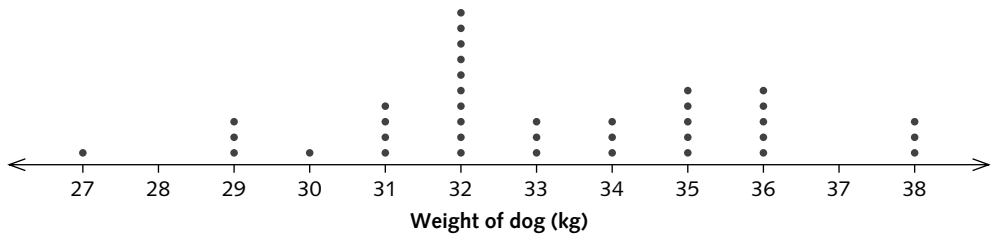
## Exercise 20A

**1** **a**

| Weight (correct to nearest kg) | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Tally | I | | III | I | IIII | 𝍸 | III | III | 𝍸 | 𝍸 | | III |
| Frequency | 1 | 0 | 3 | 1 | 4 | 10 | 3 | 3 | 5 | 5 | 0 | 3 |

**b** 38

**c**






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**2** **a**

| Score | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Tally | 𝍸 | | | | | 𝍸 | 𝍸 | 𝍸 𝍸 | | 𝍸 | 𝍸 | | 𝍸 | | | | 𝍸 |
| Frequency | 3 | 0 | 0 | 0 | 0 | 1 | 2 | 7 | 0 | 2 | 2 | 0 | 2 | 0 | 0 | 0 | 1 |

**b** 88 **c** 79

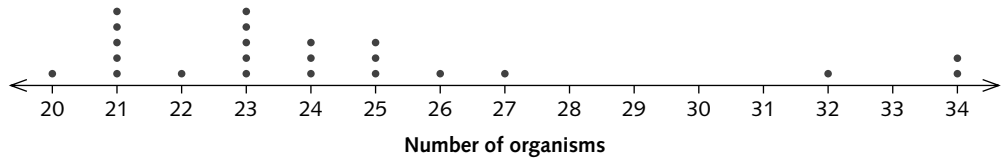
**d**




**3** **a**

| Number of organisms | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Tally | 𝍸 | 𝍸 | 𝍸 | 𝍸 | 𝍸 | 𝍸 | 𝍸 | 𝍸 | | | | | 𝍸 | | 𝍸 |
| Frequency | 1 | 5 | 1 | 5 | 3 | 3 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 2 |

**b**




## Exercise 20B

**1** **a**

| Pets | Cat | Dog | Mouse | Bird | Pig |
|---|---|---|---|---|---|
| Frequency | 8 | 12 | 3 | 1 | 1 |

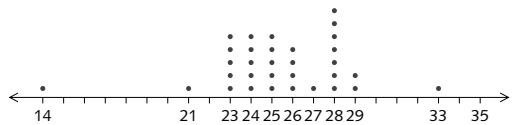
**b** dog

**2** **a**

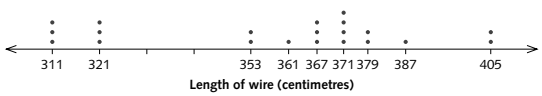
| Number of chairs | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Tally | 𝍸 | | | | | | | 𝍸 | | 𝍸 | 𝍸 | 𝍸 | 𝍸𝍸 | 𝍸 | 𝍸 𝍸 | 𝍸 | | | | 𝍸 |
| Frequency | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 5 | 5 | 5 | 4 | 1 | 7 | 2 | 0 | 0 | 0 | 1 |

**b** 28 chairs

**c**




**3**




## Exercise 20C

**1** **a**

| Stem | Leaves |
|---|---|
| 3 | 9 |
| 4 | 5 9 9 |
| 5 | 6 |
| 6 | 7 8 |
| 7 | 3 8 8 8 |
| 8 | 5 7 7 9 |
| 9 | 8 9 9 |

**b** \$39, \$99 **c** \$78

**2** **a** 24 **b** 38 people

**3** **a**

| Stem | Leaves |
|---|---|
| 7 | 6 8 9 |
| 8 | 1 6 6 8 |
| 9 | 2 4 7 7 8 9 9 |
| 10 | 0 1 1 1 2 2 3 3 3 |
| 11 | 0 0 2 3 4 8 |
| 12 | 0 |

**b** 101 and 103

## Exercise 20D

**1** **a** 6.5 metres **b** 6 metres **c** 7 metres

**2** **a** 44 kg **b** 43 kg
**c** 42.42 kg **d** 8 kg

**3** 20 bags

**4** **a** \$7.75 **b** \$11.45
**c** Median is better, mean is affected by \$42.50
**d** \$38.50

**5** 5.1 years


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**6** **a**

| score | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| frequency | 2 | 2 | 1 | 1 | 1 | 1 | 6 | 2 | 2 | 2 |

**b** mean = 6, median = 7, mode = 7

**c** 9

**7** median = 36

| | |
|---|---|
| 2 | 0 1 3 7 7 |
| 3 | 0 1 1 2 3 4 6 6 8 8 |
| 4 | 1 1 2 8 |
| 5 | 2 3 3 4 |

**8** 1, 2, 5, 6, 6

## Review exercise

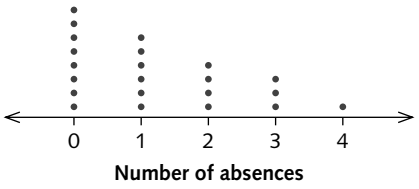
**1** **a, b**

| Stem | Leaves |
|---|---|
| 9 | 6 8 8 9 9 9 9 |
| 10 | 0 0 1 1 2 2 3 4 5 8 8 9 |
| 11 | 2 2 3 3 5 6 9 |
| 12 | 1 2 |
| 13 | 2 8 9 |
| 14 | 2 3 4 5 7 |
| 15 | 2 5 6 |
| 16 | 7 7 |
| 17 | 1 |

median $= \frac{112 + 113}{2} = 112.5$

**c** 75; the range is the difference between the highest and lowest values.

**2**




**3** **a** mean = 48.88, median = 45, range = 68

**b** mean = 92.4, median = 44, range = 551

**4** **a** \$75 000 **b** \$103 125 **c** \$248 000

**5** mean = 50.42 correct to two decimal places, median = 58

**6** **a** 7 **b** 722

**7** **a**

| Stem | Leaves |
|---|---|
| 5 | 06 56 61 98 |
| 6 | 30 |
| 7 | 21 34 63 65 72 78 90 |
| 8 | 24 43 46 52 53 64 65 82 |
| 9 | 10 12 14 35 58 |

**b** 5 **c** 824 **d** 452

**8** **a** $x = 38$ **b** median = 20

**9** **a** maximum = 78 **b** median = 58

**c** $x = 65.5$

# Chapter 21 answers

## Exercise 21A

**1** **a** polyhedron **b** not a polyhedron

**c** not a polyhedron **d** not a polyhedron

**e** polyhedron **f** not a polyhedron

**2** **a** $V = 8,\ F = 6,\ E = 12$

**b** $V = 6,\ F = 5,\ E = 9$

**c** $V = 4,\ F = 4,\ E = 6$

**d** $V = 7,\ F = 7,\ E = 12$

**e** $V = 5,\ F = 5,\ E = 8$

**3** **a** prism **b** not a prism **c** not a prism

**d** not a prism **e** prism **f** prism

**g** not a prism

**4** **a**

| Type of cross-section | Number of faces ($F$) | Number of edges ($E$) | Number of vertices ($V$) |
|---|---|---|---|
| Triangular | 5 | 9 | 6 |
| Rectangular | 6 | 12 | 8 |
| Pentagonal | 7 | 15 | 10 |
| Hexagonal | 8 | 18 | 12 |

**b** $V - E + F = 2$

**c** **i** $n + 2$ **ii** $3n$ **iii** $2n$

**d** $V - E + F = n + 2 - 3n + 2n = 2$

**5** a

| Type of pyramid | Number of faces ($F$) | Number of edges ($E$) | Number of vertices ($V$) |
|---|---|---|---|
| Triangular | 4 | 6 | 4 |
| Rectangular | 5 | 8 | 5 |
| Pentagonal | 6 | 10 | 6 |
| Hexagonal | 7 | 12 | 7 |

**b** $V - E + F = 2$

**c** **i** $n + 1$ **ii** $2n$ **iii** $n + 1$

**d** $V - E + F = n + 1 + n + 1 - 2n = 2$

**6** **a**

| Platonic solid | Number of faces ($F$) | Number of edges ($E$) | Number of vertices ($V$) |
|---|---|---|---|
| Cube | 6 | 12 | 8 |
| Tetrahedron | 4 | 6 | 7 |
| Icosahedron | 20 | 30 | 12 |
| Octahedron | 8 | 12 | 6 |
| Dodecahedron | 12 | 30 | 20 |

**b** $V - E + F = 2$



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## Exercise 21B

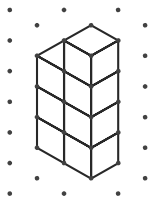
**1** **a**



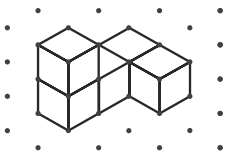
**b**



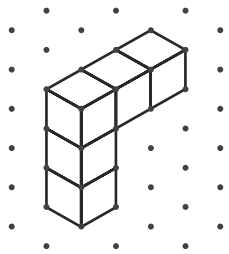
**c**



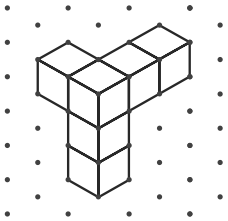
**d**



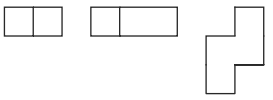
**e**



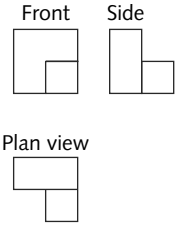
**f**



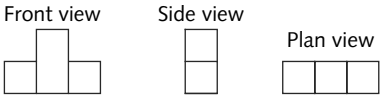
**2** **a**



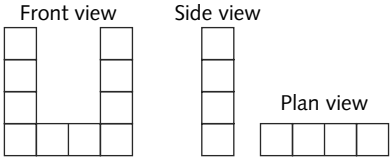

**b**



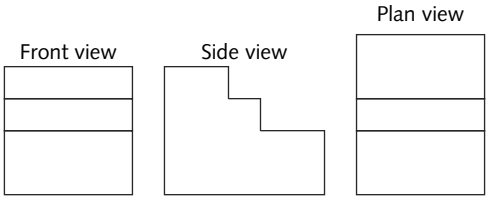

**c**



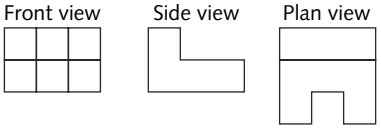

**d**



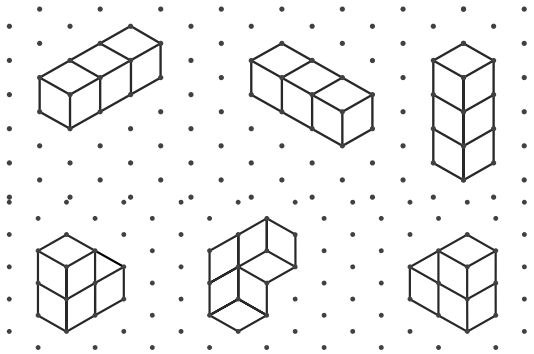

**e**




**f**

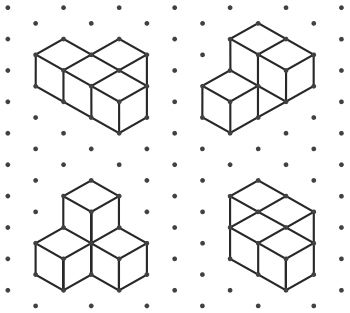



**3** **a**



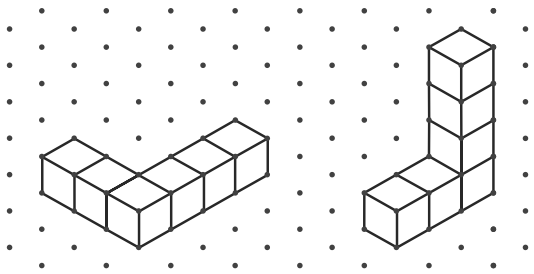
**b** no

**4** **a** There are seven such solids.

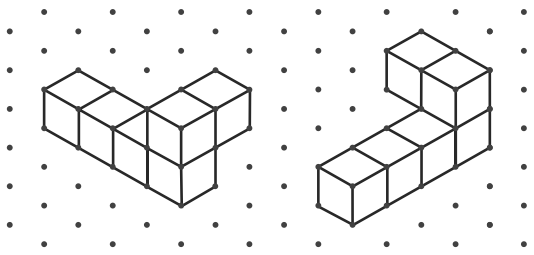


**b** 2

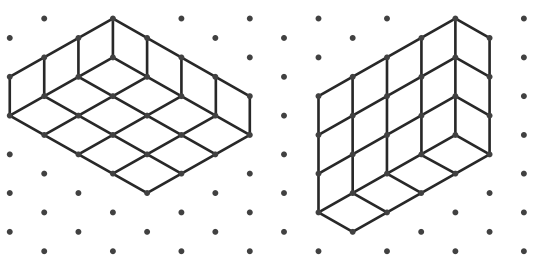
**5** **a**



**b**



**c**



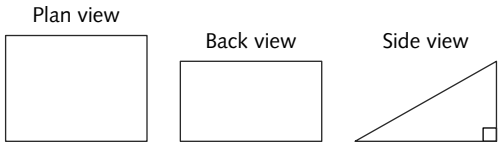

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## Review exercise

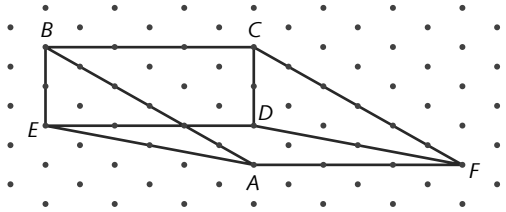
**1** **a** triangular prism

**b** *ABCF*, *CDF*, *BEA*, *AEDF*, *BCDE*

**c**

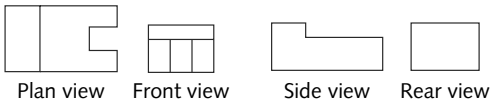



**d**



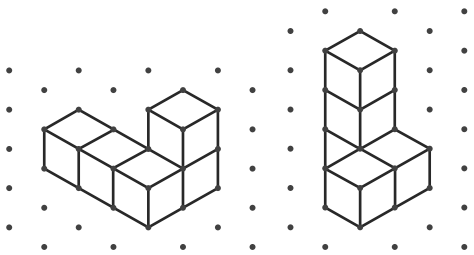

**e** $V = 6,\ E = 9,\ F = 5,\ 6 - 9 + 5 = 2$

**2** **a**

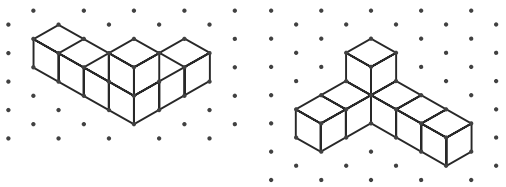



**b** no

**3** **a**



**b**



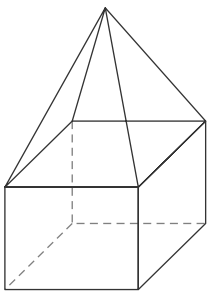
## Challenge exercise

**1** **a** square pyramid **b** triangular prism

**c** a solid with two squares and eight triangles

**2** **a** square pyramid **b** pentagonal pyramid

**3** **a**

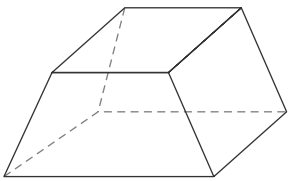


**b** nine faces and nine vertices

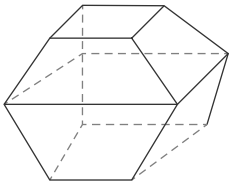
**c** $2n + 1$ faces and $4n$ edges

**d** extended square pyramid, octahedral pyramid

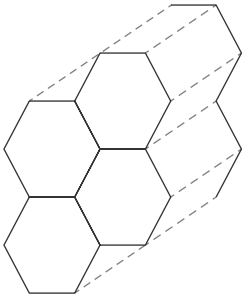
**4** **a**



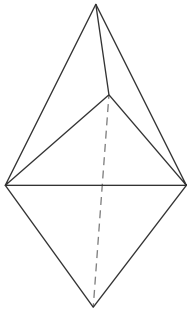
**b**



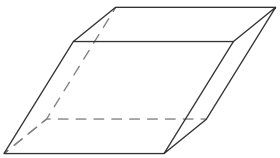
**c**



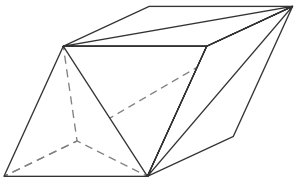
**5** **a** six-sided polyhedron — triangular dipyramid; number of faces = 6, number of vertices = 5, number of edges = 9



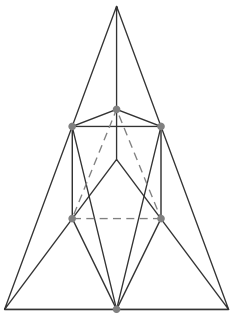
**b**



**c**



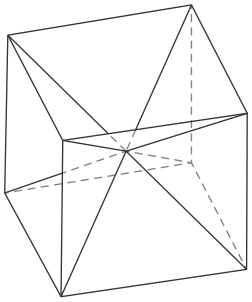
**d** octahedron





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**6**



# Chapter 22 answers

## 22A Review

### Chapter 11: Integers

**1** −5, −4, −3, −2, −1, 0, 1, 2, 3

**2** −51, −48, −45, −42

**3** **a** 5 **b** 34
**c** −7 **d** −13

**4** **a** 8 **b** 2
**c** 7 **d** −11
**e** −42 **f** 6
**g** −5 **h** −17

**5** **a** −3 **b** −12
**c** −14 **d** −7
**e** −34 **f** 20
**g** −35 **h** 11
**i** −4 **j** −14
**k** −9 **l** 50

**6** 15°C

**7** **a** −35 **b** −18
**c** −110 **d** −84
**e** −160 **f** 50
**g** −44 **h** 48

**8** **a** 6 **b** −9
**c** 5 **d** 9

**9** **a** −3 **b** −9 **c** −7

**10** **a** 81 **b** −64 **c** −4 **d** −65 **e** 33

**11** −10

**12** **a** 6 **b** −64 **c** −120
**d** 120 **e** −408 **f** −25
**g** 81 **h** 49 **i** −90

### Chapter 12: Algebra and the Cartesian plane

**1** **a** −8 **b** 4
**c** −2 **d** −7
**e** −5 **f** −64
**g** 64 **h** 16
**i** 7 **j** 11
**k** −3 **l** −14

**2** **a** 1 **b** −65
**c** 55 **d** 300
**e** −360 **f** 12
**g** 1800 **h** −10

**3** **a** −8 **b** −15
**c** −9 **d** 36
**e** −12 **f** $\frac{1}{4}$

**4** **a** −21 **b** 29
**c** −5 **d** 5
**e** −125 **f** 625

**5** **a** $\frac{x}{5}$ m

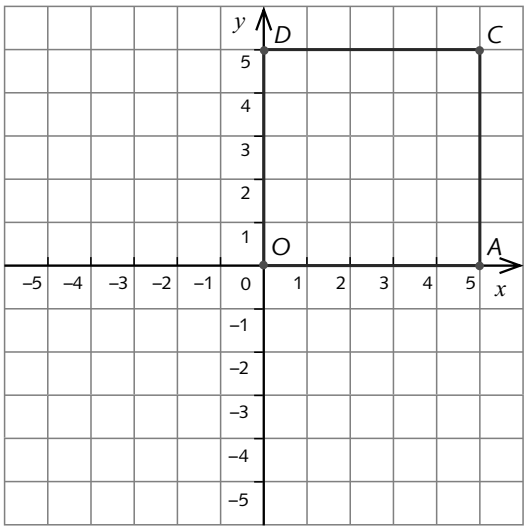
**b** **i** 4 m **ii** $8\frac{2}{5}$ m **iii** $19\frac{1}{5}$ m

**6** **a** **i** $(2000 - x)$ **ii** $(2000 - 5x)$

**b** **i** \$1500 **ii** \$1000 **iii** −\$250

**7** $A(1, 4)$, $B(3, 1)$, $C(4, -3)$, $D(-3, -2)$, $E(-2, -4)$, $F(2, -2)$, $G(-2, 2)$

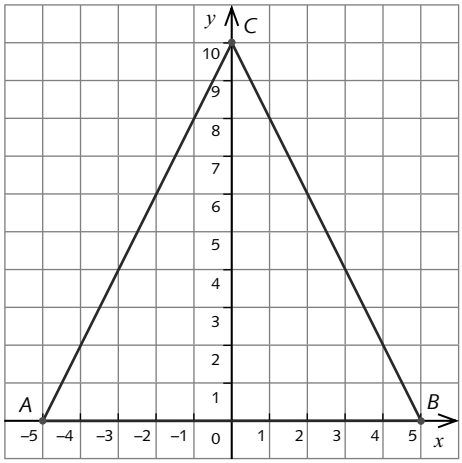
**8** **a**




square

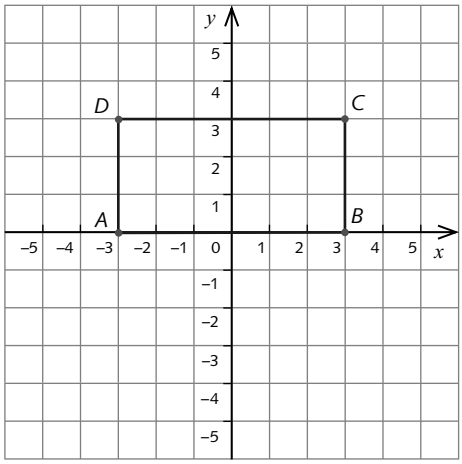

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**b**



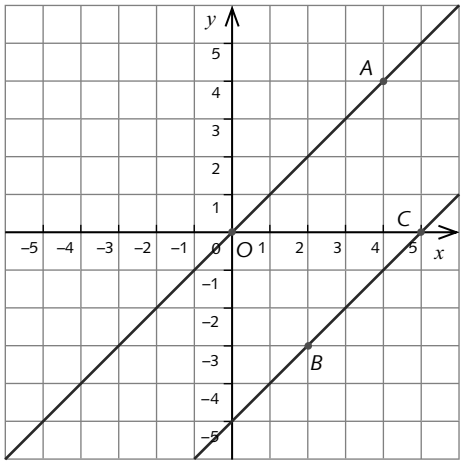

isosceles triangle

**c**




rectangle

**d**

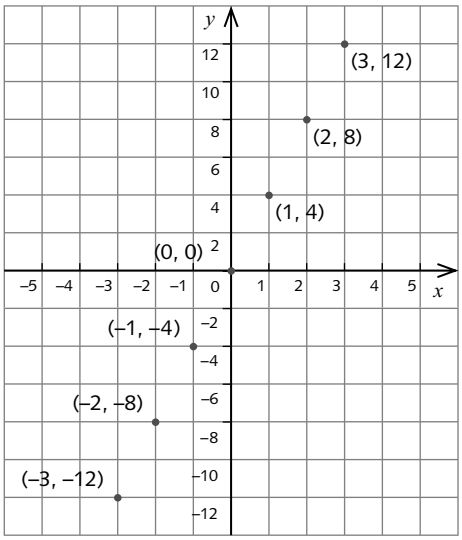



parallel lines

**9** **a** $y = 4x$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | −12 | −8 | −4 | 0 | 4 | 8 | 12 |

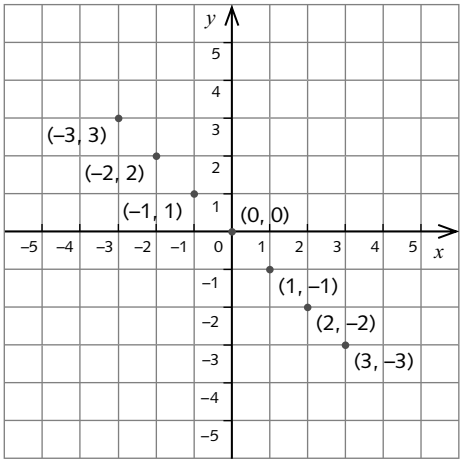
Points are: $(-3, -12)$, $(-2, -8)$, $(-1, -4)$, $(0, 0)$, $(1, 4)$, $(2, 8)$, $(3, 12)$




**b** $y = -x$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | 3 | 2 | 1 | 0 | −1 | −2 | −3 |

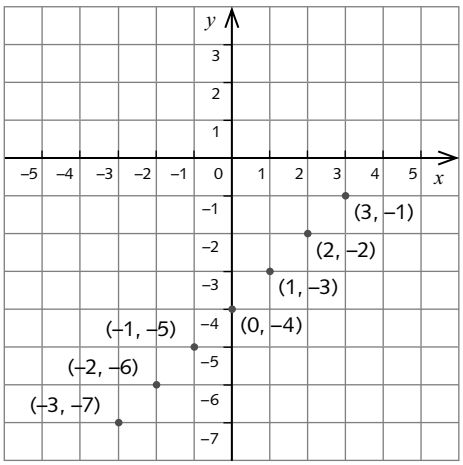
Points are: $(-3, 3)$, $(-2, 2)$, $(-1, 1)$, $(0, 0)$, $(1, -1)$, $(2, -2)$, $(3, -3)$




**c** $y = x - 4$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | −7 | −6 | −5 | −4 | −3 | −2 | −1 |

Points are: $(-3, -7)$, $(-2, -6)$, $(-1, -5)$, $(0, -4)$, $(1, -3)$, $(2, -2)$, $(3, -1)$






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## Chapter 13: Triangles and constructions

**1** **a** $\alpha = 98°, \beta = 82°, \gamma = 98°$
**b** $\alpha = 85°, \beta = 40°$
**c** $\alpha = 145°$
**d** $\alpha = 70°, \beta = 80°, \gamma = 30°$
**e** $\alpha = 52°, \beta = 70°, \gamma = 58°$
**f** $\alpha = 67°, \beta = 64°, \gamma = 43°, \theta = 43°$
**g** $\alpha = 49°, \beta = 43°, \gamma = 88°, \theta = 43°$
**h** $\alpha = 149°$
**i** $\alpha = 18°$
**j** $\beta = 50°, \alpha = 70°$
**k** $\alpha = 75°$
**l** $\alpha = 40°$
**m** $\alpha = 32°, \beta = 111°$
**n** $\alpha = 73°, \beta = 65°$
**o** $\alpha = 113°$
**p** $\alpha = 70°, \beta = 105°$
**q** $\alpha = 60°, \beta = 120°$

## Chapter 14: Negative fractions and decimals

**1** $-3\frac{1}{4}, -2\frac{1}{2}, -1\frac{3}{4}, 0, 2, 2\frac{1}{2}, 3\frac{1}{2}$

**2** **a** $3\frac{1}{4}$ **b** $2\frac{1}{6}$ **c** $\frac{1}{5}$ **d** $-\frac{1}{4}$
**e** $-2\frac{1}{2}$ **f** $-1\frac{2}{5}$ **g** $-2\frac{3}{5}$ **h** $-25$

**3** **a** $1\frac{7}{20}$ **b** $-\frac{3}{20}$ **c** $-\frac{3}{8}$ **d** $-\frac{3}{110}$
**e** $1\frac{13}{24}$ **f** $\frac{1}{10}$ **g** $\frac{3}{20}$ **h** $-\frac{11}{56}$

**4** **a** $-\frac{1}{2}$ **b** $-\frac{2}{3}$ **c** $2$ **d** $\frac{15}{32}$
**e** $\frac{5}{12}$ **f** $-\frac{7}{8}$ **g** $1\frac{1}{7}$ **h** $-2\frac{4}{9}$

**5** **a** $-2\frac{1}{4}$ **b** $\frac{3}{4}$ **c** $-3\frac{3}{4}$ **d** $-3$
**e** $\frac{27}{64}$ **f** $\frac{27}{64}$ **g** $3\frac{3}{4}$ **h** $7$

**6** **a** $-1\frac{5}{8}$ **b** $-7\frac{1}{4}$ **c** $-1\frac{13}{32}$ **d** $-3\frac{1}{8}$
**e** $7\frac{1}{32}$ **f** $-9\frac{1}{2}$ **g** $-9$ **h** $-5$

## Chapter 15: Percentages and ratios

**1** **a** 90% **b** 85% **c** 120% **d** 48%

**2** **a** 78% **b** 9.5% **c** 97% **d** 135%

**3** **a** 30% **b** 125% **c** $33\frac{1}{3}\%$

**4** **a** $4\frac{1}{2}\%, \frac{12}{32}, 0.39$ **b** $0.6, 64\%, \frac{2}{3}$

**5** $\frac{3}{8}$, 37.5%; 0.3, $33\frac{1}{3}\%$; $\frac{3}{4}$, 0.75; 2.75, 275%; 3, 3

**6** **a** $\frac{5}{8}$ **b** $\frac{7}{8}$ **c** $\frac{1}{80}$ **d** $\frac{21}{400}$

## Chapter 16: Solving equations

**1** **a** $x = 2$ **b** $x = 11$ **c** $x = 12$
**d** $x = 5$ **e** $x = 2\frac{1}{3}$ **f** $z = 22$
**g** $x = 4$ **h** $x = 12$ **i** $z = 8$

**2** **a** $x = 2$ **b** $x = 5$ **c** $x = 6$
**d** $x = 4$ **e** $x = 4$ **f** $z = 3\frac{2}{3}$
**g** $x = 12$ **h** $m = 12$ **i** $z = 50$

**3** **a** $x = -1$ **b** $x = -3$ **c** $x = -6$
**d** $x = 35$ **e** $x = 1\frac{1}{2}$ **f** $x = -\frac{4}{9}$
**g** $x = 3$ **h** $x = 0$ **i** $x = 1\frac{2}{3}$

**4** **a** $x + 3 = 6, x = 3$ **b** $x - 5 = 10, x = 15$
**c** $7x = 84, x = 12$ **d** $\frac{x}{4} = 15, x = 60$
**e** $4x - 3 = 17, x = 5$ **f** $3(x + 6) = 28, x = 3\frac{1}{3}$

**5** 50 cm, 10 cm **6** 19, 1

**7** **a** $x = -17$ **b** $x = -7$ **c** $x = 1$
**d** $x = -2\frac{2}{5}$ **e** $x = 4$ **f** $x = -3$
**g** $x = -80$ **h** $x = 4$ **i** $x = -3$
**j** $x = -3$ **k** $x = 11$ **l** $x = -\frac{1}{2}$

**8** **a** $x = \frac{1}{3}$ **b** $x = \frac{3}{4}$ **c** $x = -\frac{1}{4}$
**d** $x = -3$ **e** $x = 2$ **f** $x = \frac{3}{8}$
**g** $x = 5$ **h** $x = \frac{2}{3}$ **i** $x = 1\frac{1}{11}$

**9** $-6$

## Chapter 17: Probability

**1** **a** $\frac{3}{5}$ **b** $\frac{2}{5}$

**2** **a** $\frac{1}{3}$ **b** $\frac{5}{12}$ **c** $\frac{1}{2}$
**d** $\frac{5}{24}$ **e** $\frac{3}{4}$ **f** $\frac{7}{12}$
**g** $\frac{1}{6}$ **h** $\frac{3}{8}$ **i** $\frac{2}{3}$

**3** **a** $\frac{1}{4}$ **b** $\frac{3}{4}$ **c** $\frac{1}{2}$

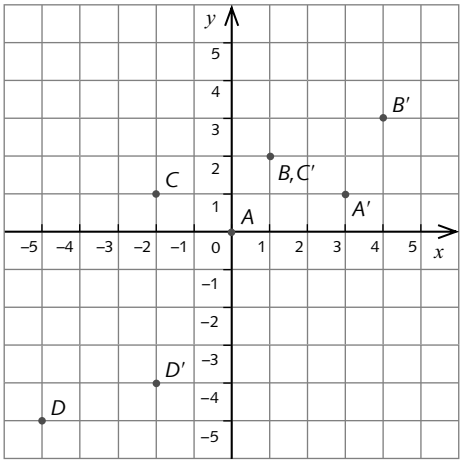
**4** **a** $\frac{2}{19}$ **b** $\frac{3}{19}$ **c** $\frac{1}{19}$ **d** $\frac{7}{19}$
**e** $\frac{12}{19}$ **f** $\frac{17}{19}$ **g** $\frac{5}{19}$

**5** **a** $\frac{1}{2}$ **b** $\frac{5}{18}$ **c** $\frac{1}{6}$


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## Chapter 18: Transformations and symmetry

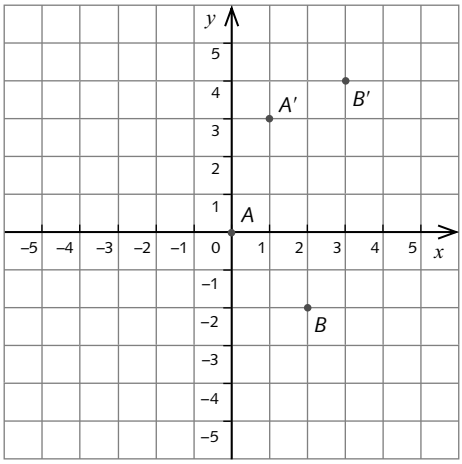
**1**




**a** $A'(3, 1)$ **b** $B'(4, 3)$

**c** $C'(1, 2)$ **d** $D'(-2, -4)$

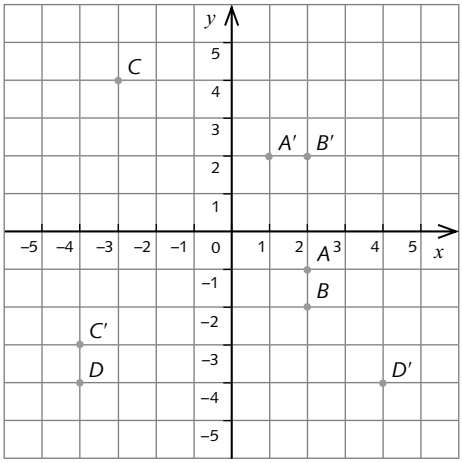
**2**




**a** 1 unit right, 3 units up

**b** 1 unit right, 6 units up

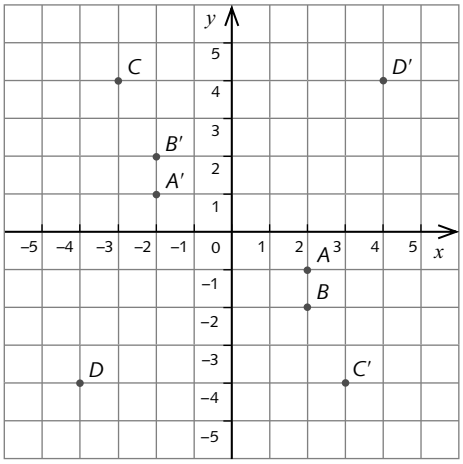
**3**




**a** $A'(1, 2)$ **b** $B'(2, 2)$

**c** $C'(-4, -3)$ **d** $D'(4, -4)$

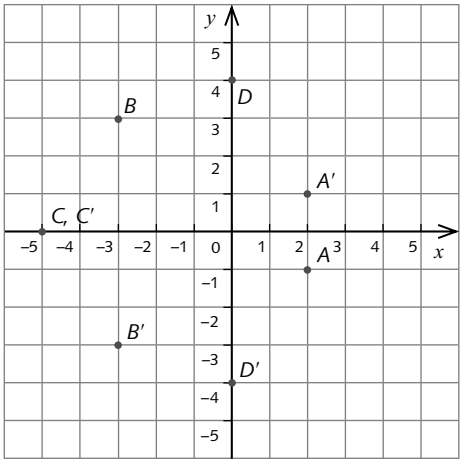
**4**




**a** $A'(-2, 1)$ **b** $B'(-2, 2)$

**c** $C'(3, -4)$ **d** $D'(4, 4)$

**5**




**a** $A'(2, 1)$

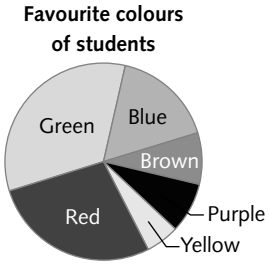
**b** $B'(-3, -3)$

**c** $C'(-5, 0)$

**d** $D'(0, -4)$

## Chapter 19: Graphs and tables

**1** $\frac{1}{3}$, 120°; $\frac{5}{18}$, 100°; $\frac{1}{18}$, 20°; $\frac{1}{12}$, 30°; $\frac{1}{12}$, 30°




**2** **a** 144° **b** 72°

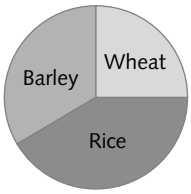
**3** **a** July **b** January and December

**c** 17°C **d** 9°C



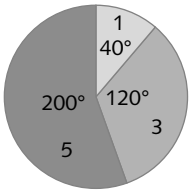
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**4** **a** **Crop production**




**b** wheat: 12 000 t; barley: 16 000 t; rice: 20 000 t

**5** **Division of a sum of money**




**6**




## Chapter 20: Statistics

**1** **a**

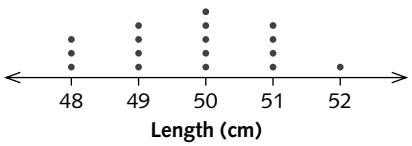
| Stem | Leaves |
|---|---|
| 2 | 2 3 4 4 6 7 |
| 3 | 1 3 5 6 8 8 9 |
| 4 | 0 1 1 1 4 6 |

**b** 36 **c** 24

**2** **a** 13.7 **b** 14 **c** 15

**3** **a** 86 **b** 29

**4**




**5** $x = 17,\ y = 22$; $x = 18,\ y = 21$; $x = 19,\ y = 20$

**6** **a** median = 18, range = 12

**b** median = 12, range = 26

**7** **a** mean = 85.64 correct to two decimal places

**b** median = 84

**c** range = 27

**d**

| Stem | Leaves |
|---|---|
| 7 | 4 6 6 6 |
| 8 | 0 1 1 2 2 2 4 4 5 6 8 |
| 9 | 1 2 2 6 6 9 |
| 10 | 1 |

## Chapter 21: Polyhedra and three-dimensional drawing

**1** **a** **i** $F = 6,\ V = 8,\ E = 12$

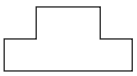
**ii** $F = 5,\ V = 6,\ E = 9$

**iii** $F = 10,\ V = 8,\ E = 16$

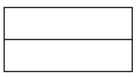
**iv** $F = 14,\ V = 24,\ E = 36$

**b** **i** 2 **ii** 2 **iii** 2 **iv** 2

**2** **a** Front view



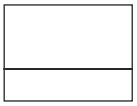
Side view



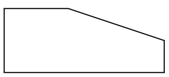
Plan view



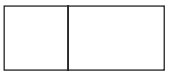
**b** Front view



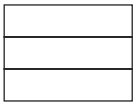
Side view



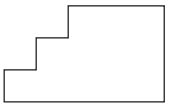
Plan view



**c** Front view



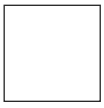
Side view



Plan view

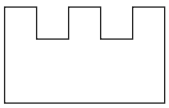


**d** Front view

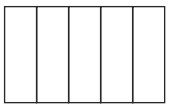



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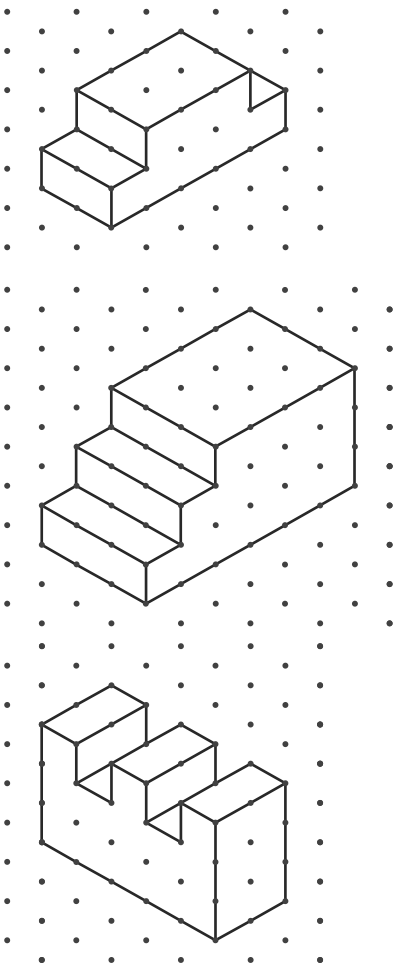
Side view



Plan view



**3** **a**



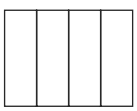
**4** Front view



Side view



Plan view



## 22B Tessellations

### *Activity 1*

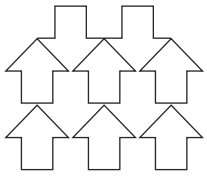
| Polygon | Number of sides | Interior angle sum | Size of each angle |
|---|---|---|---|
| **Triangle** | 3 | 180° | 60° |
| **Quadrilateral** | 4 | 360° | 90° |
| **Pentagon** | 5 | 540° | 108° |
| **Hexagon** | 6 | 720° | 120° |
| **Heptagon** | 7 | 900° | $128\frac{4}{7}°$ |
| **Octagon** | 8 | 1080° | 135° |
| **Nonagon** | 9 | 1260° | 140° |
| **Decagon** | 10 | 1440° | 144° |
| **Dodecagon** | 12 | 1800° | 150° |

The square, equilateral triangle and regular hexagon are the only polygons with angle sizes that divide 360°, thus enabling several such shapes to meet at a vertex without leaving any spaces.

### *Activity 3*

The interior angles of a pentagon add up to 540°.
$\angle A + \angle B + \angle D =$ 360° and 4 Cairo tiles fit together to form a hexagon which tessellates the plane.

### *Activity 4*



### *Activity 5*

The other seven codes
[6, 3, 6, 3], [8, 8, 4], [4, 3, 4, 3, 3], [12, 12, 3],
[4, 4, 3, 3, 3], [12, 6, 4], [6, 3, 3, 3, 3]


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